2026

# AP® Calculus BC

## Free-Response Questions

# CALCULUS BC
## SECTION II PART A
## TIME – 30 MINUTES

**Directions:**

Section II, Part A has 2 free-response questions and lasts 30 minutes.

**A graphing calculator is required for the questions on this part of the exam.** You may use a handheld graphing calculator or the calculator available in this application. **Make sure your calculator is in radian mode.**

You may use the available paper for scratch work and planning, but only work written in the free-response booklet will be scored. Any work done on scratch paper will not be scored. In the free-response booklet, write your solution to each part of each question in the space provided for that part. For questions that have sub-parts, be sure to label those clearly in your solution. Use a pencil or a pen with black or dark blue ink.

You are permitted to use your calculator to solve an equation, find the derivative of a function at a point, or calculate the value of a definite integral. However, you must clearly indicate the setup of your question, namely the equation, function, or integral you are using. If you use other built-in features or programs, you must show the mathematical steps necessary to produce your results.

Show all of your work, even though a question may not explicitly remind you to do so. Clearly label any functions, graphs, tables, or other objects that you use. Justifications require that you give mathematical reasons and that you verify the needed conditions under which relevant theorems, properties, definitions, or tests are applied. Your work will be scored on the correctness and completeness of your methods as well as your answers. Answers without supporting work will usually not receive credit.

Your work must be expressed in standard mathematical notation rather than calculator syntax. For example, $\int_1^5 x^2 dx$ may not be written as $\text{fnInt}(X^2, X, 1, 5)$.

Unless otherwise specified, answers (numeric or algebraic) need not be simplified. If you use decimal approximations in calculations, your work will be scored on accuracy. Unless otherwise specified, your final answers should be accurate to three places after the decimal point.

Unless otherwise specified, the domain of a function $f$ is assumed to be the set of all real numbers $x$ for which $f(x)$ is a real number.

You can go back and forth between questions in this part until time expires. The clock will turn red when 5 minutes remain—**the proctor will not give you any time updates or warnings.**

Note: This exam was originally administered digitally. It is presented here in a format optimized for teacher and student use in the classroom.

**1.** Male birds of a certain species arrive at a nesting area over a thirty-day period. The rate at which the male birds arrive at the nesting area at time $t$ days is modeled by a differentiable function $M$, where $M(t)$ is measured in number of birds per day. Selected values of $M(t)$ are shown in the table.

| $t$ (days) | 0 | 5 | 10 | 15 | 20 | 25 | 30 |
|---|---|---|---|---|---|---|---|
| $M(t)$ (birds per day) | 2 | 7 | 16 | 6 | 5 | 2 | 0 |

**Part A**

Approximate $M'(7.5)$ using the average rate of change of $M$ over the interval $5 \le t \le 10$. Show the work that leads to your answer, and indicate units of measure.

**Part B**

i. Use a midpoint Riemann sum with the three subintervals $[0, 10]$, $[10, 20]$, and $[20, 30]$ to approximate $\int_0^{30} M(t)\,dt$. Show the work that leads to your answer.

ii. Interpret the meaning of $\int_0^{30} M(t)\,dt$ in the context of the problem.

**Part C**

The rate at which female birds of the same species arrive at the same nesting area, in birds per day, is modeled by the function $F$ defined as follows.

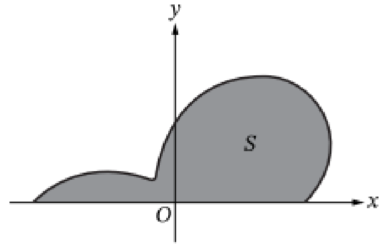
$$F(t) = \begin{cases} 0 & \text{for } 0 \le t < 15 \\ 18 + 16 \sin\left(\frac{\pi}{20}(t + 15)\right) & \text{for } 15 \le t \le 45 \end{cases}$$

How many female birds of this species arrive at the nesting area from $t = 15$ to $t = 45$? Show the setup for your calculations, and round your answer to the nearest integer.

**Part D**

On the interval $15 < t < 30$, the difference in the rates at which male and female birds of this species arrive at the nesting area can be modeled by the differentiable function $D(t) = M(t) - F(t)$, where $F$ is the function defined in part C. Is there a time $t$ in the interval $15 < t < 20$ when $D(t) = 0$? Justify your answer.

