# 06-Solar Resource Part 3 

ECEGR 4530

Renewable Energy Systems

## Overview

- Effect of the Atmosphere
- Clearness Index
- Irradiation
- Irradiance Algorithm
- Air Mass Ratio


## What Influences Angle of Incidence?

- Declination ( $\delta$ )
- Latitude ( $\phi$ )
- Tilt ( $\beta$ )
- Time of day (hour angle) ( $\omega$ )


## What Influences Angle of Incidence?

- See Lecture 06-Solar Resource Part 2 for derivation


## What Influences Angle of Incidence?

Extraterrestrial irradiance accounting for the tilt, latitude and declination of a surface at solar noon:

$$
\begin{aligned}
& \mathbf{G}_{0 \mathrm{~T}}=\mathbf{G}_{0 \mathrm{n}} \cos (\theta)= \mathbf{G}_{0 \mathrm{n}} \cos (\phi-\delta-\beta) \\
&=\mathbf{G}_{0 \mathrm{n}}[\cos (\phi) \cos (\delta) \cos (\beta) \\
& \quad-\cos (\phi) \sin (\delta) \sin (\beta) \\
&+\sin (\phi) \sin (\beta) \cos (\delta) \\
&+\sin (\phi) \cos (\beta) \sin (\delta)] \quad \text { Important result }
\end{aligned}
$$

## What Influences Angle of Incidence?

- Now also accounting for time of day
- $\cos (\theta)=\sin (\delta) \sin (\phi) \cos (\beta)$

```
-sin(\delta)}\operatorname{cos}(\phi)\operatorname{sin}(\beta
    +\operatorname{cos}(\delta)\operatorname{cos}(\phi)\operatorname{cos}(\beta)\operatorname{cos}(\omega)
    +\operatorname{cos}(\delta)\operatorname{sin}(\phi)\operatorname{sin}(\beta)\operatorname{cos}(\omega)\quad\mathrm{ Important result}
```


## Simplifications

- If $\beta=0$ (no tilt), then $\theta_{z}=\theta$ and
- $\cos (\theta)=\sin (\delta) \sin (\phi)+\cos (\delta) \cos (\phi) \cos (\omega)$
- For surfaces tilted at their latitude
- $\cos (\theta)=\cos (\delta) \cos (\omega)$
- For surfaces at solar noon
- $\cos (\theta)=\cos (\phi-\delta-\beta)$


## What Influences Angle of Incidence?

- Try to maximize $\cos (\theta)$

$$
\begin{aligned}
\mathbf{G}_{0 \mathrm{~T}}=\mathbf{G}_{0 \mathrm{n}} \cos (\theta)= & \mathbf{G}_{0 \mathrm{n}} \cos (\phi-\delta-\beta) \\
= & \mathbf{G}_{0 \mathrm{n}}[\cos (\phi) \cos (\delta) \cos (\beta) \\
& -\cos (\phi) \sin (\delta) \sin (\beta) \\
& +\sin (\phi) \sin (\beta) \cos (\delta) \\
& +\sin (\phi) \cos (\beta) \sin (\delta)]
\end{aligned}
$$

Adjust tilt to minimize $\phi-\delta-\beta$

## What Influences Angle of Incidence?

- Try to maximize $\cos (\theta)$

```
\operatorname{cos}(0)=\operatorname{sin}(\delta)\operatorname{sin}(\phi)\operatorname{cos}(\beta)
    -sin(\delta)\operatorname{cos}(\phi)\operatorname{sin}(\beta)
    +\operatorname{cos(\delta)}\operatorname{cos}(\phi)\operatorname{cos}(\beta)\operatorname{cos}(\omega)
    +\operatorname{cos}(\delta)\operatorname{sin}(\phi)\operatorname{sin}(\beta)\operatorname{cos}(\omega)
```

Dependence on $\cos (\omega)$ means that we should try to track sun east-west to minimize $\omega$

## Introduction

- Previous lectures have focused on extraterrestrial irradiance
- Now we examine irradiance on Earth's surface
- Resolve GHI into beam and diffuse components
- Irradiance on a surface has 3 components:
- A: Direct
- B: Diffuse (from the sky)
- C: Reflected (reflected from the ground)
- $G_{T}=A+B+C$
- Empirical formulae


## Components of Irradiance

reflected

## Atmospheric Effects

- Atmosphere: reflects, absorbs and scatters solar radiation
- Net result: reduction in irradiance at the Earth's surface and a non-uniform attenuation of the energy density spectrum
- On average, 30\% of incident solar irradiance is reflected back into space


