06-Three Phase Circuits

Text: Chapter 8.1-8.2

ECEGR 3500

Electrical Energy Systems

Professor Henry Louie

Overview

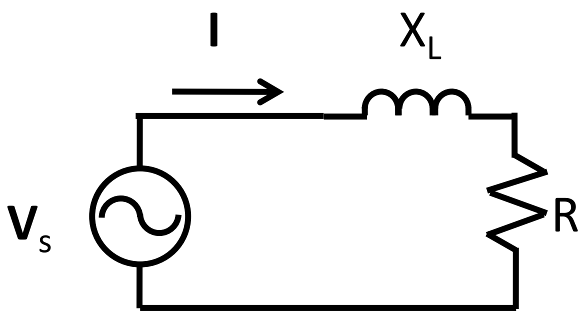
- Generation of Three-Phase Voltage
- Three Phase Voltage
- Delta, Wye Connections
- Load Connections

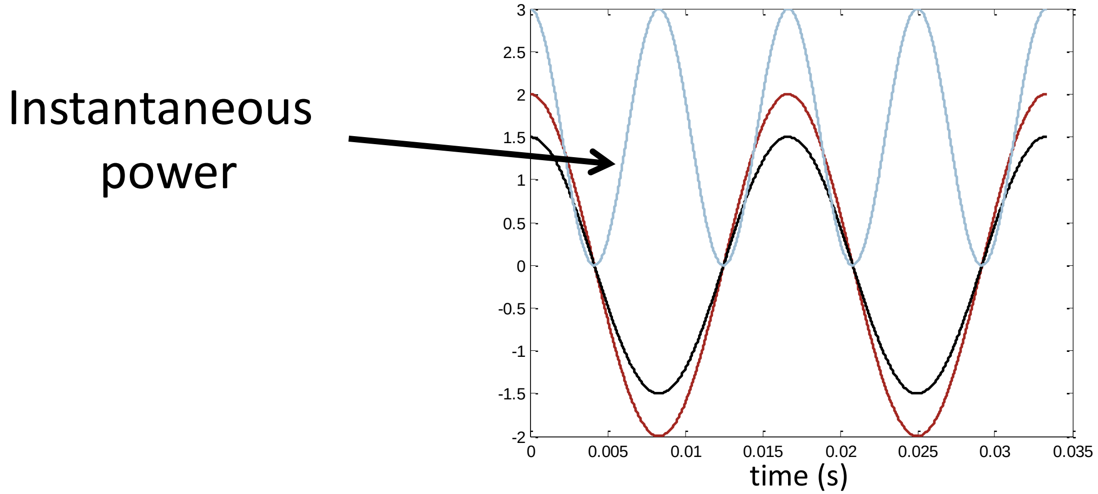
Questions

- How is three phase different from single phase?
- How can circuit elements be connected to make three phase systems?
- Why do some electrical panels say 208/120 or 480/277?

Single Phase

- We have analyzed single phase circuits
- Recall:
  - Power pulsates at twice the frequency of voltage, current
  - Two conductors are needed





Transmission Lines

Have you ever noticed that transmission lines have conductors in multiples of three?

Generating AC Voltage

- As demonstrated in class, we can induce a voltage in a coil by passing a magnet across a coil
- The voltage induced is in accordance with Lenz's Law:

$$e = -\frac{d\Phi}{dt}$$

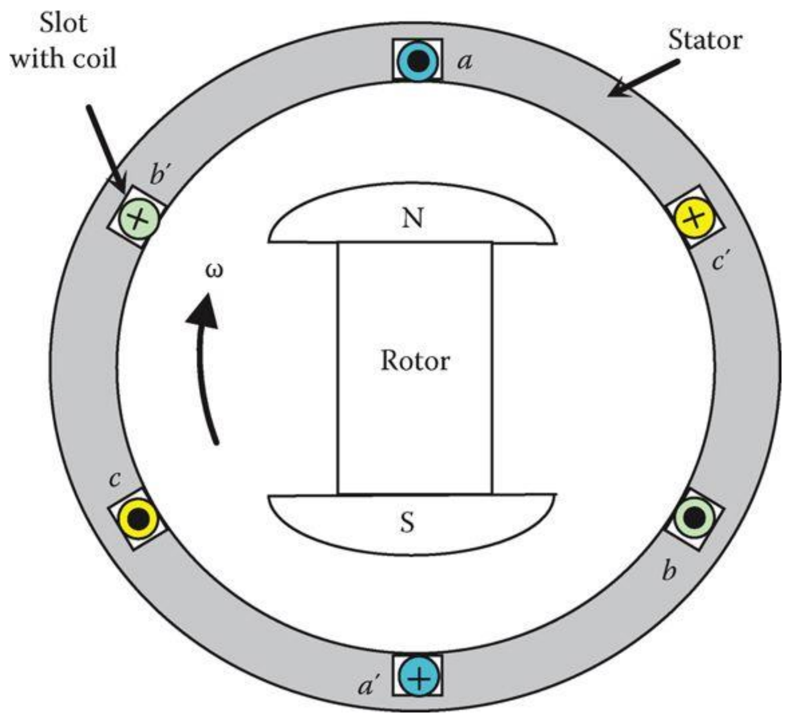
  - e: induced voltage in the coil, V
  - ϕ: magnetic flux, Wb

We will delve into Lenz's Law later in the course

- Voltage is induced when the coil experiences a <u>change</u> in flux

Simple Three-Phase Generator

- Generators are cylindrical in shape
- Three coils arranged around the stator
  - Each coil has two conductors shown in the cross section (a, a'; b, b'; c, c')
  - Coils are spatially offset by 120 degrees from each other
- Rotor contains a magnet that rotates
- Air gap separates rotor from stator



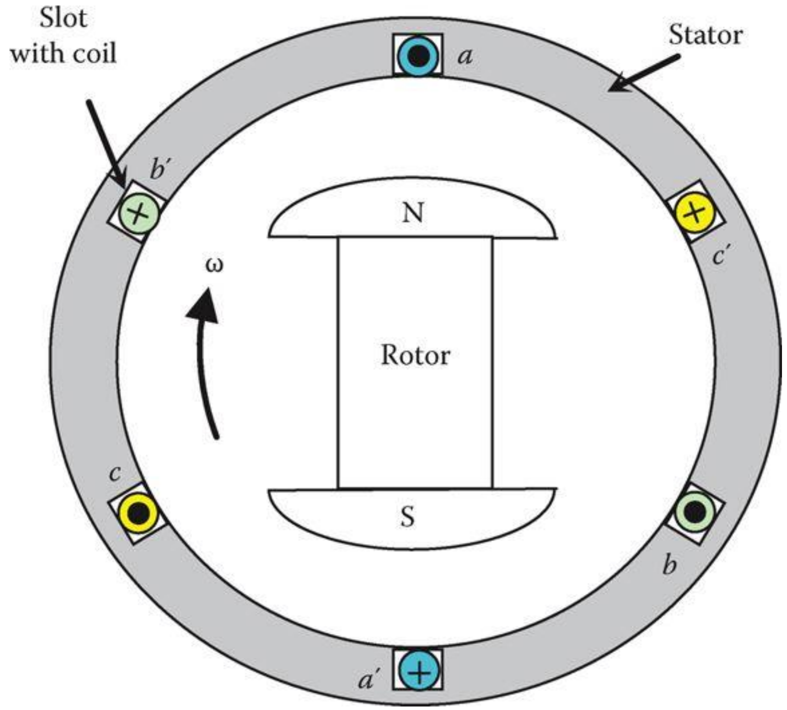
Cross-section of a simple three-phase generator

Simple Three-Phase Generator

- As the rotor rotates, the flux ϕ passing through each coil varies sinusoidally
- Derivative of a sinusoid is also a sinusoid, so the induced voltage is sinusoidal

$$e = -\frac{d\Phi}{dt}$$

- Since the coils are physically offset by 120 degrees, the voltages are offset from each other in phase by 120 degrees
- Each coil will have the same maximum voltage, v_{max}



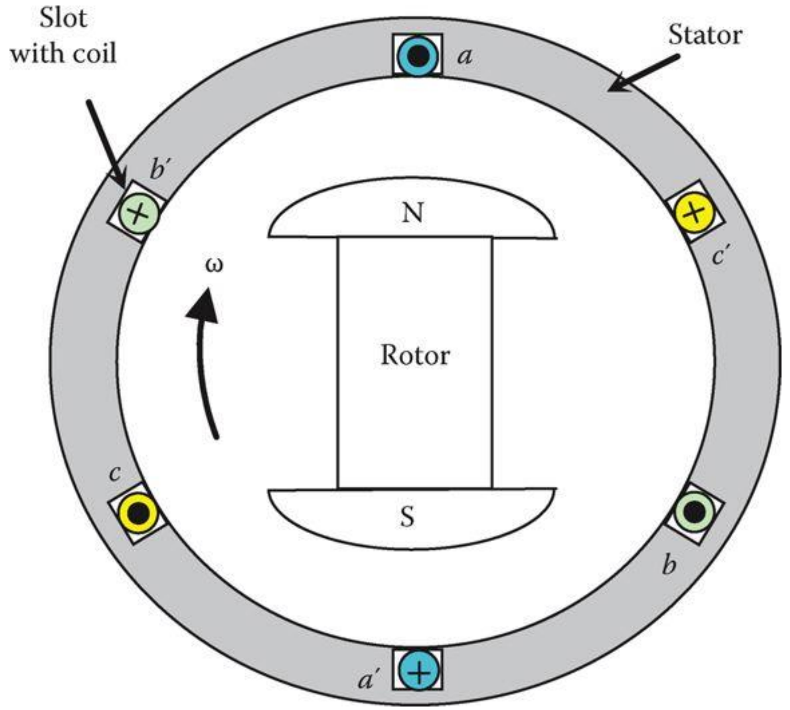
Phase Voltage

- The voltage induced in each coil is known as the "phase voltage"
- There are three voltages produced by the generator (one per coil):

$$v_{\mathrm{aa'}}(t)$$

$$v_{\mathrm{bb'}}(t)$$

$$v_{\mathrm{cc'}}(t)$$



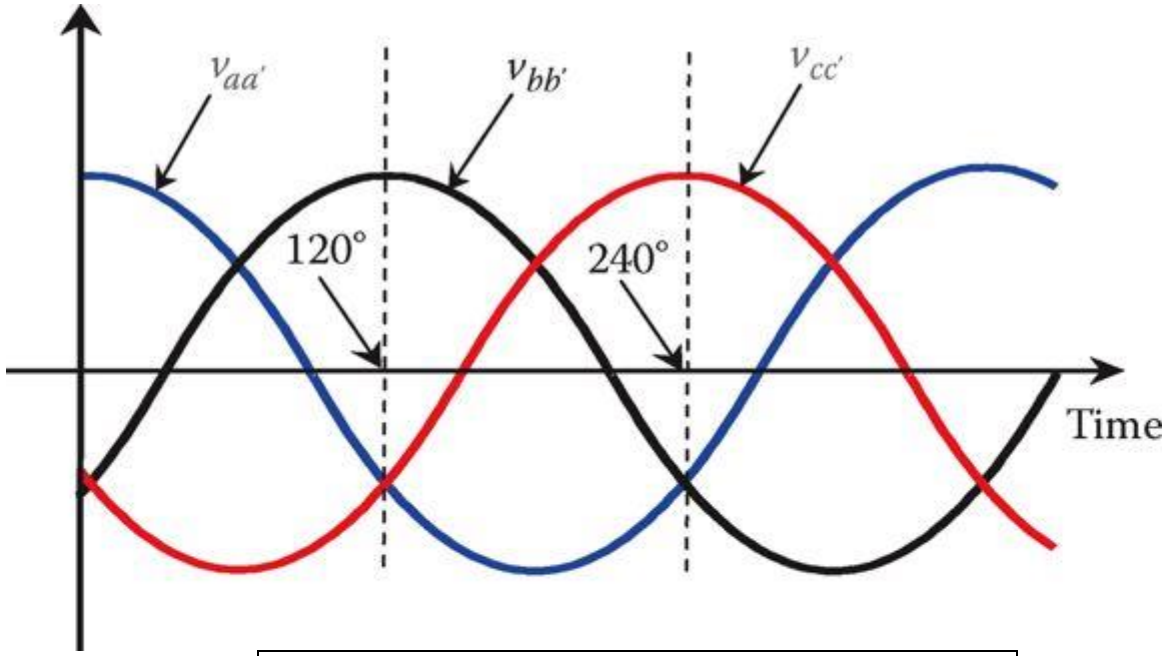
Phase Voltage in the Time Domain

The phase voltages are expressed in the time domain as:

$$v_{aa'}(t) = v_{max}\cos(\omega t)$$

$$v_{bb'}(t) = v_{max}\cos(\omega t - 120^o)$$

$$v_{cc'}(t) = v_{max}\cos(\omega t + 120^o)$$



Note: instantaneous values
$v_{aa'}(t) + v_{bb'}(t) + v_{cc'}(t) = 0$ for all t

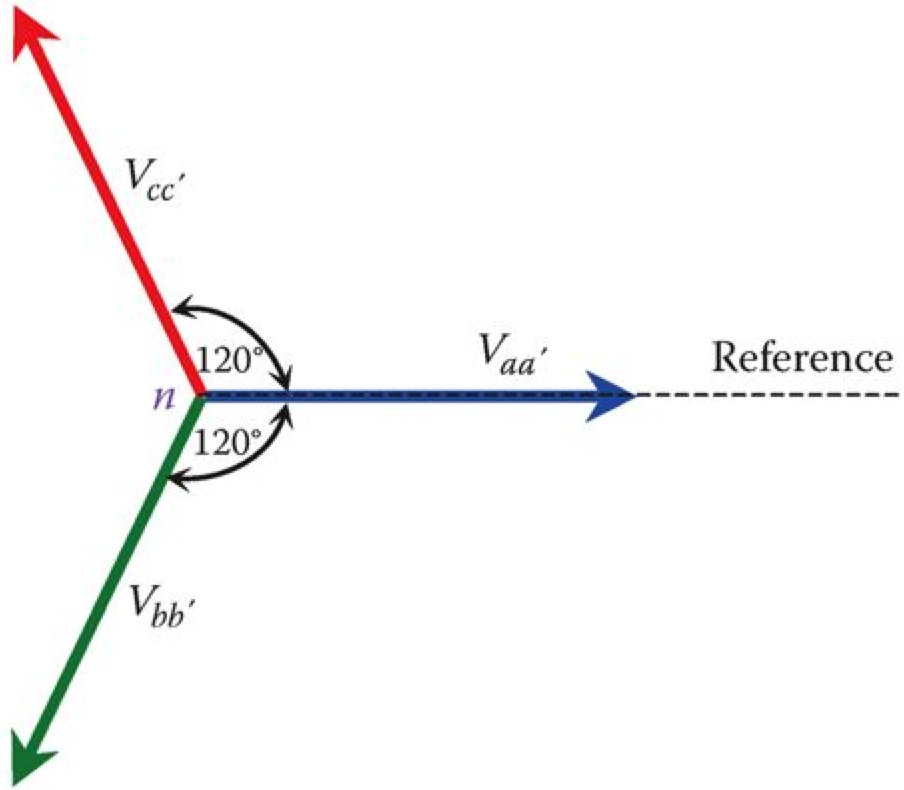
Phase Voltage in the Phasor Domain

Each phase voltage is a sinusoid, so they can be expressed in phasor form as:

$$\boldsymbol{V}_{\mathrm{aa'}} = \frac{V_{\max}}{\sqrt{2}}\angle 0^{\circ}$$

$$\boldsymbol{V}_{\mathrm{bb'}} = \frac{V_{\max}}{\sqrt{2}}\angle -120^{\circ}$$

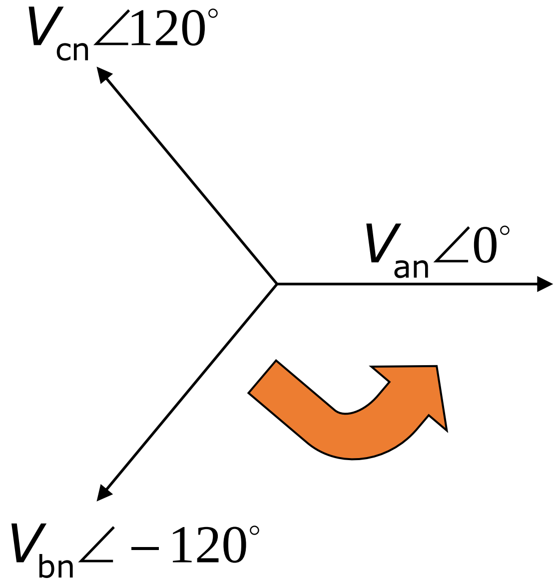
$$\boldsymbol{V}_{\mathrm{cc'}} = \frac{V_{\max}}{\sqrt{2}}\angle 120^{\circ}$$