**2.** Let $S$ be the shaded region bounded by the graph of the polar curve $r(\theta) = 3 + 2\sin(2\theta) + \cos(2\theta)$ for $0 \le \theta \le \pi$, as shown in the figure.




It can be shown that $r'(\theta) = 4\cos(2\theta) - 2\sin(2\theta)$.

**Part A**

Find the area of region $S$. Show the setup for your calculations.

**Part B**

There is a point on the curve at which the slope of the line tangent to the curve is $-\frac{3}{7}$. At this point, $\frac{dy}{d\theta} = \frac{3\sqrt{2}}{2}$. Find $\frac{dx}{d\theta}$ at this point. Show the work that leads to your answer.

**Part C**

i. Find the value of $\theta$ in the interval $0 < \theta < \frac{\pi}{2}$ at which $r$ has a critical point.

ii. Use a derivative test to determine whether the critical point is the location of a relative minimum, a relative maximum, or neither for $r$.

**Part D**

Find the average distance from the origin to a point on the polar curve $r(\theta) = 3 + 2\sin(2\theta) + \cos(2\theta)$ for $\frac{\pi}{2} \le \theta \le \pi$. Show the setup for your calculations.

# CALCULUS BC
# SECTION II PART B
# TIME – 1 HOUR

**Directions:**

Section II, Part B has 4 free-response questions and lasts 1 hour.

**A calculator is not allowed for this part of the exam.**

You may use the available paper for scratch work and planning, but only work written in the free-response booklet will be scored. Any work done on scratch paper will not be scored. In the free-response booklet, write your solution to each part of each question in the space provided for that part. For questions that have sub-parts, be sure to label those clearly in your solution. Use a pencil or a pen with black or dark blue ink.

Show all of your work, even though a question may not explicitly remind you to do so. Clearly label any functions, graphs, tables, or other objects that you use. Justifications require that you give mathematical reasons and that you verify the needed conditions under which relevant theorems, properties, definitions, or tests are applied. Your work will be scored on the correctness and completeness of your methods as well as your answers. Answers without supporting work will usually not receive credit.

Your work must be expressed in standard mathematical notation rather than calculator syntax. For example, $\int_1^5 x^2 dx$ may not be written as $\text{fnInt}(X^2, X, 1, 5)$.

Unless otherwise specified, answers (numeric or algebraic) need not be simplified. If you use decimal approximations in calculations, your work will be scored on accuracy. Unless otherwise specified, your final answers should be accurate to three places after the decimal point.

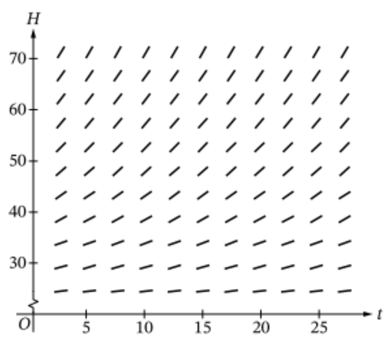
Unless otherwise specified, the domain of a function $f$ is assumed to be the set of all real numbers $x$ for which $f(x)$ is a real number.

You can go back and forth between questions in this part until time expires. The clock will turn red when 5 minutes remain—**the proctor will not give you any time updates or warnings.**

**3.** A pie is taken from a hot oven and put on a table. The internal temperature of the pie at time $t$ minutes can be modeled by the function $H$ that satisfies the differential equation $\frac{dH}{dt} = -\frac{1}{15}(H - 20)$, where $H(t)$ is measured in degrees Celsius and $H(0) = 75$. For $t > 0$, it is known that $20 < H(t) < 75$.

**Part A**

Explain why the following could not be a slope field for the differential equation $\frac{dH}{dt} = -\frac{1}{15}(H - 20)$.




**Part B**

Find the slope of the line tangent to the graph of $H$ at time $t = 0$. Show the work that leads to your answer.

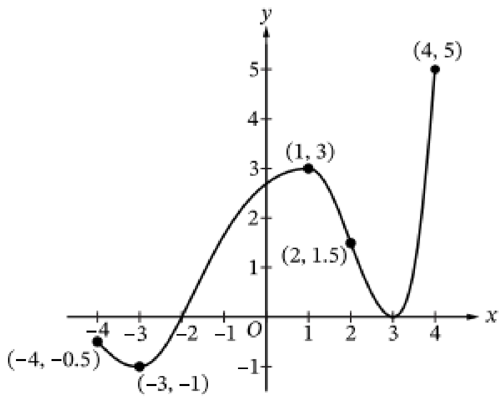
**Part C**

It can be shown that $\frac{d^2H}{dt^2} = \frac{1}{225}(H - 20)$. The line tangent to the graph of $H$ at time $t = 0$ is used to approximate $H(5)$, the internal temperature of the pie at time $t = 5$. Is this approximation an overestimate or an underestimate for the actual value of $H(5)$? Give a reason for your answer.