## Atmospheric Effects

- Recall that the atmosphere also causes diffusion of irradiance
- We have neglected $\mathbf{G}_{\mathrm{d}}$ in all previous derivations
- $\mathrm{G}_{\mathrm{d}}$ is difficult to compute, due to the complex geometry of clouds, etc
- Use empirical formula to determine the ratio of $\mathbf{G}_{\mathrm{d}}$ and $\mathbf{G}_{\mathrm{b}}$ for a given $\mathbf{G H I}$ and $\mathbf{G}_{0}$


## Clearness Index

- Basic idea: use ratio of irradiance on surface to irradiance at the top of the atmosphere as a proxy for how cloudy it is.
- Clouds imply higher $\mathbf{G}_{\mathrm{d}}$ component of $\mathbf{G}_{\mathrm{GHI}}$
- clear day: $\mathbf{G}_{\mathrm{GHI}} / \mathbf{G}_{0}$ is closer to 1
- Smaller $\mathbf{G}_{\mathrm{d}}$ component of $\mathbf{G}_{\mathrm{GHI}}$
- cloudy day: $\mathrm{G}_{\mathrm{GHI}} / \mathrm{G}_{0}$ is closer to 0
- Larger $\mathbf{G}_{\mathrm{d}}$ component of $\mathbf{G}_{\mathrm{GHI}}$


## Clearness Index

- Clearness Index $k_{t}$ : ratio of global irradiance received on a horizontal surface to the extraterrestrial irradiance ( $\mathrm{G}_{\mathrm{GHI}} / \mathrm{G}_{0}$ )
- Recall: $G_{o}$ is the irradiance on a surface (with the same angle of incidence as the horizontal surface) without the atmosphere accounted for (extraterrestrial)
- Average $k_{\mathrm{t}}$ values are available for many locations
- Usually, clearness index uses the ratio of radiations over an hour, day or month


## Computing Diffuse Irradiance

- Go can be computed (see previous lectures)
- Need to know either $G_{G H I}$ or $k_{t}$
- A simple empirical formula is
- $\mathrm{G}_{\mathrm{d}} / \mathrm{G}_{\mathrm{GHI}}=1-1.13 \mathrm{k}_{\mathrm{t}}$
- Reasonably valid for $0.3<k_{\mathrm{t}}<0.8$
- $\mathbf{G}_{\mathrm{b}}$ can then be computed from
- $G_{b}=G_{G H I}-G_{d}$


## Computing Beam Irradiance

- Find $\mathbf{G}_{\mathrm{b}}$ and $\mathbf{G}_{\mathrm{d}}$ if $\mathbf{G}_{\mathrm{GHI}}=800 \mathrm{~W} / \mathrm{m}^{2}$ and $\mathbf{G}_{\mathrm{o}}=1350 \mathrm{~W} / \mathrm{m}^{2}$


## Computing Beam Irradiance

- Find $\mathbf{G}_{\mathrm{b}}, \mathbf{G}_{\mathrm{d}}$ if $\mathbf{G}_{\mathrm{GHI}}=800 \mathrm{~W} / \mathrm{m}^{2}$ and $\mathrm{G}_{\mathrm{o}}=1350$ $\mathrm{W} / \mathrm{m}^{2}$
- $\mathrm{k}_{\mathrm{t}}=800 / 1350=0.593$
- $\mathrm{G}_{\mathrm{d}} / \mathrm{G}_{\mathrm{GHI}}=1-1.13 \mathrm{k}_{\mathrm{t}}$
- $\mathbf{G}_{\mathrm{d}}=\left(1-1.13 \mathrm{k}_{\mathrm{t}}\right) \mathbf{G}_{\mathrm{GHI}}=(0.3304) \mathbf{G}_{\mathrm{GHI}}=264 \mathrm{~W} / \mathrm{m}^{2}$
- $\mathrm{G}_{\mathrm{b}}=800-264=536 \mathrm{~W} / \mathrm{m}^{2}$


## Computing Beam Irradiance

- Given $\mathrm{G}_{\mathrm{b}}$ how do we find beam irradiance for a tilted surface, $\mathbf{G}_{\mathrm{b} T}$ ?
- remember, $\mathrm{G}_{\mathrm{b}}$ is for a horizontal surface
- $\mathbf{G}_{\mathrm{b}}=\mathrm{Gcos}_{\mathrm{z}}$