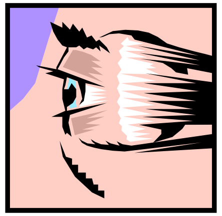


Phase Rotation

- Power systems use "three phase"
- We are concerned with balanced three phase
- Balanced circuit conditions:
  - impedances are equal for each phase
  - voltage source phasors have equal magnitude and have a 120 degree phase shift
  - a, b, c phase rotation

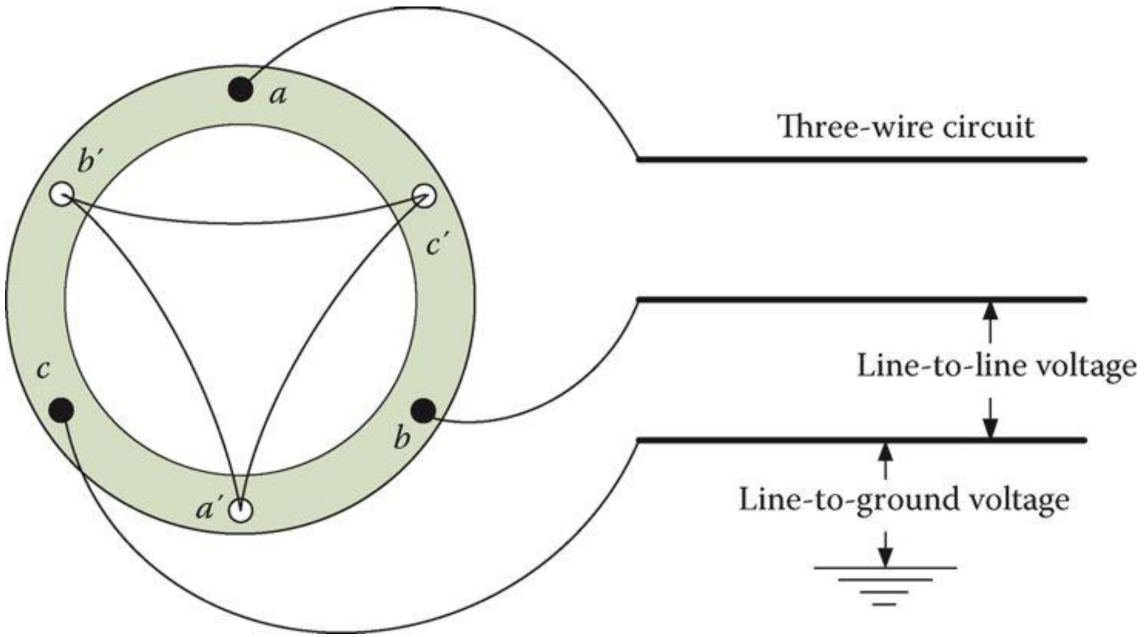
Phase Rotation



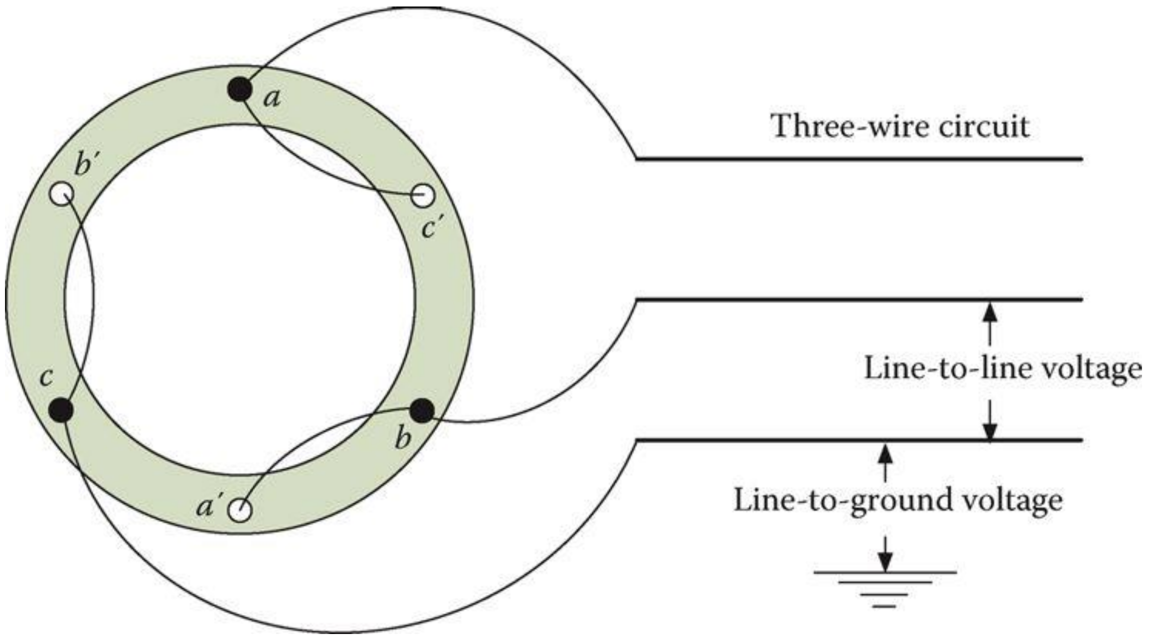


Connections of Three-Phase Sources

WYE (Y)

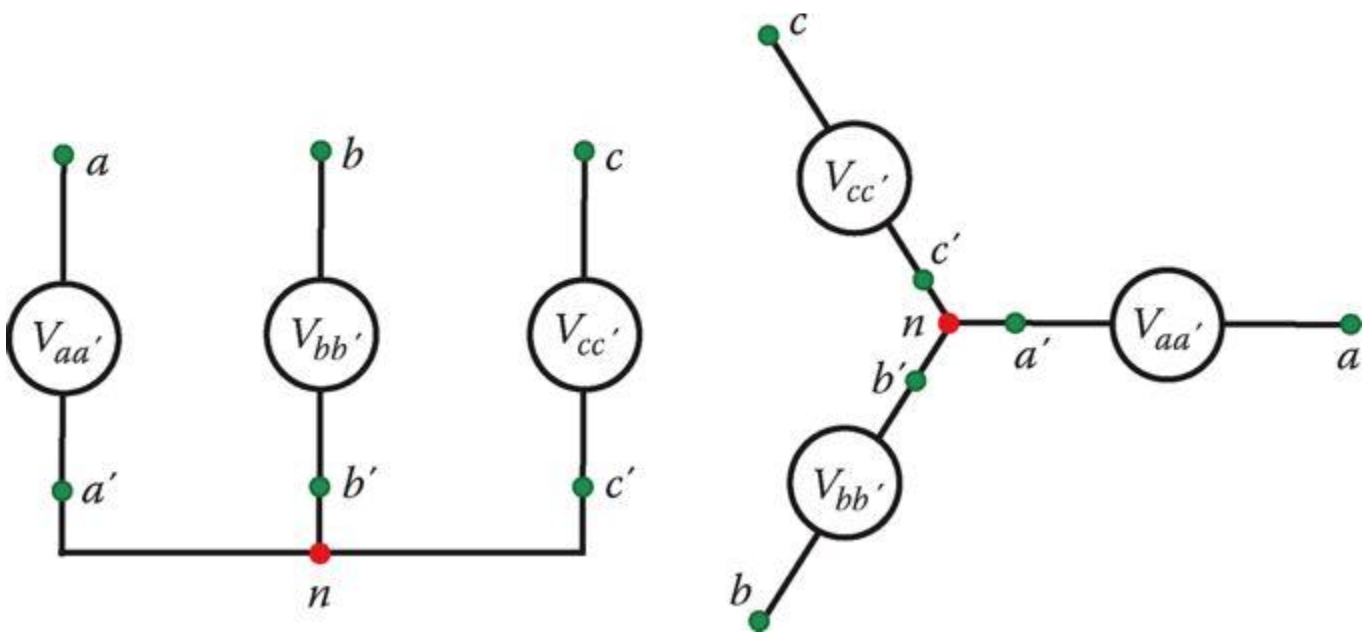


DELTA (Δ)



Wye-Connected Sources

- Wye-connected generator can be modelled as three voltage sources connected to a common point *n* (neutral)
- Each voltage source represents a coil of the generator

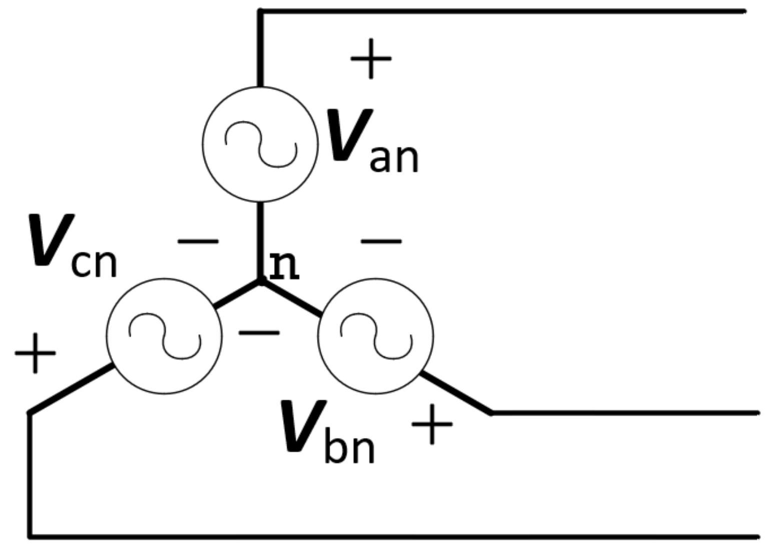


Wye-Connected Sources

- Let's clean up the notation
  - subscript n notes that voltages are referenced to a common node (neutral)
- The voltage across the coils are now written as:

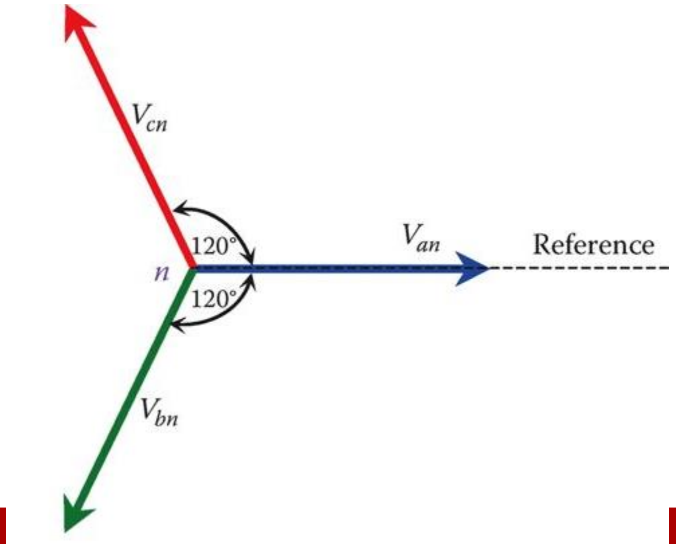
  $\boldsymbol{V}_{\mathrm{an}}$
  $\boldsymbol{V}_{\mathrm{bn}}$
  $\boldsymbol{V}_{\mathrm{cn}}$
- Let's also define the magnitude of the phase voltages as

  V_{ph} : magnitude of the phase voltage



Phasor Diagram of Wye Sources

- Magnitude of the voltage to neutral is equal for any phase
- Each voltage source is displaced by 120 degrees
- The phase voltage is equal to the magnitude of any of the voltage sources



$$V_{ph} = V_{an} = V_{bn} = V_{cn} = |\boldsymbol{V}_{an}| = |\boldsymbol{V}_{bn}| = |\boldsymbol{V}_{cn}|$$

$$\boldsymbol{V}_{an} = V_{an}\angle 0^\circ = V_{ph}\angle 0^\circ$$

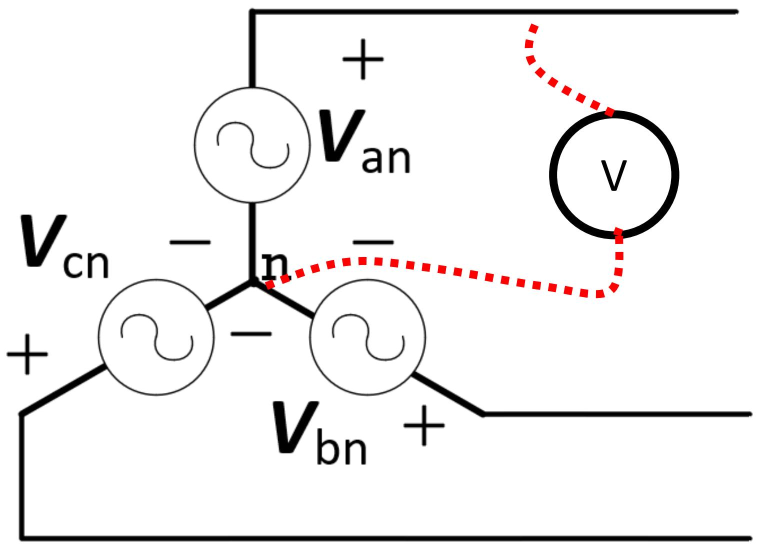
$$\boldsymbol{V}_{bn} = V_{bn}\angle -120^\circ = V_{ph}\angle -120^\circ$$

$$\boldsymbol{V}_{cn} = V_{cn}\angle 120^\circ = V_{ph}\angle 120^\circ$$

Line-Neutral Voltage

We are often interested in the voltage between one of the lines (or end of coil) and the neutral point. In a wye-connected system, this is referred to as the "Line-Neutral" voltage,

which is the same as the phase voltage



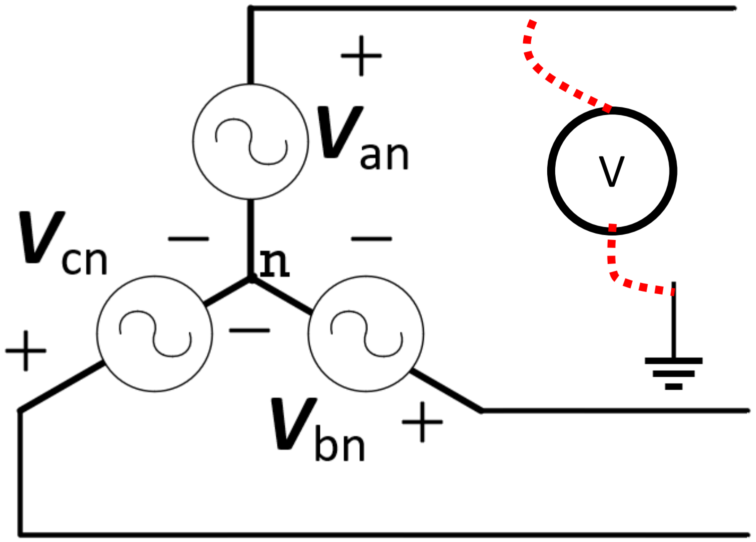
Example: if $\boldsymbol{V}_{an} = 277\angle 0^{\circ}$, then we might say that the a-phase line-to-neutral voltage is 277 volts at zero degrees

