



$$\frac{d}{dx}(x^n) = nx^{n-1}$$

*Real
Science.
Brighter
Futures.*

Mathematics 6.5 – The Quotient Rule WORKSHEET

*Practise
Apply
Improve
Succeed*

1 Simple quotients (polynomials)

Differentiate the following.

- | | |
|----------------------------|----------------------------|
| a) $y = \frac{x}{x+1}$ | d) $y = \frac{3x-1}{2x+5}$ |
| b) $y = \frac{2x+3}{x}$ | e) $y = \frac{x^2-4}{x+1}$ |
| c) $y = \frac{x^2+1}{x-2}$ | f) $y = \frac{5-x}{x^2+3}$ |

2 Quotients with trigonometric functions

Differentiate the following.

- | | |
|-------------------------------|------------------------------|
| a) $y = \frac{\sin x}{x}$ | d) $y = \frac{x}{\cos x}$ |
| b) $y = \frac{\cos x}{x+1}$ | e) $y = \frac{2x+1}{\sin x}$ |
| c) $y = \frac{\sin x + 1}{x}$ | f) $y = \frac{\cos x}{x-1}$ |

3 More polynomial quotients

Differentiate the following.

- | | |
|-----------------------------|-------------------------------|
| a) $y = \frac{x^2+1}{3x-2}$ | d) $y = \frac{4-x^2}{2x-1}$ |
| b) $y = \frac{2x+1}{x^2+4}$ | e) $y = \frac{x^2+3x}{x^2+1}$ |
| c) $y = \frac{x^3-1}{x+2}$ | f) $y = \frac{3x+5}{x^2-4}$ |

4 Quotients with composite functions (use the chain rule)

Differentiate the following.

- | | |
|---------------------------------|----------------------------------|
| a) $y = \frac{x^2+1}{\sin x}$ | d) $y = \frac{2x+1}{\sin(3x)}$ |
| b) $y = \frac{\sin(2x)}{x^2+1}$ | e) $y = \frac{1+\cos(x^2)}{x-2}$ |
| c) $y = \frac{\cos(x^2)}{x}$ | f) $y = \frac{\sin(x^2)}{x^2+1}$ |

5 Mixed functions

Differentiate the following. (Use the product rule where needed.)

- | | |
|--------------------------------|---------------------------------|
| a) $y = \frac{x^2+1}{x-1}$ | d) $y = \frac{x^2+3x+1}{x^2+1}$ |
| b) $y = \frac{\sin(2x)}{x+1}$ | e) $y = \frac{\cos(2x)}{x-3}$ |
| c) $y = \frac{2x-1}{\sin(3x)}$ | f) $y = \frac{3x^2-2x}{\sin x}$ |

6 Problem solving and applications

- a) Given $y = \frac{x^2+1}{x-1}$, find $\frac{dy}{dx}$ and simplify your answer.
- b) The function $y = \frac{\sin x}{x+2}$. Find the gradient of the curve at the point $x=0$.
- c) The function $y = \frac{2x+1}{\cos x}$. Find the gradient of the curve at the point $x = \frac{\pi}{4}$.
- d) Given $y = \frac{x^2-4}{x+1}$, find the values of x for which y is not defined, and state any restrictions on x .

7 Challenge

Differentiate the following.

- | | |
|-------------------------------------|---------------------------------------|
| a) $y = \frac{x^2 + \sin x}{x-2}$ | d) $y = \frac{x^2+1}{\sin(2x)}$ |
| b) $y = \frac{\sin(3x) + x}{x^2+1}$ | e) $y = \frac{2x + \cos x}{x^2-3x+1}$ |
| c) $y = \frac{x \cos x}{x^2-1}$ | f) $y = \frac{1 + \sin(x^2)}{x+1}$ |

8 Key skills checked

- ☐ use the quotient rule with u (numerator) and v (denominator)
- ☐ differentiate polynomials
- ☐ differentiate trigonometric functions
- ☐ use the chain rule where needed (from 6.3)
- ☐ use the product rule where needed (from 6.4)
- ☐ simplify your answers
- ☐ identify any restrictions on the domain (values where $v=0$)
- ☐ apply differentiation to solve problems
- ☐ check your answers for sensible form and sign

*"Practice today,
greater possibilities tomorrow."*