Recall for horizontal surfaces, $\theta=\theta_{z}$


## Computing Beam Irradiance

- $\mathrm{G}_{\mathrm{bT}}=\mathrm{G} \cos \theta$
- $\mathrm{G}_{\mathrm{b}}=\mathrm{G} \cos \theta_{\mathrm{z}}$
- $\mathbf{G}=\mathbf{G}_{\mathrm{b}}\left(1 / \cos \theta_{\mathrm{Z}}\right)$

Here we know $\mathrm{G}_{\mathrm{b}}$ and can compute $\theta$ and $\theta_{z}$, but we don't know $G$, so we need to eliminate it

- Then:
- $\mathrm{G}_{\mathrm{bT}}=\mathrm{G}_{\mathrm{b}} \mathrm{R}_{\mathrm{b}}$
- Where
- $\mathrm{R}_{\mathrm{b}}=\cos \theta / \cos \theta_{\mathrm{z}}$



## Computing Beam Irradiance

- We now have solved for $A$
- $\mathrm{G}_{\mathrm{T}}=\mathrm{A}+\mathrm{B}+\mathrm{C}$
- $\mathrm{A}=\mathrm{G}_{\mathrm{b}} \mathrm{R}_{\mathrm{b}}$


## Exercise

- Compute the beam irradiance on a surface tilted at $15^{\circ}$ at $30^{\circ}$ N on April $15^{\text {th }}$, with a clearness index of 0.75 at solar noon.


## Exercise

- Compute the beam irradiance on a surface tilted at $15^{\circ}$ at $30^{\circ}$ N on April $15^{\text {th }}$, with a clearness index of 0.75 at solar noon.
- $\mathrm{G}_{\mathrm{d}} / \mathrm{G}_{\mathrm{GHI}}=1-1.13 \mathrm{k}_{\mathrm{t}}$
- We have $k_{t}$ and can compute $G_{0}$ to find $G_{G H I}$ using $k_{t}=$ $\mathbf{G}_{\mathrm{GHI}} / \mathbf{G}_{0}$


## Exercise

- Compute the beam irradiance on a surface tilted at $15^{\circ}$ at $30^{\circ}$ N on April $15^{\text {th }}$, with a clearness index of 0.75 at solar noon.

$$
\phi=30^{\circ}
$$

$$
\beta=15^{\circ}
$$

$$
\omega=0^{\circ}
$$

$$
\begin{aligned}
& \delta=\delta_{0} \sin \left(\frac{360^{\circ}(284+d)}{365}\right)=23.5^{\circ} \sin \left(\frac{360^{\circ}(284+105)}{365}\right)=9.4^{\circ} \\
& G_{\text {on }}(d)=G_{s c}\left[1+0.033 \cos \left(2 \pi\left(\frac{105}{365}\right)\right)\right]=1356 \mathrm{~W} / \mathrm{m}^{2}
\end{aligned}
$$

## Exercise

- Now that we have the angles, we can compute $G_{o}=G_{o n} \cos \theta_{z}$
- Solving for the cosine of the zenith angle ( $\beta=0$ for zenith angle)

```
\operatorname{cos}(\mp@subsup{0}{z}{})=\operatorname{sin}(\delta)\operatorname{sin}(\phi)+\operatorname{cos}(\delta)\operatorname{cos}(\phi)\operatorname{cos}(\omega)
    = sin(9.4})\operatorname{sin}(3\mp@subsup{0}{}{\circ})+\operatorname{cos}(9.\mp@subsup{4}{}{\circ})\operatorname{cos}(3\mp@subsup{0}{}{\circ})\operatorname{cos}(\mp@subsup{0}{}{\circ}
    = 0.936 [or using cos( }\mp@subsup{0}{z}{})=\operatorname{cos}(\phi-\delta)
```

- Therefore, for a horizontal surface at the top of the atmosphere: $G_{o}=G_{o n} \cos \theta_{z}=1356 \times 0.936=1270 \mathrm{~W} / \mathrm{m}^{2}$