A voltmeter measuring the voltage between one line (a-phase) of the system and the neutral point

Line-to-Ground Voltage

In some cases, we are interested in the voltage between one of the lines (or end of a coil) and ground

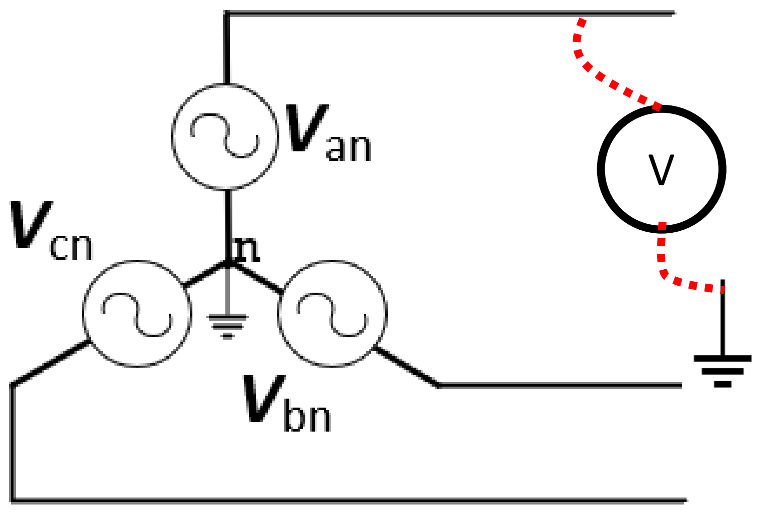
This is known as the “line-to-ground” voltage



A voltmeter measuring the voltage between one line (a-phase) and ground

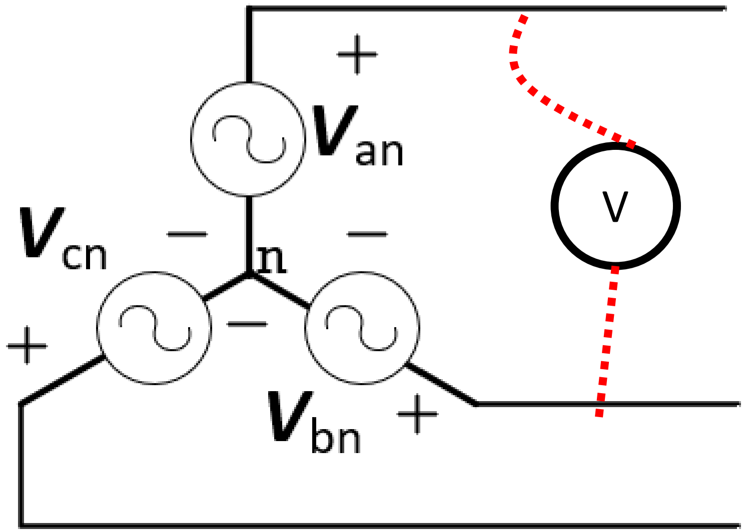
Line-to-Ground Voltage

The neutral point is often grounded so in most cases "line-to-neutral" and "line-to-ground" voltages are the same



Line-Line Voltage

The voltage between any two lines (or two terminals of a generator or load) is known as the "line-line" voltage (or simply "line voltage")

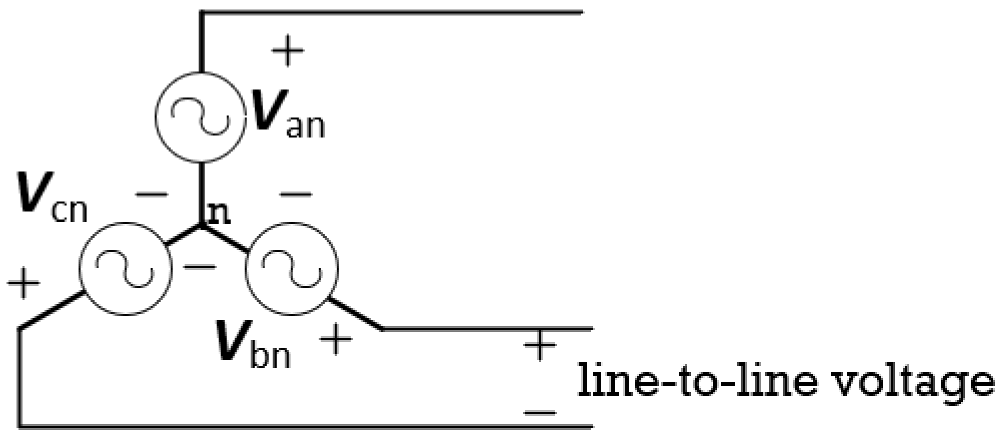


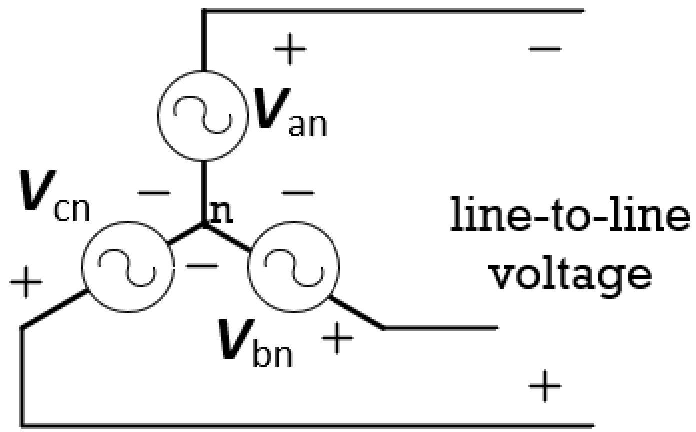
The magnitude of the line-line voltage is defined as:

V_{ll}: magnitude of the line voltage

Line-Line Voltage

Other examples of line voltage:





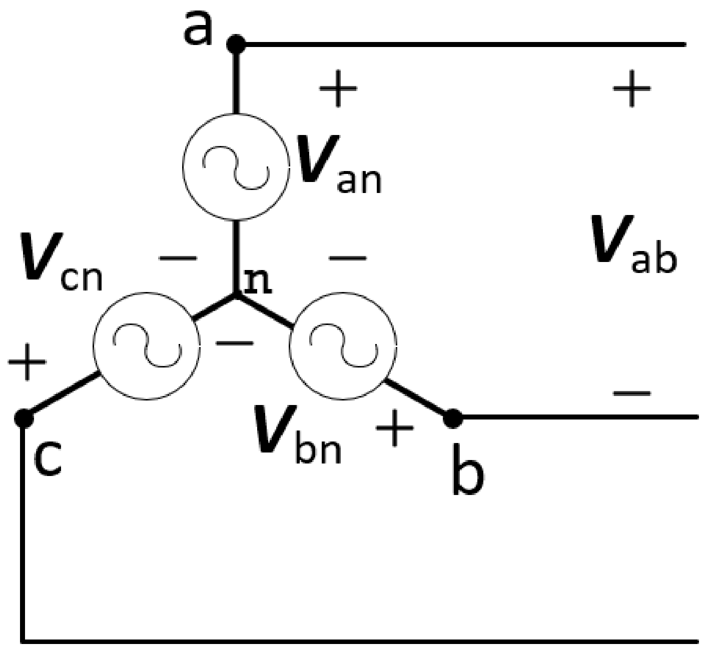
Line-Line Voltages

Consider the line-line voltage between a-phase and b-phase, denoted as $\boldsymbol{V}_{\mathrm{ab}}$

$$\boldsymbol{V}_{\mathrm{ab}} = \boldsymbol{V}_{\mathrm{an}} - \boldsymbol{V}_{\mathrm{bn}}$$

By KVL

$$\boldsymbol{V}_{\mathrm{ab}} = V_{\mathrm{ph}}\angle 0^{\circ} - V_{\mathrm{ph}}\angle -120^{\circ} = \sqrt{3}V_{\mathrm{ph}}\angle 30^{\circ}$$

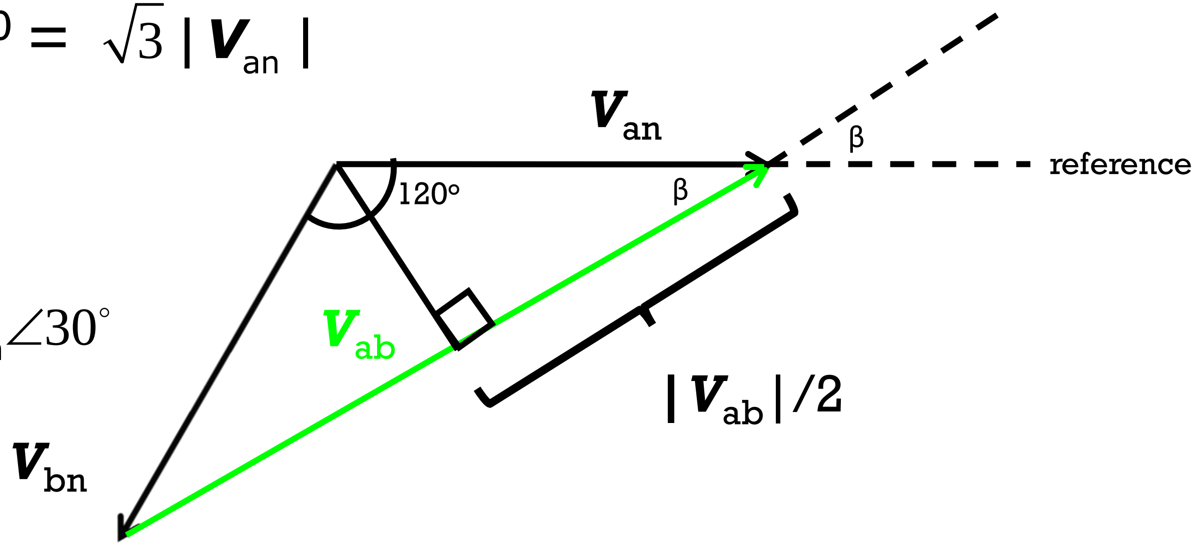


Phase Voltage and Line Voltage Relationship

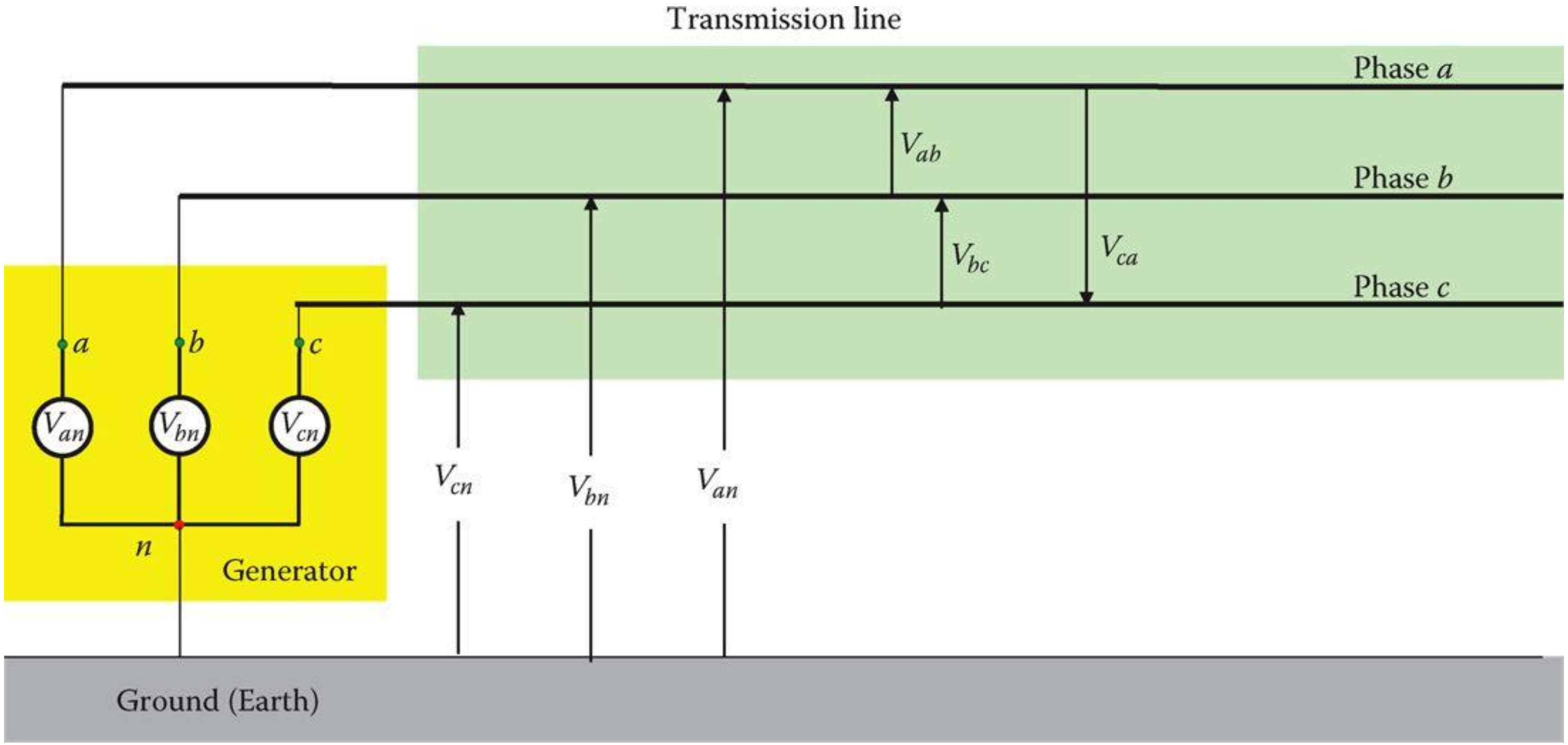
- Generically:
  - $\beta = 30°$
  - $|\boldsymbol{V}_{ab}| = 2|\boldsymbol{V}_{an}|\cos 30^0 = \sqrt{3}\,|\boldsymbol{V}_{an}|$
- So that

$$\boldsymbol{V}_{ab} = (\sqrt{3}\angle 30^\circ)\boldsymbol{V}_{an}$$

$$\boldsymbol{V}_{ab} = (\sqrt{3}\angle 30^\circ)V_{ph}\angle 0^\circ = \sqrt{3}V_{ph}\angle 30^\circ$$



Phase and Line Voltages in Wye Systems



Line-Line Voltage for Wye Sources

We can show by KVL that the line-line voltages for wye sources are:

$$\boldsymbol{V}_{ab} = \boldsymbol{V}_{an} - \boldsymbol{V}_{bn} = \boldsymbol{V}_{an}(\sqrt{3}\angle 30^\circ) = \sqrt{3}\mathrm{V}_{ph}\angle 30^\circ$$

$$\boldsymbol{V}_{bc} = \boldsymbol{V}_{bn} - \boldsymbol{V}_{cn} = \boldsymbol{V}_{bn}(\sqrt{3}\angle 30^\circ) = \sqrt{3}\mathrm{V}_{ph}\angle -90^\circ$$

$$\boldsymbol{V}_{ca} = \boldsymbol{V}_{cn} - \boldsymbol{V}_{an} = \boldsymbol{V}_{cn}(\sqrt{3}\angle 30^\circ) = \sqrt{3}\mathrm{V}_{ph}\angle 150^\circ$$

Using:

$$\boldsymbol{V}_{an} = V_{ph}\angle 0^\circ$$

$$\boldsymbol{V}_{bn} = V_{ph}\angle -120^\circ$$

$$\boldsymbol{V}_{cn} = V_{ph}\angle 120^\circ$$

Exercise

The line-line voltage of a wye-connected source always has a greater magnitude than the line-neutral voltage.

