

2004 AP[®] CALCULUS BC FREE-RESPONSE QUESTIONS

5. A population is modeled by a function P that satisfies the logistic differential equation

$$\frac{dP}{dt} = \frac{P}{5} \left(1 - \frac{P}{12} \right).$$

- (a) If $P(0) = 3$, what is $\lim_{t \rightarrow \infty} P(t)$?

If $P(0) = 20$, what is $\lim_{t \rightarrow \infty} P(t)$?

- (b) If $P(0) = 3$, for what value of P is the population growing the fastest?

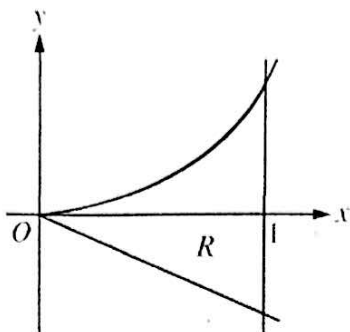
- (c) A different population is modeled by a function Y that satisfies the separable differential equation

$$\frac{dY}{dt} = \frac{Y}{5} \left(1 - \frac{t}{12} \right).$$

Find $Y(t)$ if $Y(0) = 3$.

- (d) For the function Y found in part (c), what is $\lim_{t \rightarrow \infty} Y(t)$?

2014 AP[®] CALCULUS BC FREE-RESPONSE QUESTIONS



5. Let R be the shaded region bounded by the graph of $y = xe^{x^2}$, the line $y = -2x$, and the vertical line $x = 1$, as shown in the figure above.

(a) Find the area of R .

(b) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when R is rotated about the horizontal line $y = -2$.

(c) Write, but do not evaluate, an expression involving one or more integrals that gives the perimeter of R .

2016 AP[®] CALCULUS BC FREE-RESPONSE QUESTIONS

4. Consider the differential equation $\frac{dy}{dx} = x^2 - \frac{1}{2}y$.

(a) Find $\frac{d^2y}{dx^2}$ in terms of x and y .

(b) Let $y = f(x)$ be the particular solution to the given differential equation whose graph passes through the point $(-2, 8)$. Does the graph of f have a relative minimum, a relative maximum, or neither at the point $(-2, 8)$? Justify your answer.

(c) Let $y = g(x)$ be the particular solution to the given differential equation with $g(-1) = 2$. Find $\lim_{x \rightarrow -1} \left(\frac{g(x) - 2}{3(x + 1)^2} \right)$. Show the work that leads to your answer.

(d) Let $y = h(x)$ be the particular solution to the given differential equation with $h(0) = 2$. Use Euler's method, starting at $x = 0$ with two steps of equal size, to approximate $h(1)$.

2017 AP[®] CALCULUS BC FREE-RESPONSE QUESTIONS

5. Let f be the function defined by $f(x) = \frac{3}{2x^2 - 7x + 5}$.

(a) Find the slope of the line tangent to the graph of f at $x = 3$.

(b) Find the x -coordinate of each critical point of f in the interval $1 < x < 2.5$. Classify each critical point as the location of a relative minimum, a relative maximum, or neither. Justify your answers.

(c) Using the identity that $\frac{3}{2x^2 - 7x + 5} = \frac{2}{2x - 5} - \frac{1}{x - 1}$, evaluate $\int_5^{\infty} f(x) \, dx$ or show that the integral diverges.

(d) Determine whether the series $\sum_{n=5}^{\infty} \frac{3}{2n^2 - 7n + 5}$ converges or diverges. State the conditions of the test used for determining convergence or divergence.