

2017 AP® CALCULUS BC FREE-RESPONSE QUESTIONS

$$f(0) = 0$$

$$f'(0) = 1$$

$$f^{(n+1)}(0) = -n \cdot f^{(n)}(0) \text{ for all } n \geq 1$$

6. A function f has derivatives of all orders for $-1 < x < 1$. The derivatives of f satisfy the conditions above. The Maclaurin series for f converges to $f(x)$ for $|x| < 1$.

- (a) Show that the first four nonzero terms of the Maclaurin series for f are $x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4}$, and write the general term of the Maclaurin series for f .

- (b) Determine whether the Maclaurin series described in part (a) converges absolutely, converges conditionally, or diverges at $x = 1$. Explain your reasoning.

- (c) Write the first four nonzero terms and the general term of the Maclaurin series for $g(x) = \int_0^x f(t) dt$.

- (d) Let $P_n\left(\frac{1}{2}\right)$ represent the n th-degree Taylor polynomial for g about $x = 0$ evaluated at $x = \frac{1}{2}$, where g is

the function defined in part (c). Use the alternating series error bound to show that

$$\left| P_4\left(\frac{1}{2}\right) - g\left(\frac{1}{2}\right) \right| < \frac{1}{500}.$$

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6. The Maclaurin series for $\ln(1+x)$ is given by

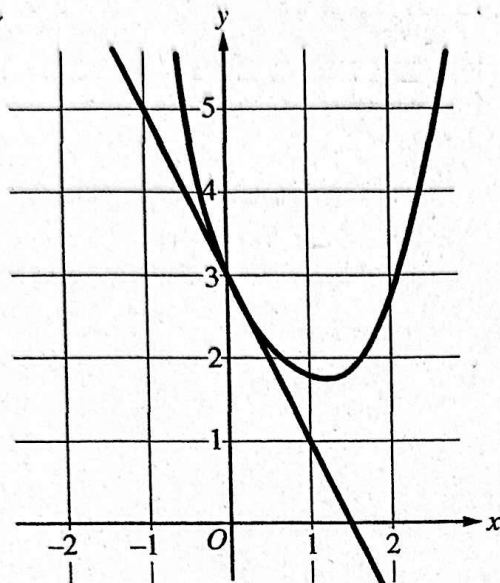
$$x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots + (-1)^{n+1} \frac{x^n}{n} + \cdots$$

On its interval of convergence, this series converges to $\ln(1+x)$. Let f be the function defined by

$$f(x) = x \ln\left(1 + \frac{x}{3}\right).$$

- (a) Write the first four nonzero terms and the general term of the Maclaurin series for f .
- (b) Determine the interval of convergence of the Maclaurin series for f . Show the work that leads to your answer.
- (c) Let $P_4(x)$ be the fourth-degree Taylor polynomial for f about $x = 0$. Use the alternating series error bound to find an upper bound for $|P_4(2) - f(2)|$.

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n	$f^{(n)}(0)$
2	3
3	$-\frac{23}{2}$
4	54

6. A function f has derivatives of all orders for all real numbers x . A portion of the graph of f is shown above, along with the line tangent to the graph of f at $x = 0$. Selected derivatives of f at $x = 0$ are given in the table above.

(a) Write the third-degree Taylor polynomial for f about $x = 0$.

(b) Write the first three nonzero terms of the Maclaurin series for e^x . Write the second-degree Taylor polynomial for $e^x f(x)$ about $x = 0$.

(c) Let h be the function defined by $h(x) = \int_0^x f(t) dt$. Use the Taylor polynomial found in part (a) to find an approximation for $h(1)$.

(d) It is known that the Maclaurin series for h converges to $h(x)$ for all real numbers x . It is also known that the individual terms of the series for $h(1)$ alternate in sign and decrease in absolute value to 0. Use the alternating series error bound to show that the approximation found in part (c) differs from $h(1)$ by at most 0.45.