- True
- False

Exercise

The line-line voltage of a wye-connected source always has a greater magnitude than the line-neutral voltage.

- True
- False

The line-neutral voltage is the same as the phase voltage in a wye-connected source. The line-line voltage is square root of three times greater than the phase voltage in wye-connected sources.

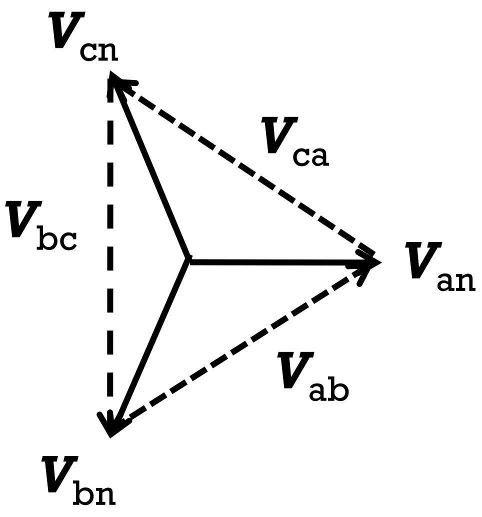
Three Phase Voltage

$$\boldsymbol{V}_{an} = \boldsymbol{V}_{bn}(1\angle 120^\circ)$$
$$\boldsymbol{V}_{bn} = \boldsymbol{V}_{cn}(1\angle 120^\circ)$$
$$\boldsymbol{V}_{cn} = \boldsymbol{V}_{an}(1\angle 120^\circ)$$

$$\mathbf{V}_{ab} = \mathbf{V}_{bc}(1\angle 120^\circ)$$
$$\mathbf{V}_{bc} = \mathbf{V}_{ca}(1\angle 120^\circ)$$
$$\mathbf{V}_{ca} = \mathbf{V}_{ab}(1\angle 120^\circ)$$

Balanced sets

Phasor Diagram



Observations: Wye Connected Sources

1. Magnitude of the line-line voltage of the transmission line is greater than the magnitude of its phase voltage by a factor of square root of three

   Generically: $V_{ll} = V_{ph}\sqrt{3}$

2. Line-line voltage of the transmission line leads its phase voltage by 30°. ($\boldsymbol{V}_{ab}$ leads $\boldsymbol{V}_{an}$ by 30°, $\boldsymbol{V}_{bc}$ leads $\boldsymbol{V}_{bn}$ by 30° and $\boldsymbol{V}_{ca}$ leads $\boldsymbol{V}_{cn}$ by 30°)

Phase and Line Voltage Conversion (Wye Sources)

We can convert to and from phase and line-line voltages in wye-connected sources (or loads) as:

$$\mathbf{V}_{an} = \frac{\mathbf{V}_{ab}}{\sqrt{3}} \angle -30°$$

$$\mathbf{V}_{bn} = \frac{\mathbf{V}_{bc}}{\sqrt{3}} \angle -30°$$

$$\mathbf{V}_{cn} = \frac{\mathbf{V}_{ca}}{\sqrt{3}} \angle -30°$$

Phase Voltages

$$\mathbf{V}_{ab} = \mathbf{V}_{an}(\sqrt{3}\angle 30^\circ)$$

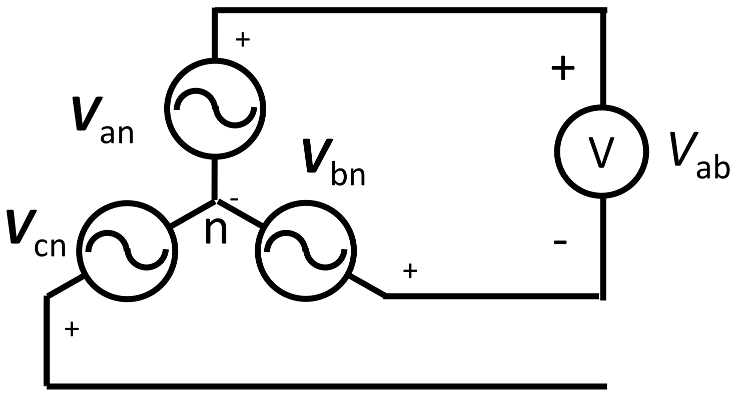
$$\mathbf{V}_{bc} = \mathbf{V}_{bn}(\sqrt{3}\angle 30^\circ)$$

$$\mathbf{V}_{ca} = \mathbf{V}_{cn}(\sqrt{3}\angle 30^\circ)$$

Line-Line Voltages

Exercise

- Consider a voltmeter placed as shown
- If $|\boldsymbol{V}_{an}| = 120$ V, then what value is displayed on the voltmeter?



Exercise

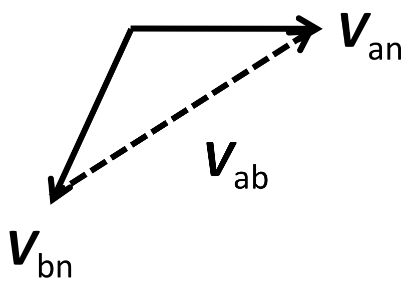
- Analytically:

$$\boldsymbol{V}_{ab} = \boldsymbol{V}_{an} - \boldsymbol{V}_{bn} = 120\angle 0^\circ - 120\angle -120^\circ$$

$$\boldsymbol{V}_{ab} = (120 + j0) - (-60 - j103.92) = 180 + j103.92 = 120\angle -120^\circ = 208\angle 30^\circ \text{ V}$$

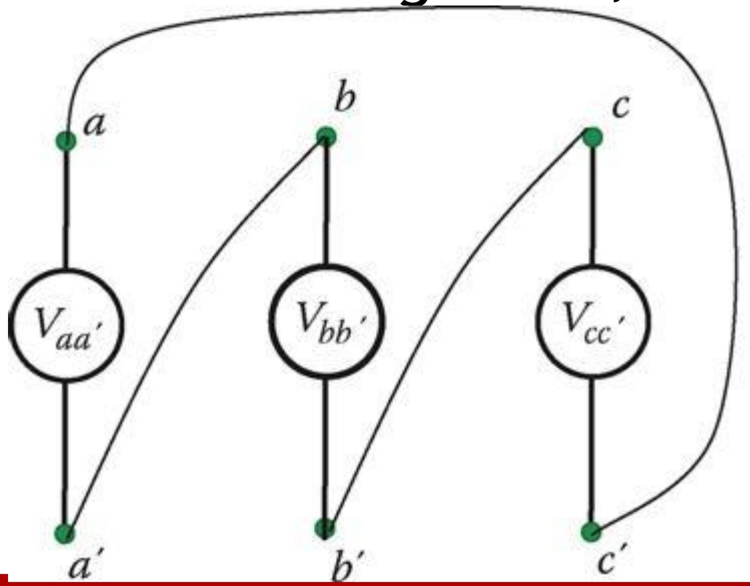
- Magnitude is 208V, and the phasor leads $\boldsymbol{V}_{an}$ by 30 degrees
- By inspection:

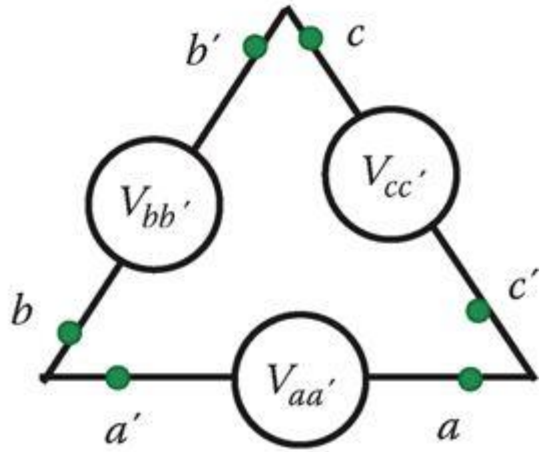
$$|\boldsymbol{V}_{ab}| = 2V_{an}\cos 30^\circ = \sqrt{3}\,|\boldsymbol{V}_{an}|$$



Delta-Connected Sources

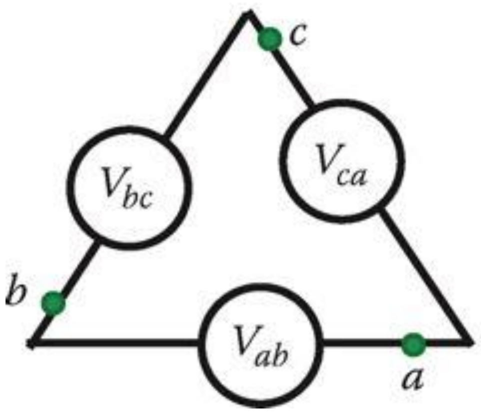
- Consider the connection of three, single phase voltage sources connected "end to end"
- Each phase can be thought of as a different coil in a generator
- No direct connection to ground; no neutral point

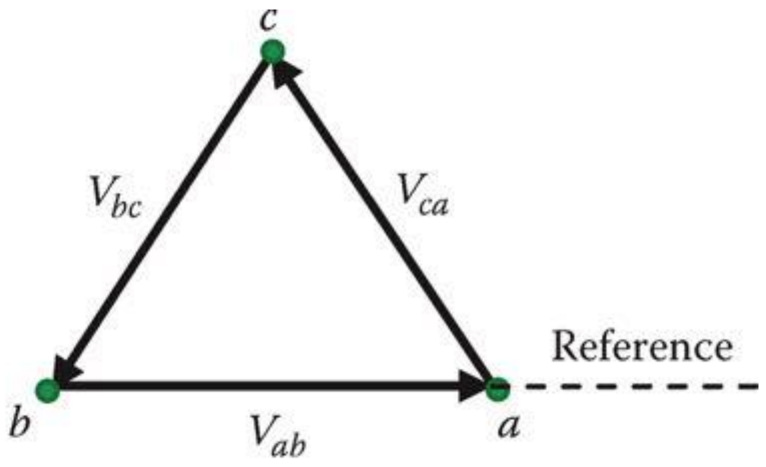




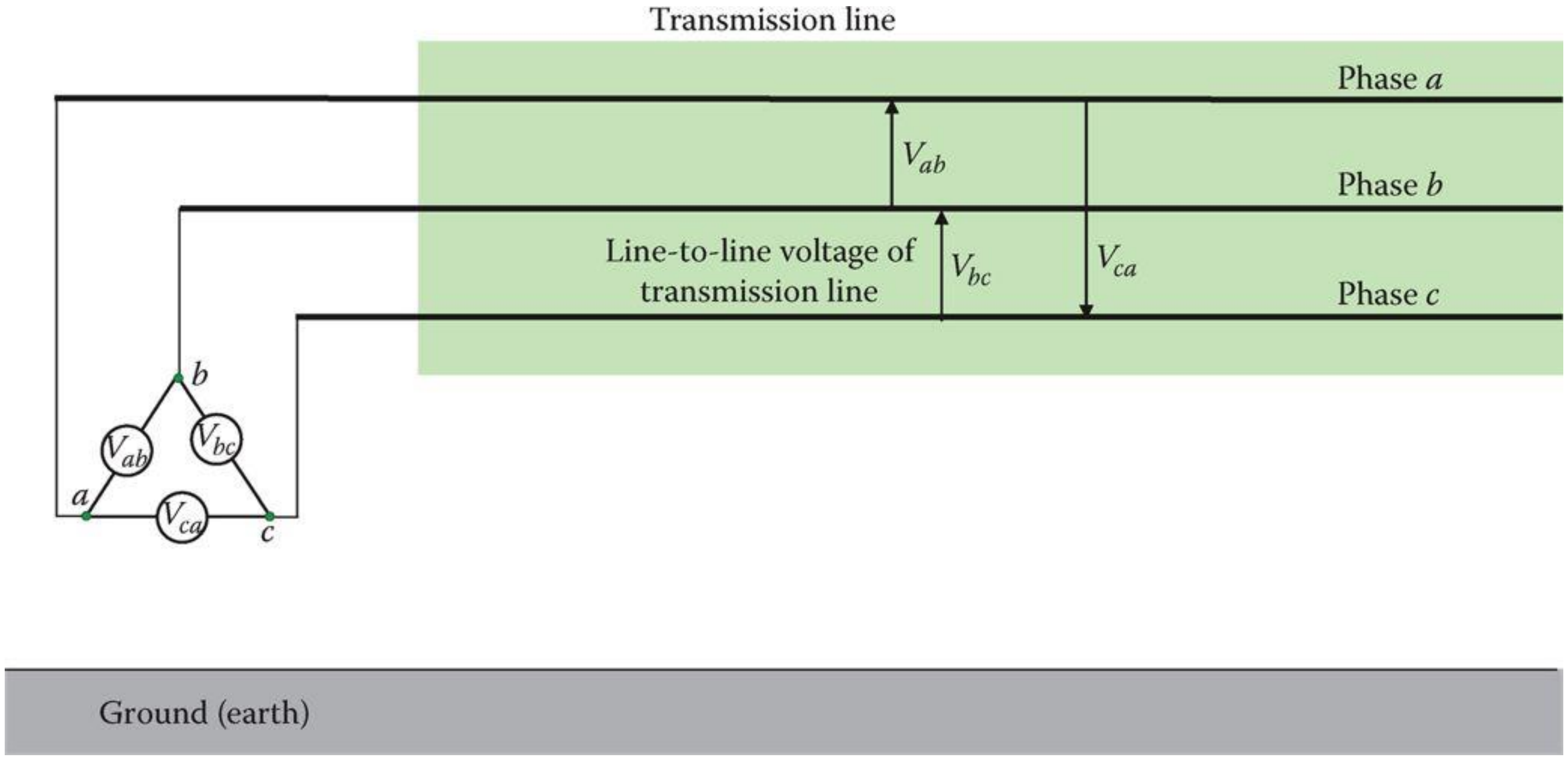
Delta-Connected Sources

- The voltage across the sources (coils) are the line-line voltage
- All line-line voltages have the same magnitude (V_{ll}) and are offset by 120 degrees