## Exercise

- Now, solving for GHI:
- $\mathrm{k}_{\mathrm{t}}=\mathrm{G}_{\mathrm{GHI}} / \mathrm{G}_{0}$
- $\mathbf{G}_{\mathrm{GHI}}=952.5 \mathrm{~W} / \mathrm{m}^{2}$
- Computing the Diffuse Irradiance:
- $\mathrm{G}_{\mathrm{d}} / \mathrm{G}_{\mathrm{GHI}}=1-1.13 \mathrm{k}_{\mathrm{t}}$
- $\mathbf{G}_{\mathrm{d}}=145.25 \mathrm{~W} / \mathrm{m}^{2}$
- Solving for $G_{b}$
- $\mathbf{G}_{\mathrm{b}}=\mathbf{G}_{\mathrm{GHI}}-\mathbf{G}_{\mathrm{d}}=807.25 \mathrm{~W} / \mathrm{m}^{2}$


## Exercise

- The $G_{b}$ we computed is for a Horizontal surface, we need to find $G_{b T}$, the beam irradiance on a tilted surface
- We can relate $G_{b}$ and $G_{b T}$ by: $G_{b T}=G_{b} R_{b}$
- Where
- $\mathrm{R}_{\mathrm{b}}=\cos \theta / \cos \theta_{\mathrm{z}}$
- We already know $\cos \theta_{z}$, so we need to calculate $\cos \theta$


## Exercise

- Solving for the cosine of the angle of incidence:

```
\operatorname{cos}(0)=\operatorname{sin}(\delta)\operatorname{sin}(\phi)\operatorname{cos}(\beta)
- sin}(\delta)\operatorname{cos}(\phi)\operatorname{sin}(\beta
+ \operatorname{cos}(\delta)\operatorname{cos}(\phi)\operatorname{cos}(\beta)\operatorname{cos}(\omega)
+ cos(\delta)\operatorname{sin}(\phi)\operatorname{sin}(\beta)\operatorname{cos}(\omega)
cos(0)=\operatorname{sin}(9.\mp@subsup{4}{}{\circ})\operatorname{sin}(3\mp@subsup{0}{}{\circ})\operatorname{cos}(1\mp@subsup{5}{}{\circ})\quad[or using \operatorname{cos}(0)=\operatorname{cos}(\phi-\delta-\beta)]
- sin}(9.\mp@subsup{4}{}{\circ})\operatorname{cos}(3\mp@subsup{0}{}{\circ})\operatorname{sin}(1\mp@subsup{5}{}{\circ}
+ cos(9.4 ) cos(30})\operatorname{cos}(1\mp@subsup{5}{}{\circ})\operatorname{cos}(\mp@subsup{0}{}{\circ}
+\operatorname{cos}(9.4}\mp@subsup{4}{}{\circ})\operatorname{sin}(3\mp@subsup{0}{}{\circ})\operatorname{sin}(1\mp@subsup{5}{}{\circ})\operatorname{cos}(\mp@subsup{0}{}{\circ})=0.99
```


## Exercise

- $\mathrm{R}_{\mathrm{b}}=\cos \theta / \cos \theta_{z}=1.06$
- So that the beam irradiance on the tilted surface is: $G_{b T}=$ $\mathrm{G}_{\mathrm{b}} \mathrm{R}_{\mathrm{b}}=858.1 \mathrm{~W} / \mathrm{m}^{2}$


## Atmospheric Effects



## Atmospheric Effects

$G_{d}$ is also dependent on the tilt of the surface

## Computing Diffuse Irradiance

- We already know $\mathrm{G}_{\mathrm{d}}$
- Now we need to be able to account for the tilting of a surface
- When $\beta=0^{\circ}$, all of $\mathbf{G}_{\mathrm{d}}$ is received
- When $\beta=90^{\circ}$, half of $\mathbf{G}_{\mathrm{d}}$ is received
- When $\beta=180^{\circ}$, no $G_{d}$ is received (surface is facing the ground)
- $G_{d T}=1 / 2 G_{d}(1+\cos \beta)$
- This assumes isotropic conditions


## Computing Beam Irradiance

- We now have solved for $A$ and $B$
- $\mathrm{G}_{\mathrm{T}}=\mathrm{A}+\mathrm{B}+\mathrm{C}$
- $A=G_{b} R_{b}$
- $B=1 / 2 G_{d}(1+\cos \beta)$


## Ground Reflectance

- Final component is the diffuse irradiance that reflects off the ground
- Ground albedo ( $\rho$ )
- Usually 0.2 , but can be up to 0.8 for snow, ice
- We assume that this is proportional to the $\mathbf{G}_{\mathrm{GHI}}$
- $\mathbf{G}_{\text {gnd }}=\rho \mathbf{G}_{\mathrm{GHI}}$
- Amount received depends on the tilt of the surface


