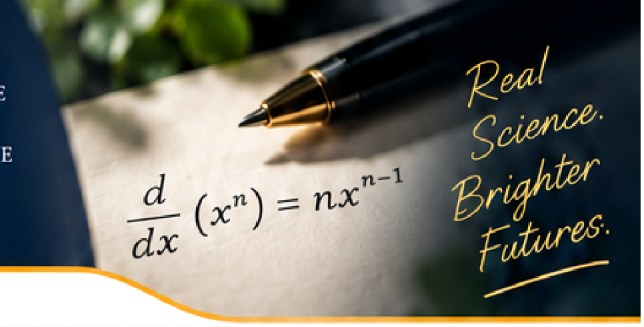

Mathematics 6.3 – The Chain Rule

WORKSHEET

Practice Progress Success

1 Derivatives of sin x and cos x

Differentiate the following.

a) $y = \sin x$

b) $y = \cos x$

c) $y = \sin(2x)$

d) $y = \cos(3x)$

e) $y = \sin(x + 1)$

f) $y = \cos(4x - 2)$

2 The chain rule (polynomials)

Differentiate the following, using the chain rule.

a) $y = (2x + 1)^3$

b) $y = (5x - 3)^3$

c) $y = (3x + 4)^5$

d) $y = (x^2 + 1)^4$

e) $y = (4 - 3x)^6$

f) $y = (2x - 5)^7$

3 Chain rule with trigonometric functions

Differentiate the following.

a) $y = \sin(3x)$

b) $y = \cos(5x)$

c) $y = \sin(2x + 1)$

d) $y = \cos(4x - 3)$

e) $y = \sin(x^2)$

f) $y = \cos(3x^2)$

4 Mixed functions

Differentiate the following.

a) $y = (x^2 + 1)^3$

b) $y = (3x - 2)^4$

c) $y = \sin(x^2 + 1)$

d) $y = \cos(3x - 1)$

e) $y = \sin(2x + 3)^5$

f) $y = (1 + \cos x)^3$

5 More trigonometric examples

Differentiate the following.

a) $y = \sin(5x^2)$

b) $y = \cos(2x^2 + 1)$

c) $y = \sin(1 - 2x^2)$

d) $y = \cos(x^2 - 3x)$

e) $y = \sin(3x - 1)^4$

f) $y = \cos(4 - 3x^2)$

6 Composite functions

Differentiate the following.

a) $y = (x^2 + 3x + 1)^4$

b) $y = (3x^2 - 2x)^5$

c) $y = \sin(3x^2 - 2x + 1)$

d) $y = \sin(2x^2 + 3x + 1)$

e) $y = \cos(5x^2 - x + 2)$

f) $y = (4 - x^2)^6$

7 Problem solving

a) Given $y = \sin(ax + b)$, where a and b are constants, find $\frac{dy}{dx}$ in terms of a and b.

b) Given $y = (kx + 1)^n$, where k and n are constants, find $\frac{dy}{dx}$ in terms of k, n and x.

c) The function $y = \sin(x^2 + 2x)$.
Find the gradient of the curve at the point $x = 1$.

d) The function $y = \cos(2x^2 - 3x)$.
Find the gradient of the curve at the point $x = 0$.

8 Extension

a) Find $\frac{dy}{dx}$ for $y = \sin(x^2 + 1)^3$.

b) Find $\frac{dy}{dx}$ for $y = \cos(3x^2 + x)^4$.

c) Find $\frac{dy}{dx}$ for $y = \sin(2x^2 - 4x + 3)$.

d) Find $\frac{dy}{dx}$ for $y = (1 + \cos(2x))^5$.

e) Find the value of $\frac{dy}{dx}$ at $x = 2$ for $y = \sin(x^2 - 3x)$.

f) Given $y = \cos(x^2 + 1)$, find the gradient of the curve at the point where $x = -1$.

Key skills practised:

- ☐ differentiate sine and cosine functions
- ☐ use the chain rule with the substitution u
- ☐ differentiate trigonometric functions using the chain rule
- ☐ differentiate composite polynomial functions using the chain rule
- ☐ apply the chain rule in more complex contexts

"Practice builds understanding."