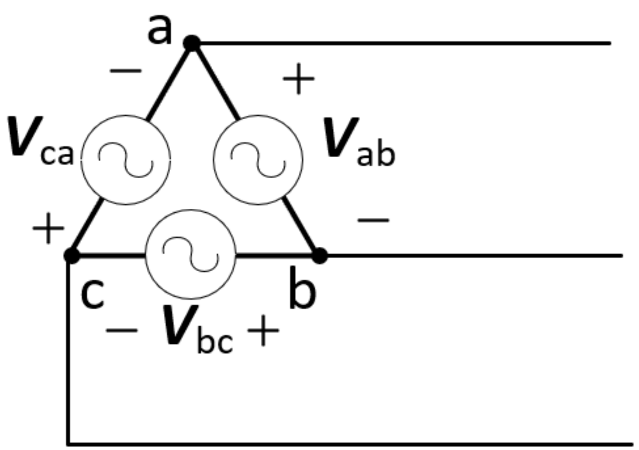


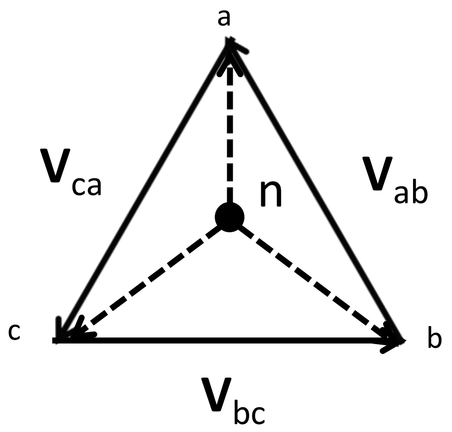
Line-Line Voltages in Delta Systems



Where is the neutral?

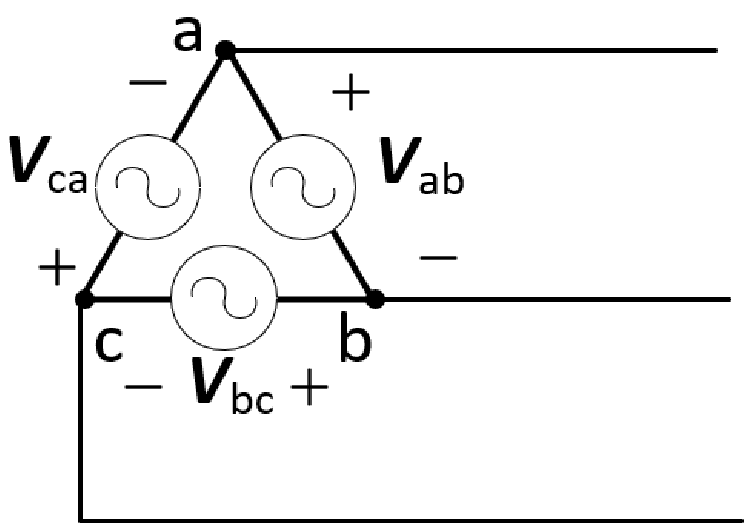
- No neutral point in Delta-connected sources, however a theoretical neutral is:
  - Equidistant between points a, b and c
  - Voltage magnitude between a-n, b-n, and c-n are equal (and less than V_{ab}, V_{bc} and V_{ca})





Phase Voltage in Delta Connected Sources

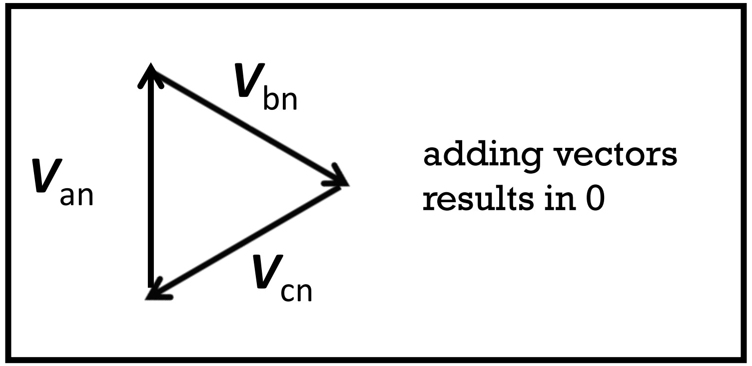
- Recall the definition of phase voltage: voltage across the coils of a generator
- In Delta-connected sources, the phase voltage is $\boldsymbol{V}_{ab}$, $\boldsymbol{V}_{bc}$, $\boldsymbol{V}_{ca}$
- But this is the same as the line-line voltage
- <u>In Delta-connected systems, the phase voltage is the same as the line-line voltage</u>

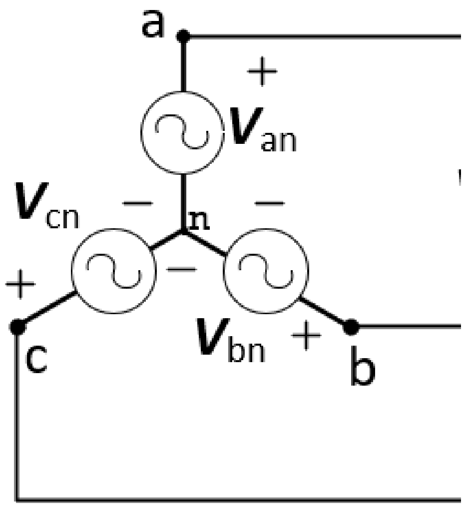


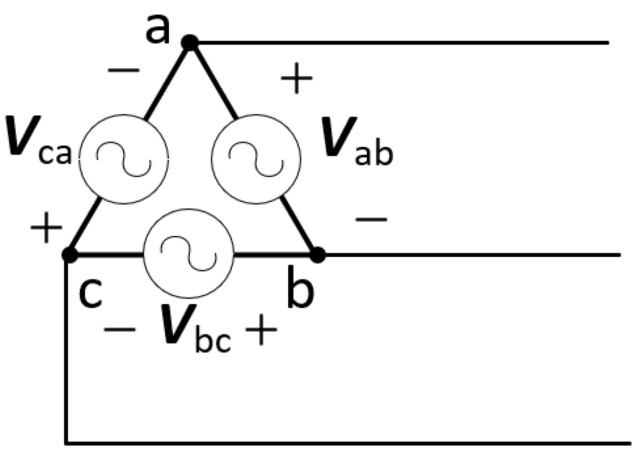
Three-Phase Voltage Observation

Voltages (line or phase) sum to zero in Wye and Delta connections

$$\boldsymbol{V}_{aa'} + \boldsymbol{V}_{bb'} + \boldsymbol{V}_{cc'} = 0$$
$$\boldsymbol{V}_{an} + \boldsymbol{V}_{bn} + \boldsymbol{V}_{cn} = 0$$
$$\boldsymbol{V}_{ab} + \boldsymbol{V}_{bc} + \boldsymbol{V}_{ca} = 0$$

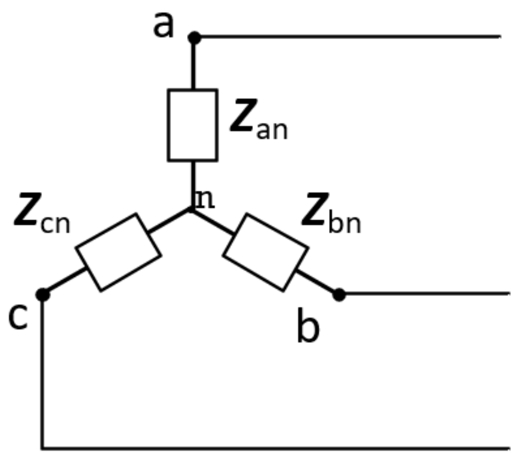




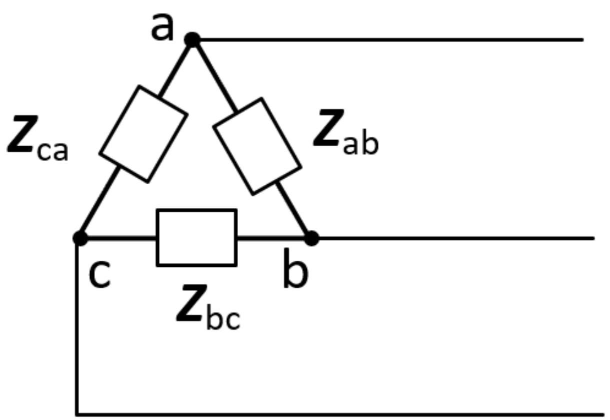


Three-Phase Loads

- Three-phase sources are connected to three-phase loads in two common configurations
  - Y (wye)
  - Delta



Wye



Delta

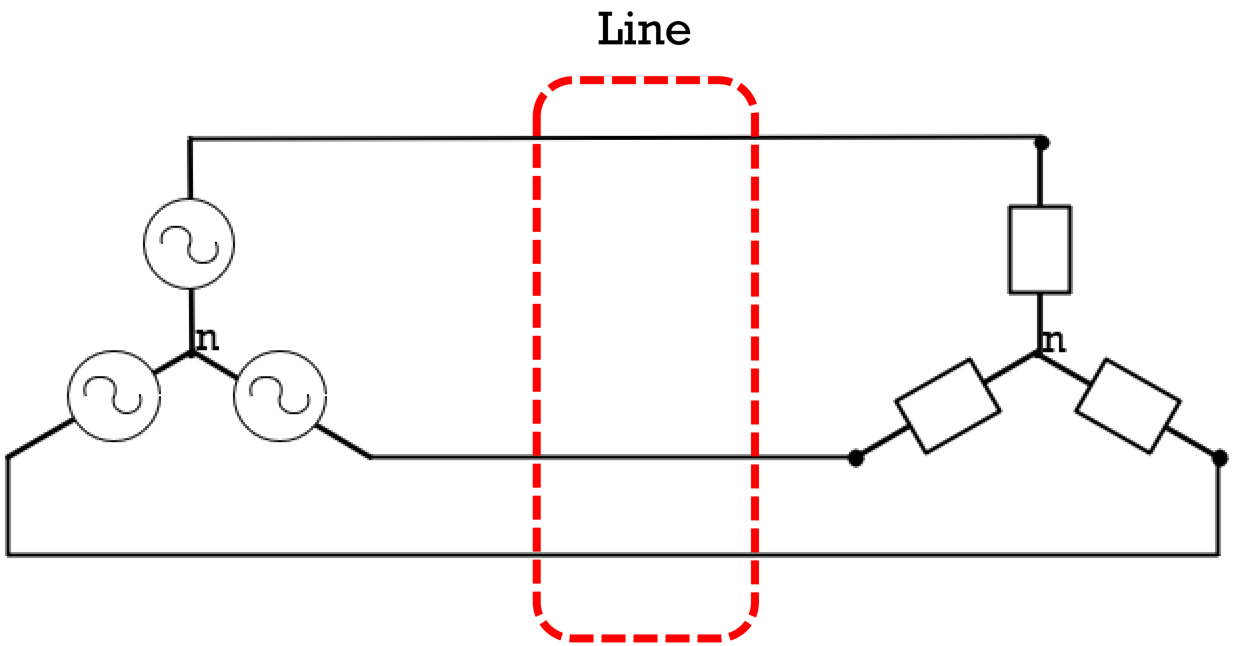
Three-Phase Loads

- Y sources can be connected to delta and/or Y loads
- Delta sources can be connected to delta and/or Y loads

wye source connected to a wye load

Three-Phase Loads

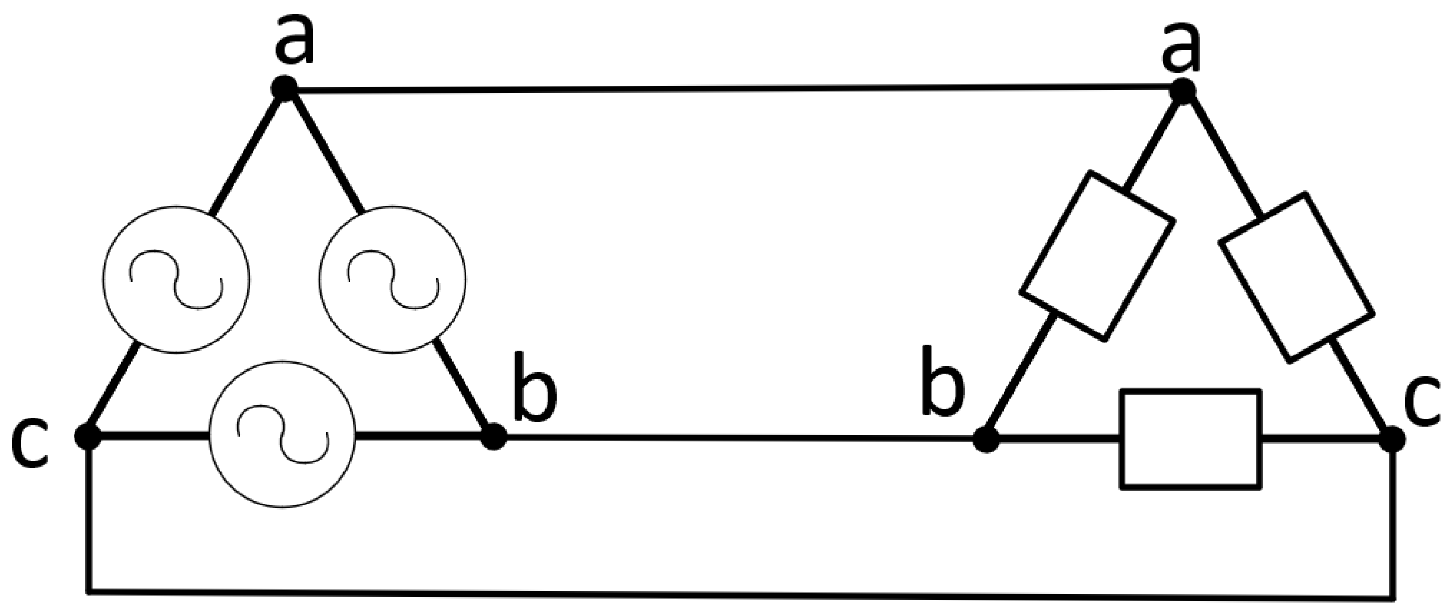
The conductor connecting the source to the load is referred to as a "line" (as in "power line", "distribution line"), "cable", or "conductor"



