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'ROM GCSE TO GRADUATION

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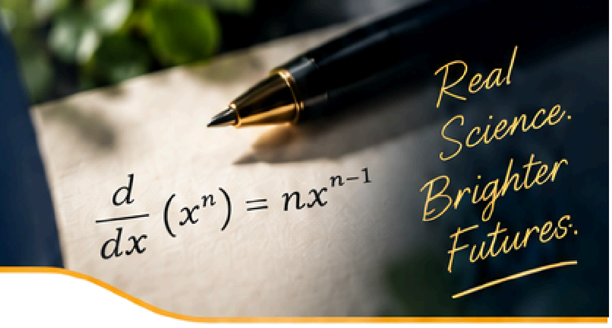

Mathematics 6.1
Differentiation
REVISION NOTES

What is differentiation?

Differentiation finds the gradient (rate of change) of a function.

If $y = f(x)$ then $f'(x)$ means "the derivative of $f(x)$", which gives the gradient of the curve at the point x.

Small steps. Big progress.

1 The derivative as a gradient

If $y = f(x)$ then $f'(x)$ is the gradient of the tangent to the curve $y = f(x)$ at the point x.

Gradient of tangent $= f'(x)$

Example For $f(x) = x^2$,
$f'(x) = 2x$.

So the gradient at $x = 3$ is $f'(3) = 6$.

2 Differentiation from first principles

The derivative can be defined by

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Example Find $f'(x)$ from first principles for $f(x) = x^2$.

$$f(x+h) = (x+h)^2 = x^2 + 2xh + h^2$$

$$f'(x) = \lim_{h \to 0} \frac{(x^2 + 2xh + h^2) - x^2}{h}$$

$$= \lim_{h \to 0} (2x + h) = 2x$$

3 Basic differentiation rules

For n a constant:

$\frac{d}{dx}(x^n) = nx^{n-1}$ (power rule)

$\frac{d}{dx}(k) = 0$ (constant)

$\frac{d}{dx}(kx) = k$ (constant multiple)

Example Differentiate $f(x) = 5x^3 - 4x + 7$.

$f'(x) = 15x^2 - 4$

4 More examples

Use the power rule for any real power n.

Example Differentiate:

(a) $f(x) = x^5 \Rightarrow f'(x) = 5x^4$

(b) $f(x) = x^{-2} \Rightarrow f'(x) = -2x^{-3}$

(c) $f(x) = 3x^{\frac{1}{2}} \Rightarrow f'(x) = \frac{3}{2}x^{-1\frac{1}{2}}$

(d) $f(x) = \frac{1}{x} = x^{-1} \Rightarrow f'(x) = -x^{-2}$

5 Functions of the form $ax + b$

If $f(x) = ax + b$ where a, b are constants, then $f'(x) = a$.

Example For $f(x) = 7x - 5$,
$f'(x) = 7$.

The gradient is constant for a straight line.

6 Differentiating sums and multiples

Differentiate term by term and keep any constant multiples.

Example Differentiate $f(x) = 4x^3 - 2x^2 + 5x - 1$.

$f'(x) = 12x^2 - 4x + 5$

There is no need to simplify further unless asked to do so.

7 Gradient at a specific point

1. Differentiate to find $f'(x)$.
2. Substitute the given value of x.

Example $f(x) = 2x^3 - 3x$.
Find the gradient when $x = 1$.

$f'(x) = 6x^2 - 3$

$f'(1) = 6(1)^2 - 3 = 3$

8 Equation of a tangent

The equation of the tangent at $x = a$ is

$$y - f(a) = f'(a)(x - a)$$

1. Find $f'(x)$.
2. Find $f(a)$ and $f'(a)$.
3. Substitute into the formula.

Example $f(x) = x^2$. Find the equation of the tangent at $x = 2$.

$f'(x) = 2x, \quad f(2) = 4, \quad f'(2) = 4$

$y - 4 = 4(x - 2) \Rightarrow y = 4x - 4$

Key things to remember

- $f'(x)$ means "the derivative of $f(x)$".
- Use the power rule: $\frac{d}{dx}(x^n) = nx^{n-1}$.
- The derivative of a constant is 0.
- Differentiate each term separately.
- You can differentiate negative and fractional powers.
- The gradient at a point is found by substituting the value of x into $f'(x)$.
- Use $y - f(a) = f'(a)(x - a)$ for the equation of a tangent.
- Give answers in their simplest form.

You should now be able to:

- [ ] find derivatives using the power rule
- [ ] differentiate expressions with constants and multiples
- [ ] use first principles for simple cases
- [ ] find the gradient of a curve at a given point
- [ ] find the equation of a tangent to a curve

"Practice turns confusion into confidence."

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