## Ground Reflectance

## Ground Reflectance

- Accounting for the tilting of a surface
- When $\beta=0^{\circ}$, no of $\mathbf{G}_{\text {gnd }}$ is received
- When $\beta=90^{\circ}$, half of $\mathbf{G}_{\text {gnd }}$ is received
- When $\beta=180^{\circ}$, all $G_{\text {gnd }}$ is received (surface is facing the ground)
- $\mathbf{G}_{\mathrm{gnd}, \mathrm{T}}=1 / 2 \rho \mathbf{G}_{\mathrm{GHI}}(1-\cos \beta)$


## Computing Beam Irradiance

- We now have solved for A, B and C
- $G_{T}=A+B+C$
- $G_{b T}=G_{b} R_{b}$
- $G_{d T}=1 / 2 G_{d}(1+\cos \beta)$

```
Note: B depends on \(\mathrm{G}_{\mathrm{d}}\),
whereas \(C\) depends on \(G_{G H I}\)
```

- $G_{\mathrm{gnd}, \mathrm{T}}=1 / 2 \rho \mathrm{G}_{\mathrm{GHI}}(1-\cos \beta)$


## Exercise

- Consider a surface that is tilted at $30^{\circ}$. The measured GHI is $250 \mathrm{~W} / \mathrm{m}^{2}$, and $\mathbf{G}_{0}=404 \mathrm{~W} / \mathrm{m}^{2}$. Compute $\mathbf{G}_{\mathrm{b}}, \mathbf{G}_{\mathrm{d}}$ and $\mathbf{G}_{\mathrm{T}}$. Let $\cos (\theta)=0.693$ and $\cos \left(\theta_{z}\right)=0.286$. Assume $\rho=0.2$.


## Exercise

- Consider a surface that is tilted at $30^{\circ}$. The measured GHI is $250 \mathrm{~W} / \mathrm{m}^{2}$, and $\mathbf{G}_{0}=404 \mathrm{~W} / \mathrm{m}^{2}$. Compute $\mathbf{G}_{\mathrm{b}}, \mathbf{G}_{\mathrm{d}}$ and $\mathbf{G}_{\mathrm{T}}$. Let $\cos (\theta)=0.693$ and $\cos \left(\theta_{z}\right)=0.286$. Assume $\rho=0.2$.
- $\mathrm{k}_{\mathrm{t}}=250 / 404=0.622$


## Exercise

- Consider a surface that is tilted at $30^{\circ}$. The measured GHI is $250 \mathrm{~W} / \mathrm{m}^{2}$, and $\mathbf{G}_{0}=404 \mathrm{~W} / \mathrm{m}^{2}$. Compute $\mathbf{G}_{\mathrm{b}}, \mathbf{G}_{\mathrm{d}}$ and $\mathbf{G}_{\mathrm{T}}$. Let $\cos (\theta)=0.693$ and $\cos \left(\theta_{z}\right)=0.286$. Assume $\rho=0.2$.
- $\mathrm{k}_{\mathrm{t}}=250 / 404=0.622$
- $\mathrm{G}_{\mathrm{d}} / \mathrm{G}_{\mathrm{GHI}}=1-1.13 \mathrm{k}_{\mathrm{t}}$
- $\mathbf{G}_{\mathrm{d}}=\left(1-1.13 \mathrm{k}_{\mathrm{t}}\right) \mathbf{G}_{\mathrm{GHI}}=(0.297) \mathbf{G}_{\mathrm{GHI}}=74.3 \mathrm{~W} / \mathrm{m}^{2}$


## Exercise

- Consider a surface that is tilted at $30^{\circ}$. The measured GHI is $250 \mathrm{~W} / \mathrm{m}^{2}$, and $\mathbf{G}_{0}=404 \mathrm{~W} / \mathrm{m}^{2}$. Compute $\mathbf{G}_{\mathrm{b}}, \mathbf{G}_{\mathrm{d}}$ and $\mathbf{G}_{\mathrm{T}}$. Let $\cos (\theta)=0.693$ and $\cos \left(\theta_{z}\right)=0.286$. Assume $\rho=0.2$.
- $\mathrm{k}_{\mathrm{t}}=250 / 404=0.622$
- $\mathrm{G}_{\mathrm{d}}=74.3 \mathrm{~W} / \mathrm{m}^{2}$
- $\mathrm{G}_{\mathrm{b}}=250-74.3=175.7 \mathrm{~W} / \mathrm{m}^{2}$