Three-Phase Loads

wye source connected to a delta load

Three-Phase Loads



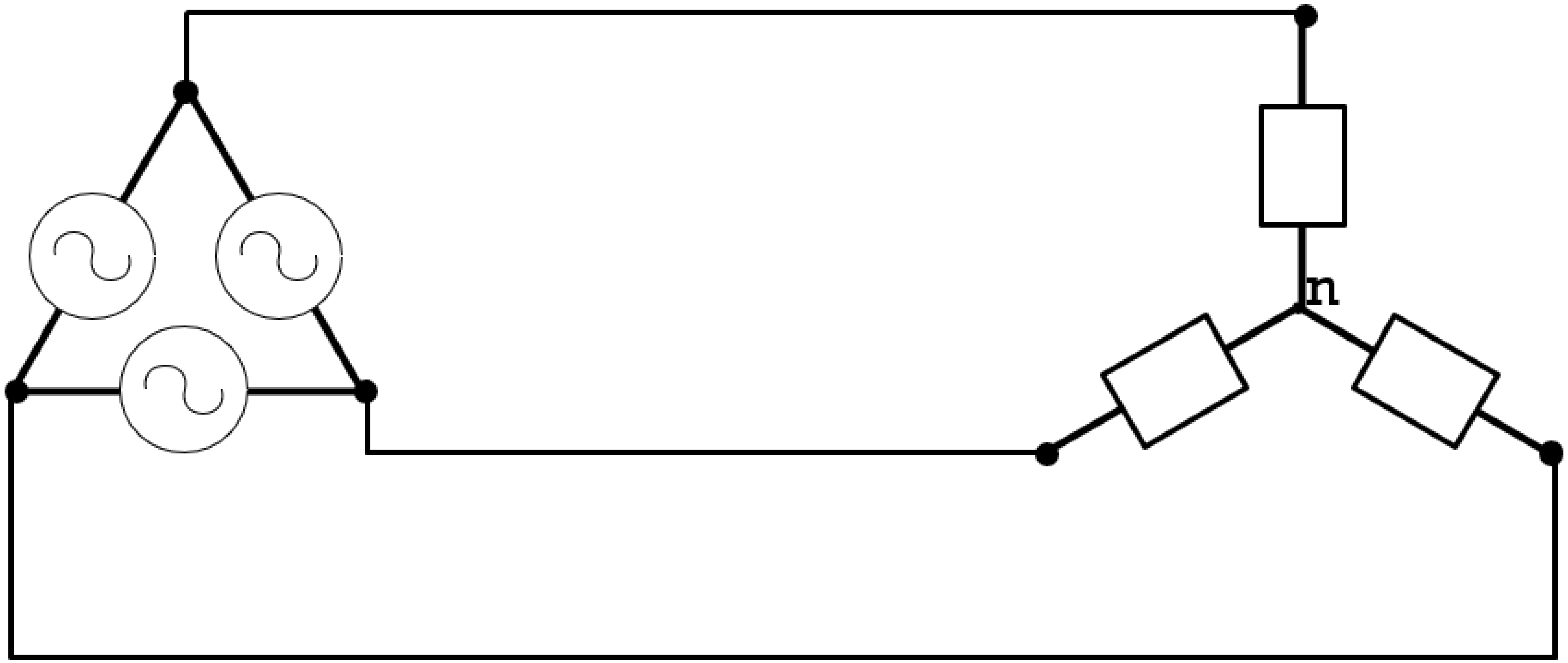
delta source connected to a delta load

Exercise

Draw a delta source connected to a wye load

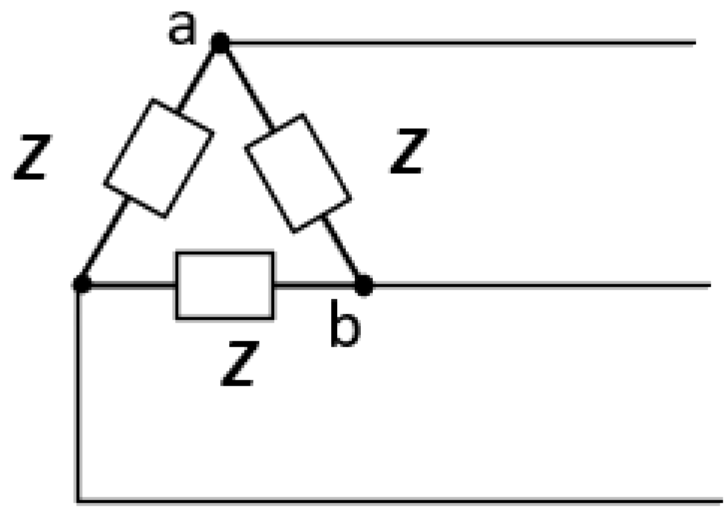
Exercise

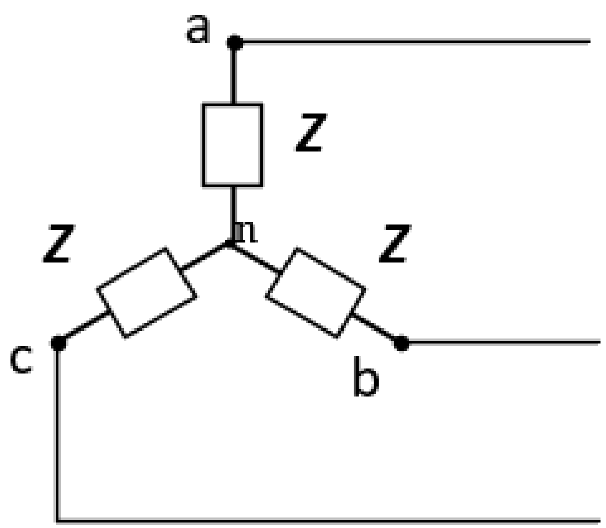
Draw a delta source connected to a wye load



Balanced Three-Phase Load

- In balanced three-phase systems, each impedance is equal
- We remove the subscripts for clarity

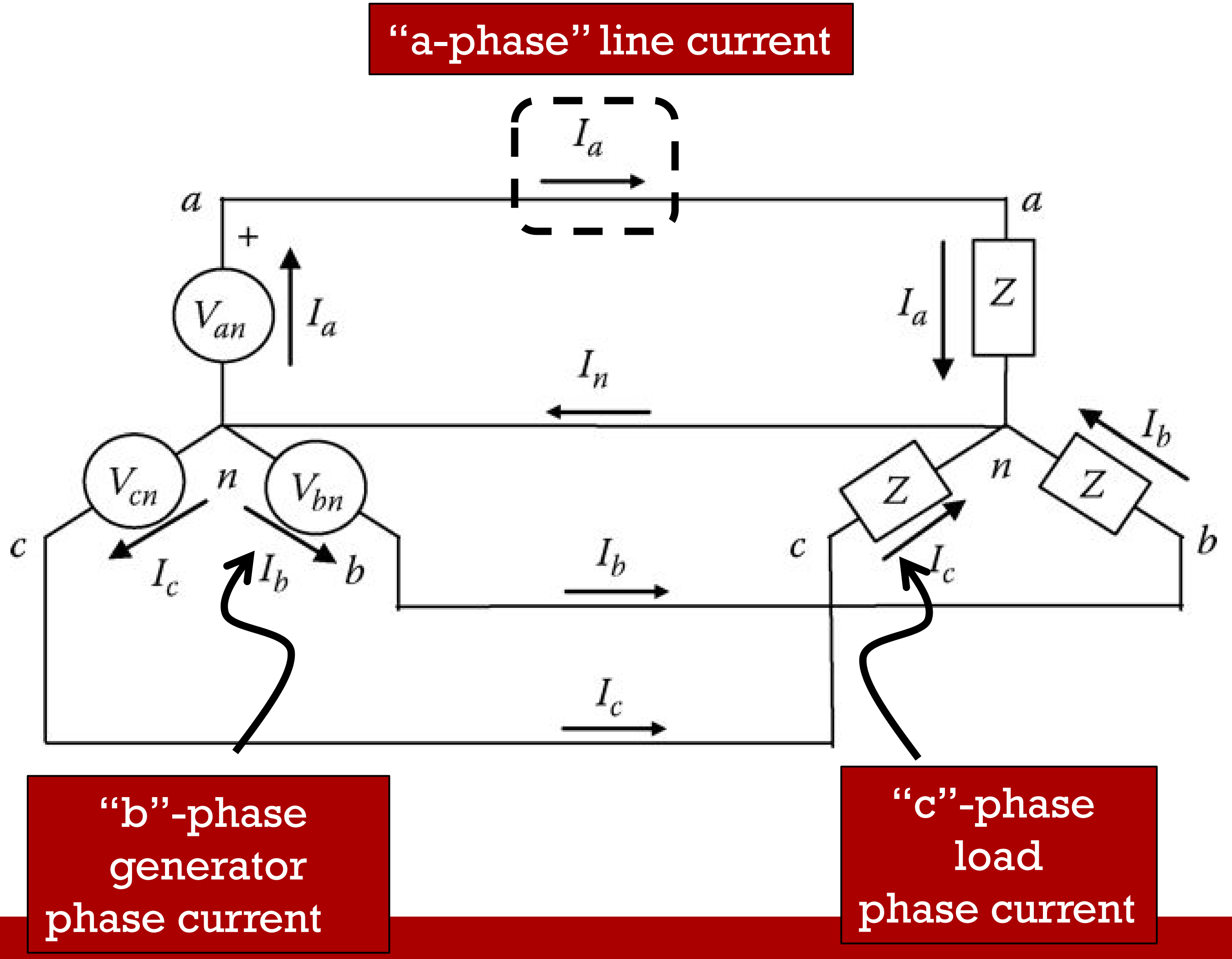




Three-Phase Current

- Current in a line is known as the "line current"
- Current through an element (load or source) is known as the "phase" current
- In Wye systems, <u>line current is the same as phase current</u>
- In Delta systems, <u>line current is NOT the same as phase current</u>

Three-Phase Current in Wye Systems



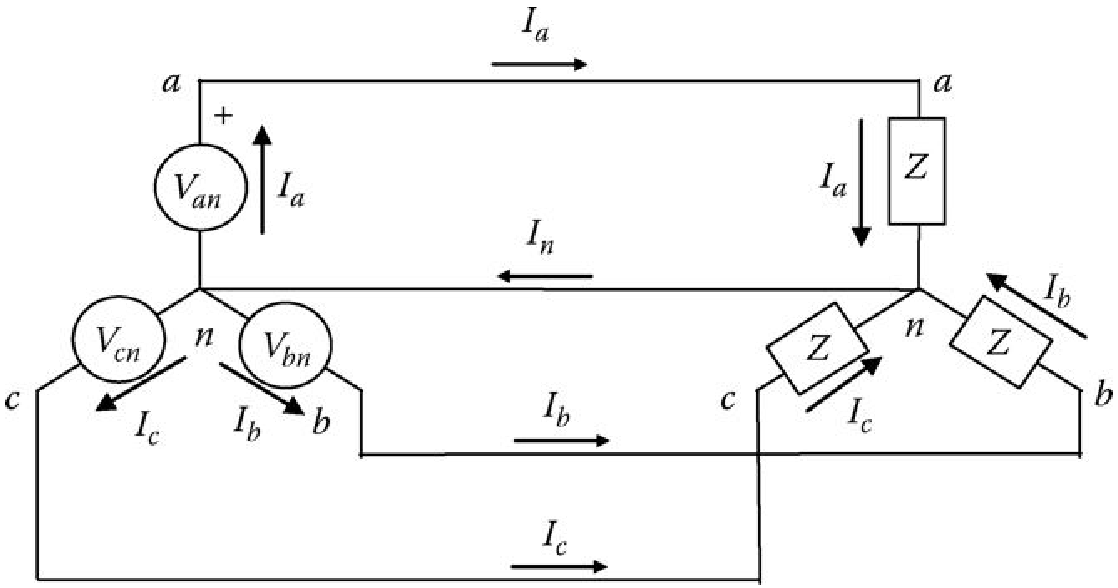
Three-Phase Current in Wye Systems

Line (and phase) current can be computed as:

$$\boldsymbol{I}_{\mathrm{a}} = \frac{\boldsymbol{V}_{\mathrm{an}}}{\boldsymbol{Z}_{\mathrm{an}}} = \frac{\boldsymbol{V}_{\mathrm{an}}}{\boldsymbol{Z}} = \frac{V_{\mathrm{ph}}\angle\theta}{Z\angle\varphi} = \frac{V_{\mathrm{ph}}}{Z}\angle(\theta-\varphi)$$

$$\boldsymbol{I}_{\mathrm{b}} = \frac{\boldsymbol{V}_{\mathrm{bn}}}{\boldsymbol{Z}_{\mathrm{bn}}} = \frac{\boldsymbol{V}_{\mathrm{bn}}}{\boldsymbol{Z}} = \frac{V_{\mathrm{ph}}\angle\theta-120^{\circ}}{Z\angle\varphi} = \frac{V_{\mathrm{ph}}}{Z}\angle\left(\theta-\varphi-120^{\circ}\right)$$

$$\boldsymbol{I}_{\mathrm{c}} = \frac{\boldsymbol{V}_{\mathrm{cn}}}{\boldsymbol{Z}_{\mathrm{cn}}} = \frac{\boldsymbol{V}_{\mathrm{cn}}}{\boldsymbol{Z}} = \frac{V_{\mathrm{ph}}\angle\theta+120^{\circ}}{Z\angle\varphi} = \frac{V_{\mathrm{ph}}}{Z}\angle\left(\theta-\varphi+120^{\circ}\right)$$



Three-Phase Current in Wye Systems

Observations:

1. Line current is equal to the phase current
2. Magnitudes of $\boldsymbol{I}_\mathrm{a}$, $\boldsymbol{I}_\mathrm{b}$, and $\boldsymbol{I}_\mathrm{c}$ are equal
3. Current phases are separated by 120°

$$\boldsymbol{I}_\mathrm{a} = \frac{V_\mathrm{ph}}{Z}\angle(\theta-\varphi)$$

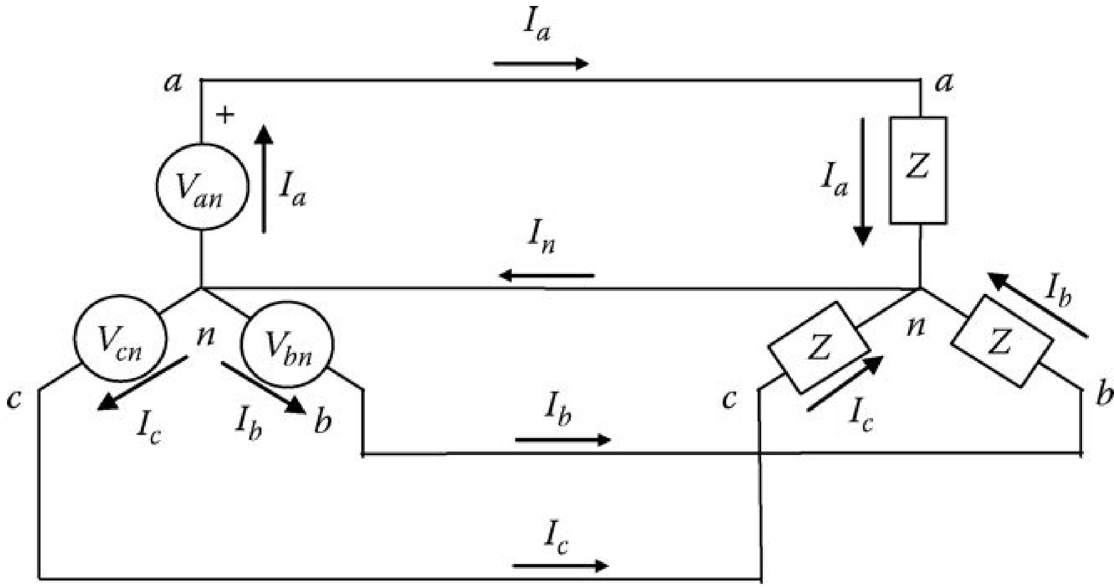
$$\boldsymbol{I}_\mathrm{b} = \frac{V_\mathrm{ph}}{Z}\angle\left(\theta-\varphi-120^\circ\right)$$

$$\boldsymbol{I}_\mathrm{c} = \frac{V_\mathrm{ph}}{Z}\angle\left(\theta-\varphi+120^\circ\right)$$