**Part D**

Use separation of variables to find an expression for $H(t)$, the particular solution to the given differential equation with initial condition $H(0) = 75$.

**4.** Let $f$ be a twice-differentiable function on the closed interval $[-4, 4]$ with $f(2) = 3$. The graph of $f'$, the derivative of $f$, is shown.




Graph of $f'$

**Part A**

For $x > 0$, the function $g$ is defined by $g(x) = f(x) - \ln x$. Find $g'(2)$. Show the work that leads to your answer.

**Part B**

Find all values of $x$ on the open interval $0 < x < 3$ at which the graph of $f$ has a point of inflection. Give a reason for your answer.

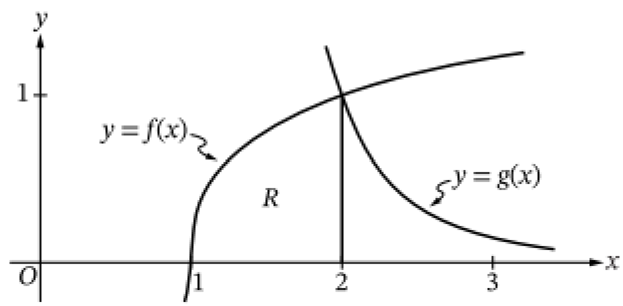
**Part C**

For $-4 \le x \le 4$, on what open intervals, if any, is the graph of $f$ both increasing and concave down? Give a reason for your answer.

**Part D**

For $-4 \le x \le 4$, find the value of $x$ at which $f$ has an absolute minimum and the value of $x$ at which $f$ has an absolute maximum. Give reasons for your answers.

**5.** Let $f$ be the function defined by $f(x) = \sqrt[3]{x-1}$, and let $g$ be the function defined by $g(x) = e^{(-2x+4)}$. The graphs of $f$ and $g$ intersect at the point $(2, 1)$. Let $R$ be the region bounded by the graph of $f$, the $x$-axis, and the vertical line $x = 2$, as shown in the figure.




### Part A

Evaluate $\int_1^2 f(x)\,dx$. Show the work that leads to your answer.

### Part B

Write, but do not evaluate, an expression involving one or more integrals that gives the volume of the solid generated when region $R$ is rotated about the $x$-axis.

### Part C

Write, but do not evaluate, an expression involving one or more integrals that gives the perimeter of region $R$.

### Part D

Evaluate $\int_2^{\infty} g(x)\,dx$. Show the work that leads to your answer.

**6.** The Maclaurin series for a function $g$ is given by

$$\sum_{n=0}^{\infty} 2\left(\frac{-x}{5}\right)^n = 2 - \frac{2}{5}x + \frac{2}{25}x^2 - \frac{2}{125}x^3 + \frac{2}{625}x^4 - \cdots + 2\left(\frac{-x}{5}\right)^n + \cdots \text{ and converges to } g(x)$$

for $|x| < 5$.

**Part A**

For $x > 0$, the Maclaurin series for $g$ is an alternating geometric series. Find $g(3)$. Show the work that leads to your answer.

**Part B**

The function $f$ is defined by $f(x) = g'(x)$. Write the first four nonzero terms of the Maclaurin series for $f$.

**Part C**

The second-degree Taylor polynomial for $f$ about $x = 0$ is used to approximate $f\left(\frac{5}{2}\right)$ as $-\frac{3}{10}$, where $f$ is the function defined in part B. Justify that $\left|f\left(\frac{5}{2}\right) - \left(-\frac{3}{10}\right)\right| \le \frac{1}{5}$.

**Part D**

The function $h$ is defined by $h(x) = 25g(x) - 2e^x$.

i. Write the first three nonzero terms of the Maclaurin series for $e^x$.

ii. Write the first three nonzero terms of the Maclaurin series for $h$.

**STOP**

**END OF EXAM**