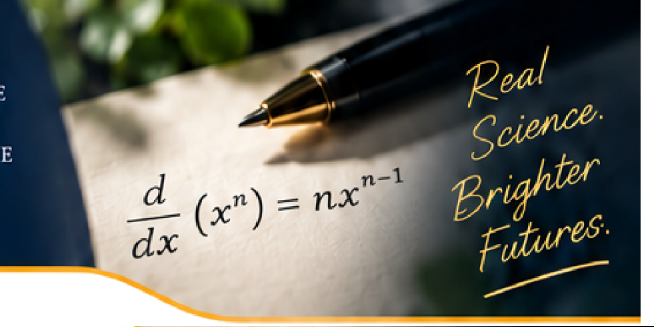

Mathematics 6.4 - The Product Rule

WORKSHEET

Practice
Apply
Improve
Succeed

1 Using the product rule (polynomials)

Differentiate the following.

a) $y = x(x+2)$

b) $y = (2x+1)(x-3)$

c) $y = (3x-2)(4x+5)$

d) $y = x^2(x+1)$

e) $y = (x-4)(x^2+3x)$

f) $y = (2x+3)(5x^2-1)$

2 Using the product rule (trigonometric functions)

Differentiate the following.

a) $y = x\sin x$

b) $y = x\cos x$

c) $y = (2x+1)\sin x$

d) $y = (3x-2)\cos x$

e) $y = x\sin(2x)$

f) $y = (x+1)\cos(3x)$

3 Trigonometric products

Differentiate the following.

a) $y = \sin x\cos x$

b) $y = x\sin(2x)$

c) $y = (\sin x)(x^2)$

d) $y = (3x-1)\cos x$

e) $y = \sin x(x+1)$

f) $y = (2x+3)\sin(3x)$

4 Products involving composite functions

Differentiate the following.

a) $y = x(x+1)^2$

b) $y = (2x-1)^3 x$

c) $y = x\sin(x^2)$

d) $y = (x^2+1)\cos(x)$

e) $y = (3x+2)\sin(x^2)$

f) $y = (x-1)^2\cos(2x)$

5 Functions requiring more than one rule

Differentiate the following, simplifying where possible.

a) $y = x^2\sin x$

b) $y = (x^2+1)\cos x$

c) $y = x\cos(x^2)$

d) $y = (2x+1)\sin(x^2)$

e) $y = (x^3+1)\cos x$

f) $y = (x^2-3x)\sin(2x)$

6 Problem solving

a) Given $y = x\sin x$, find $\frac{dy}{dx}$ and hence find the gradient of the curve at the point where $x = \frac{\pi}{2}$.

b) The function $y = (2x+1)\cos x$. Find $\frac{dy}{dx}$ and determine the gradient of the curve at the point $x = 0$.

c) Given $y = x^2\cos x$, find the equation of the tangent to the curve at the point $x = \pi$.

7 Higher level questions

Differentiate the following, giving your answer in its simplest form.

a) $y = (x^2+1)(x-3)^2$

b) $y = x^2\sin(x^2)$

c) $y = (3x-2)\cos(2x)$

d) $y = (x+1)^2\sin x$

e) $y = (2x^2-x)\cos(x^2)$

f) $y = (x^3+3x+1)\sin x$

8 Challenge

a) The function $y = (x^2+1)\cos x$ has a stationary point at $x = 0$. Determine whether this point is a maximum or a minimum.

b) A curve is defined by $y = x(3x-2)\sin x$. Find $\frac{dy}{dx}$ and determine the gradient of the curve at $x = \pi$.

c) A rectangular box has length x cm and width $(2x+1)$ cm. The area is $A = x(2x+1)$.

(i) Find $\frac{dA}{dx}$.

(ii) Hence find $\frac{dA}{dx}$ when $x = 3$.

Key skills practised:

- ☐ use the product rule to differentiate functions
- ☐ differentiate polynomials and trigonometric functions
- ☐ apply the chain rule where needed (from 6.3)
- ☐ simplify your answers and check for sensible form and sign
- ☐ solve problems involving gradients, tangents and stationary points

"Small steps today, greater success tomorrow."