Neutral Current

- Given the previous observations, we can show that:

$$\boldsymbol{I}_{\mathrm{n}} = \boldsymbol{I}_{\mathrm{a}} + \boldsymbol{I}_{\mathrm{b}} + \boldsymbol{I}_{\mathrm{c}} = 0$$



- In balanced three-phase systems, there is no current in the neutral line!
- Neutral conductor can be removed (or added) without affecting the operation of the circuit

Wye Systems without Neutral Conductor

- Consider the Y-connected source and load
- Determine $\boldsymbol{V}_{n'n}$ (the voltage between the neutral points)

Three-Phase Analysis

- Analysis is easier using admittance, ***Y***

$$\boldsymbol{Y} = \frac{1}{\boldsymbol{Z}}$$

- Line current is equal to phase current

$$\left.\begin{aligned} \boldsymbol{I}_{\mathrm{a}} &= \boldsymbol{Y}(\boldsymbol{V}_{\mathrm{an}} - \boldsymbol{V}_{\mathrm{n'n}}) \\ \boldsymbol{I}_{\mathrm{b}} &= \boldsymbol{Y}(\boldsymbol{V}_{\mathrm{bn}} - \boldsymbol{V}_{\mathrm{n'n}}) \\ \boldsymbol{I}_{\mathrm{c}} &= \boldsymbol{Y}(\boldsymbol{V}_{\mathrm{cn}} - \boldsymbol{V}_{\mathrm{n'n}}) \end{aligned}\right\} \text{ due to wye connection}$$

Wye Systems without Neutral Conductor

- Summing the line current

$$\boldsymbol{I}_{\mathrm{a}} + \boldsymbol{I}_{\mathrm{b}} + \boldsymbol{I}_{\mathrm{c}} = \boldsymbol{Y}(\boldsymbol{V}_{\mathrm{an}} + \boldsymbol{V}_{\mathrm{bn}} + \boldsymbol{V}_{\mathrm{cn}}) - 3\boldsymbol{Y}\boldsymbol{V}_{\mathrm{nn'}}$$

- Previously we showed that

$$\boldsymbol{I}_{\mathrm{a}} + \boldsymbol{I}_{\mathrm{b}} + \boldsymbol{I}_{\mathrm{c}} = 0$$

$$\boldsymbol{V}_{\mathrm{an}} + \boldsymbol{V}_{\mathrm{bn}} + \boldsymbol{V}_{\mathrm{cn}} = 0$$

- Therefore

$$0 = \boldsymbol{Y}(0) - 3\boldsymbol{Y}\boldsymbol{V}_{\mathrm{nn'}}$$

$$\Rightarrow \boldsymbol{V}_{\mathrm{nn'}} = 0$$

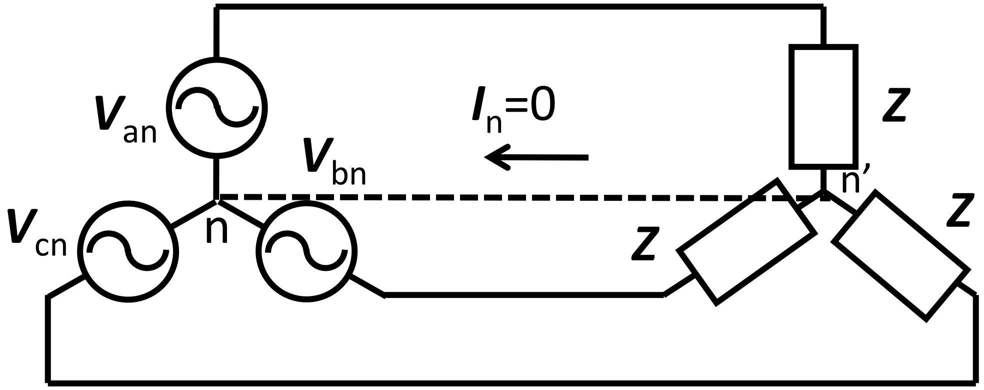
Wye Systems without Neutral Conductor

- Since $\boldsymbol{V}_{\mathrm{nn'}} = 0$, we can make a hypothetical connection without affecting the circuit
- The previous results still apply even in Wye-systems without neutral conductors

$$\boldsymbol{I}_{\mathrm{a}} = \frac{V_{\mathrm{ph}}}{Z} \angle(\theta - \varphi)$$

$$\boldsymbol{I}_{\mathrm{b}} = \frac{V_{\mathrm{ph}}}{Z} \angle(\theta - \varphi - 120^\circ)$$

$$\boldsymbol{I}_{\mathrm{c}} = \frac{V_{\mathrm{ph}}}{Z} \angle(\theta - \varphi + 120^\circ)$$



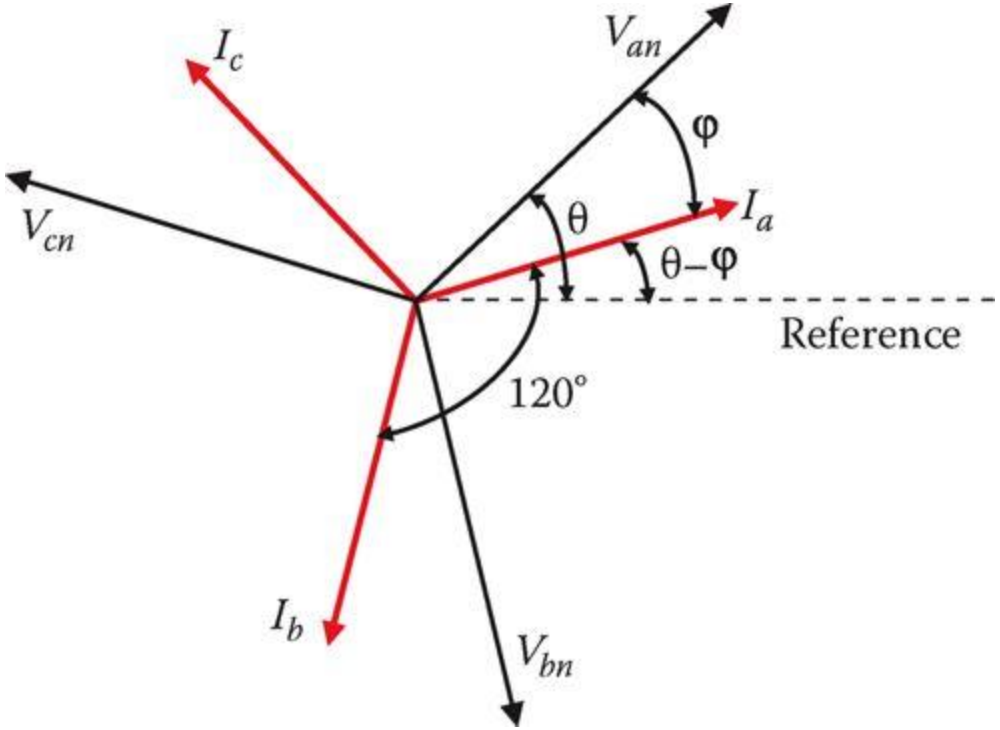
Summary: Wye Loads

1. The magnitude of the line-to-line voltage, V_{ll}, is greater than the phase voltage, V_{ph}, by a factor of √3.
2. The line-to-line voltage $\boldsymbol{V}_{ab}$ leads its phase voltage $\boldsymbol{V}_{an}$ by 30 degrees. Similar conclusions can be made for the other two phases.
3. Line currents are equal to the corresponding (a, b, c) phase currents through the load.
4. No current flows through the neutral wire, so there is no need for a wire between the neutral points; and a wire between the neutral can be added without affecting the system

Per-Phase Analysis

- Since the voltages and currents occur in three balanced sets (a, b, c), we only need to solve for one phase and then shift the result by -/+120 degrees to solve for the other phases
- <u>Phases can be conceptually decoupled</u>
- No need to solve all three phases
  - Solve for a-phase (current or voltage)
  - Shift +120° for c-phase, and -120° for b-phase
- We can therefore do a <u>per-phase analysis</u>

Example Phasor Diagram



Three-Phase Analysis

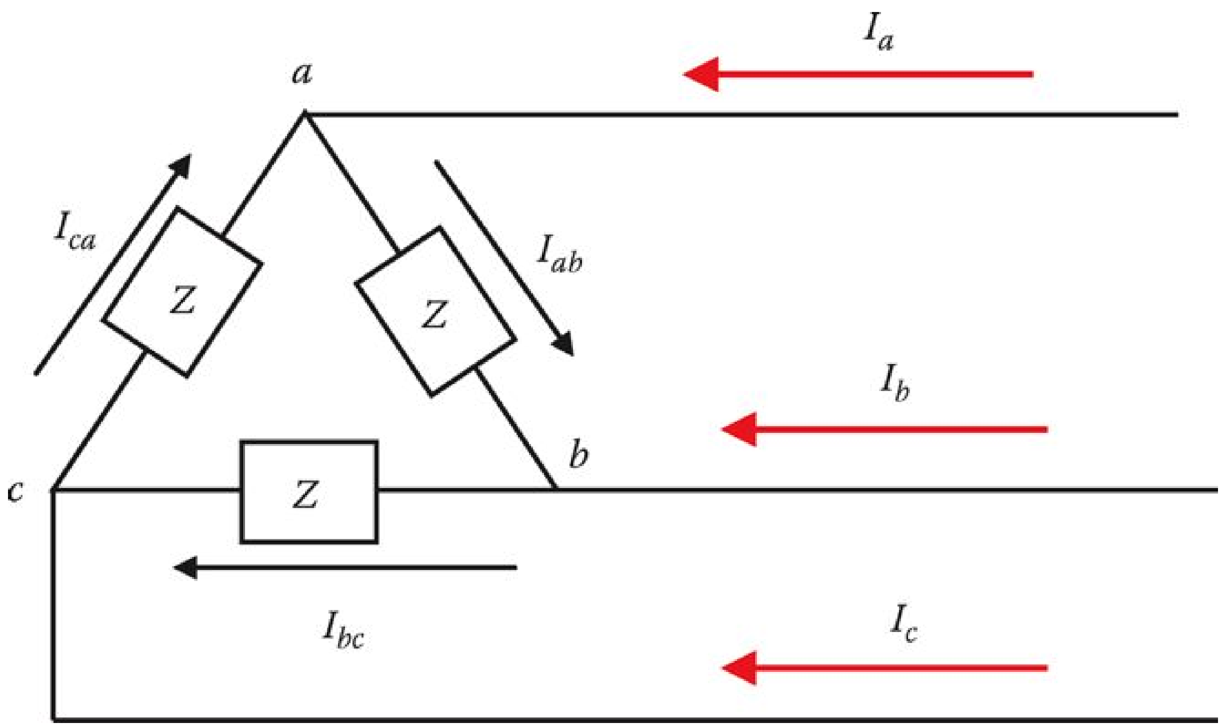
- Balanced Three-Phase Theorem
- Assume:
  - balanced three-phase system
  - all loads and sources are Y-connected (or convert them to Y-connected loads/sources)
  - no mutual inductances between phases

then

  - all neutrals have the same voltage
  - the phases are completely decoupled
  - all corresponding network variables occur in balanced sets of the same sequence as the sources

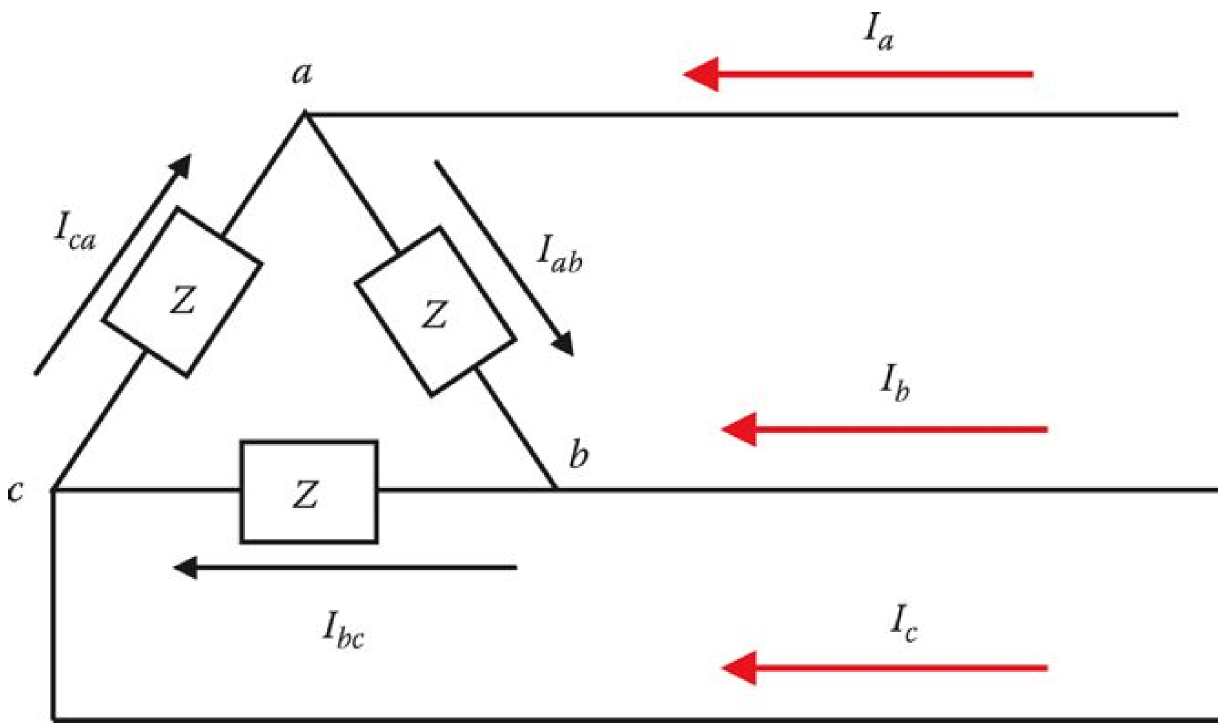
Delta Load

- No neutral point
- All loads have same impedance (balanced)
- Line-line voltage is applied to each load



Delta Load

- Line currents: $\boldsymbol{I}_a$, $\boldsymbol{I}_b$, $\boldsymbol{I}_c$
- Phase currents: $\boldsymbol{I}_{ab}$, $\boldsymbol{I}_{bc}$, $\boldsymbol{I}_{ca}$
- Line currents are NOT the same as phase currents (unlike in wye-loads)

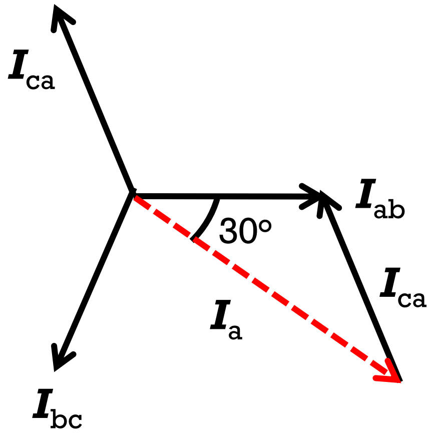


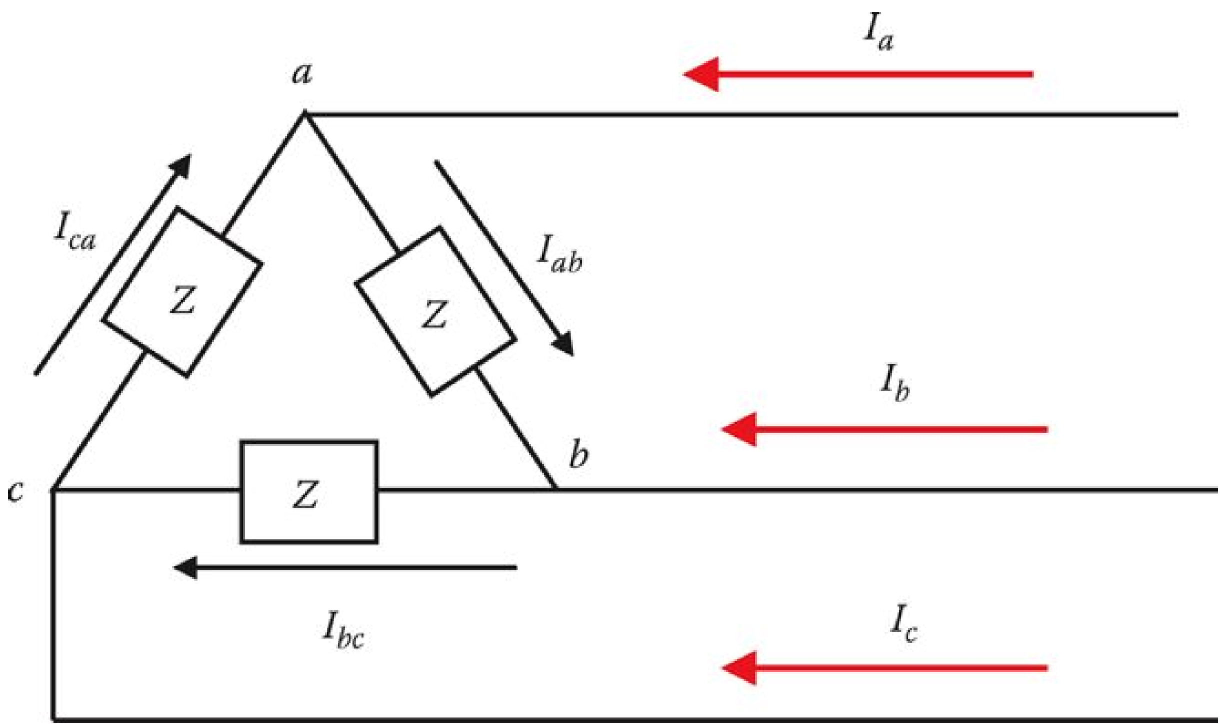
Dr. Henry Louie

Phase Currents in Delta Loads

Apply KCL to each node:

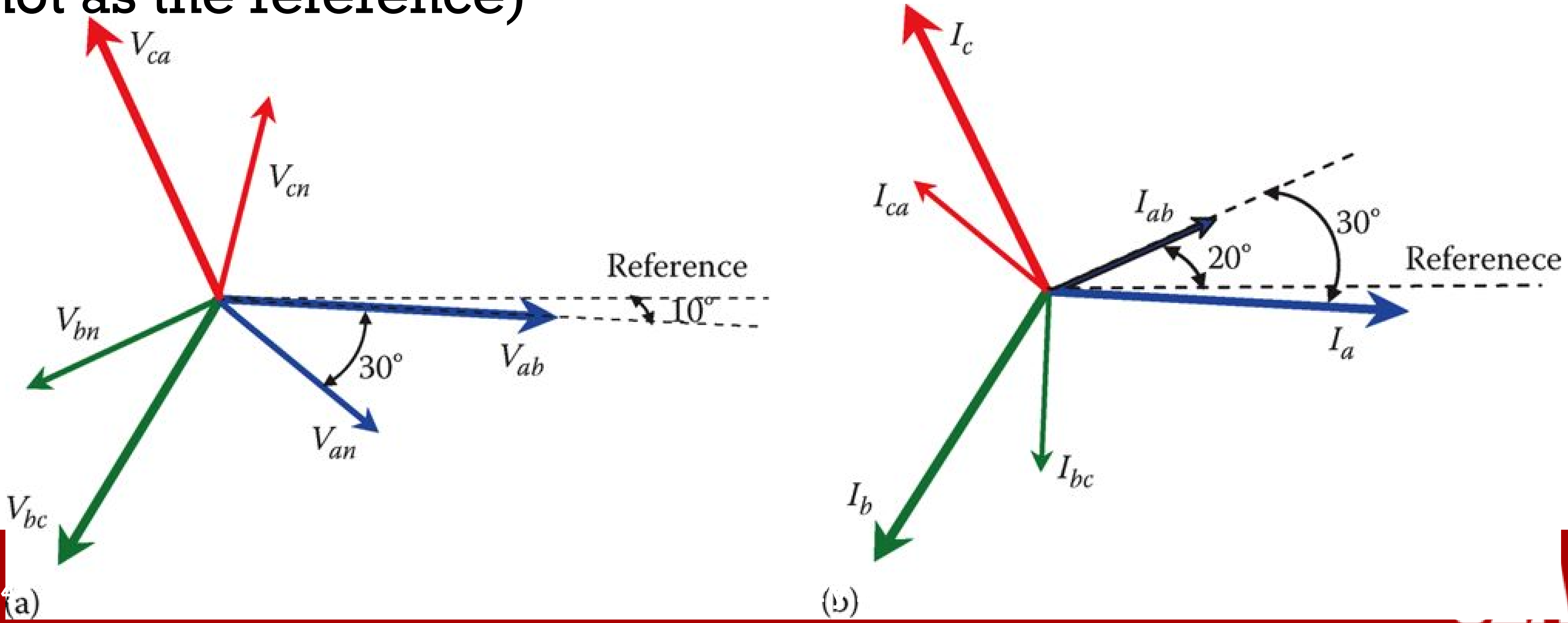
$$\boldsymbol{I}_{\text{a}} = \boldsymbol{I}_{\text{ab}} - \boldsymbol{I}_{\text{ca}} = \boldsymbol{I}_{\text{ab}}(\sqrt{3}\angle -30^{\circ})$$
$$\boldsymbol{I}_{\text{b}} = \boldsymbol{I}_{\text{bc}} - \boldsymbol{I}_{\text{ab}} = \boldsymbol{I}_{\text{bc}}(\sqrt{3}\angle -30^{\circ})$$
$$\boldsymbol{I}_{\text{c}} = \boldsymbol{I}_{\text{ca}} - \boldsymbol{I}_{\text{bc}} = \boldsymbol{I}_{\text{ca}}(\sqrt{3}\angle -30^{\circ})$$





Example Phasor Diagram

Phasor diagram of a Wye-source connected to a delta load whose impedance is -30 degrees ($\boldsymbol{V}_{an}$ is shown as -40 degrees, not as the reference)



Three Phase Loads

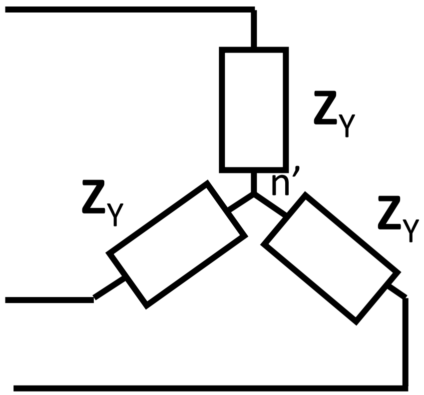
- Circuit analysis is easier if loads are connected as Y
- We can transform balanced Delta connected loads into balanced Y connected loads mathematically by

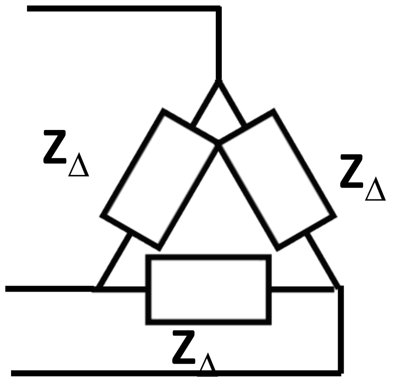
$$\mathbf{Z}_Y = \frac{\mathbf{Z}_\Delta}{3}$$

  - $\mathbf{Z}_Y$: complex impedance of Y-connected load (Ohms)
  - $\mathbf{Z}_\Delta$: complex impedance of a Delta-connected load (Ohms)
- Results only apply to terminal conditions

Exercise

Each phase of a Y-connected load has an impedance of 6 + j12. Find the impedance of the equivalent delta-connected load.

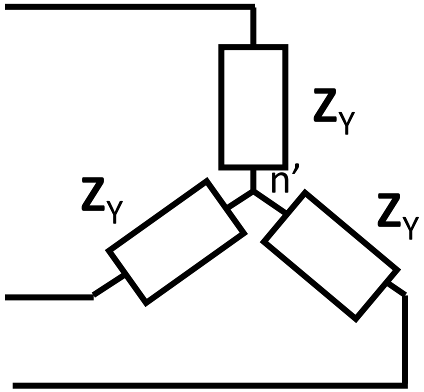


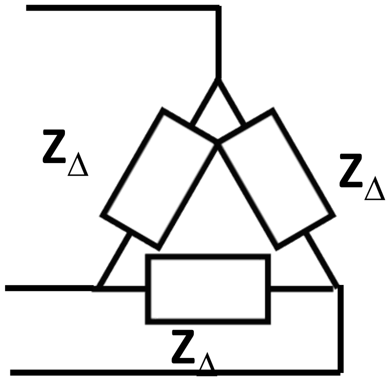


Exercise

Each phase of a Y-connected load has an impedance of 6 + j12. Find the impedance of the equivalent delta-connected load.

Answer: $Z_\Delta = 18 + j36\Omega$





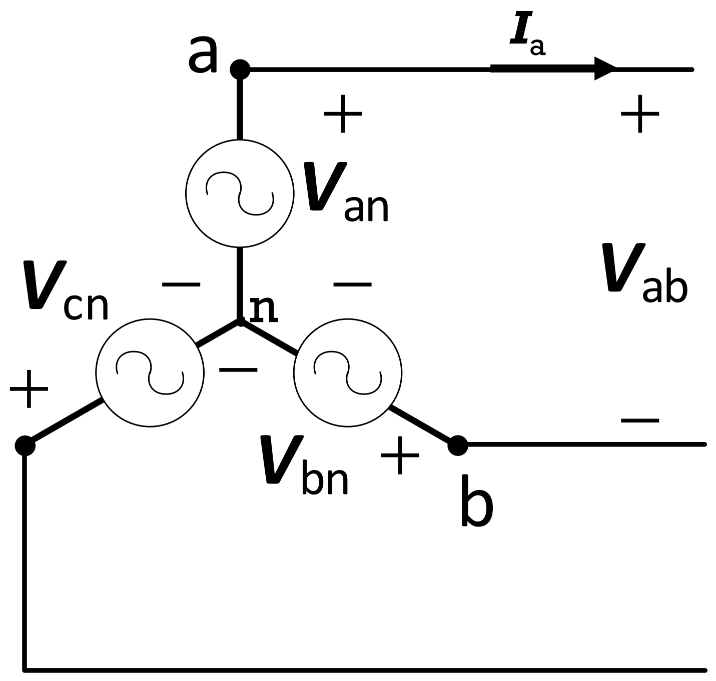
Wye Summary

Line Current = $\boldsymbol{I}_{\mathrm{a}}$

Phase Current = $\boldsymbol{I}_{\mathrm{a}}$

Line Voltage ($\boldsymbol{V}_{\mathrm{ab}}$)= $\boldsymbol{V}_{\mathrm{an}}\ \sqrt{3}\angle 30°$

Phase Voltage = $\boldsymbol{V}_{\mathrm{an}}$



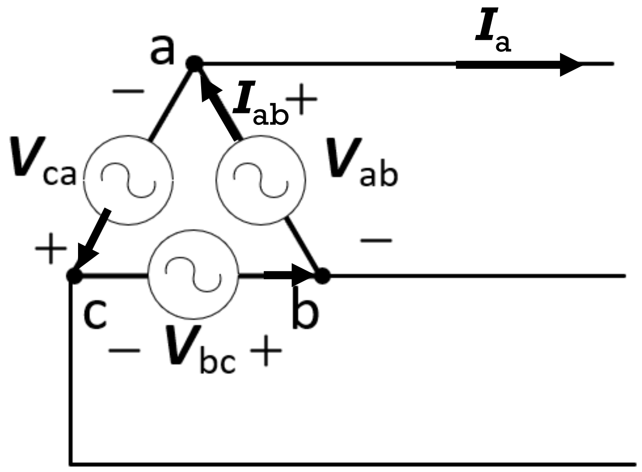
Delta Summary

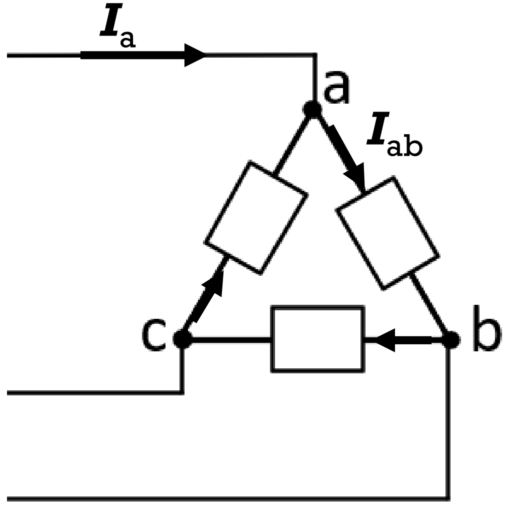
Line Current $\boldsymbol{I}_{\mathrm{a}} = \boldsymbol{I}_{\mathrm{ab}}\sqrt{3}\angle{-30^\circ}$

Phase Current = $\boldsymbol{I}_{\mathrm{ab}}$

Line Voltage = $\boldsymbol{V}_{\mathrm{ab}}$

Phase Voltage = $\boldsymbol{V}_{\mathrm{ab}}$





Summary

- Three phase systems: more efficient use of conductors; provides rotating magnetic fields
- Balanced three phase: a,b,c (voltage, current) phases displaced by 120 degrees and have equal magnitude
- Per phase analysis is useful in analyzing three-phase circuits