## Exercise

- Consider a surface that is tilted at $30^{\circ}$. The measured GHI is $250 \mathrm{~W} / \mathrm{m}^{2}$, and $\mathbf{G}_{0}=404 \mathrm{~W} / \mathrm{m}^{2}$. Compute $\mathbf{G}_{\mathrm{b}}$, $\mathbf{G}_{\mathrm{d}}$ and $\mathbf{G}_{\mathrm{T}}$. Let $\cos (\theta)=0.693$ and $\cos \left(\theta_{z}\right)=0.286$. Assume $\rho=0.2$.
- $\mathrm{k}_{\mathrm{t}}=250 / 404=0.622$
- $\mathrm{G}_{\mathrm{d}}=74.3 \mathrm{~W} / \mathrm{m}^{2}$
- $\mathrm{G}_{\mathrm{b}}=175.7 \mathrm{~W} / \mathrm{m}^{2}$
- $\mathrm{R}_{\mathrm{b}}=\cos \theta / \cos \theta_{z}=2.42$


## Exercise

- Consider a surface that is tilted at $30^{\circ}$. The measured GHI is $250 \mathrm{~W} / \mathrm{m}^{2}$, and $\mathrm{G}_{0}=404 \mathrm{~W} / \mathrm{m}^{2}$. Compute $\mathrm{G}_{\mathrm{b}}$, $\mathbf{G}_{\mathrm{d}}$ and $\mathbf{G}_{\mathrm{T}}$. Let $\cos (\theta)=0.693$ and $\cos \left(\theta_{z}\right)=0.286$. Assume $\rho=0.2$.
- $\mathrm{k}_{\mathrm{t}}=250 / 404=0.622$
- $\mathrm{G}_{\mathrm{d}}=74.3 \mathrm{~W} / \mathrm{m}^{2}$
- $\mathrm{G}_{\mathrm{b}}=175.7 \mathrm{~W} / \mathrm{m}^{2}$
- $\mathrm{R}_{\mathrm{b}}=\cos \theta / \cos \theta_{\mathrm{z}}=2.42$
- $\mathbf{G}_{\mathrm{T}}=\mathbf{G}_{\mathrm{b}} \mathrm{R}_{\mathrm{b}}+1 / 2 \mathbf{G}_{\mathrm{d}}(1+\cos \beta)+1 / 2 \rho \mathbf{G}_{\mathrm{GHI}}(1-\cos \beta)$
- $175.7(2.42)+1 / 2(74.3)(1+\cos 30)+1 / 2(0.2)(250)(1-\cos 30)$
- $\mathrm{G}_{\mathrm{T}}=425.6+69.3+3.3=498.3 \mathrm{~W} / \mathrm{m}^{2}$


## Calculating Irradiance




## Calculating Irradiance



## Irradiation

- Irradiation: irradiance received per unit area
- Computed by integrating $\mathbf{G}$ over time
- I: irradiance received over one hour, Wh/m²
- H: irradiance received over one day, Wh/m²
- Multiply irradiation by the surface area to compute radiation


## Irradiation

- Consider a surface tilted at its latitude
- $\cos (\theta)=\cos (\delta) \cos (\omega)$
- What is the extraterrestrial irradiation from 2 pm to 3 pm ?
- Let $\mathbf{G}_{0 \mathrm{n}} \cos (\delta)=1000 \mathrm{~W} / \mathrm{m}^{2}$
- Integrating:

$$
\begin{aligned}
& I_{0 T}=1000 \int_{30^{\circ}}^{-45^{\circ}} \cos (\omega) d \omega \\
& I_{0 T}=1000\left(\sin \left(\frac{\pi}{6}\right)-\sin \left(\frac{\pi}{4}\right)\right)=1000(0.207)=207 \quad \text { What are the units? }
\end{aligned}
$$

## Irradiation



## Irradiation

- Clearly the $\mathrm{I}_{\text {от }}$ does not equal 207 Wh
- We performed the integration using radians
- Units: (W/m²) x radians
- Expressed in Wh or kWh is more useful
- 24 hours $=2 \pi$ radians
- $1 \mathrm{rad}=24 /(2 \pi)=12 / \pi$ hours
- Therefore:

$$
207\left(\frac{12}{\pi}\right)=790.7 \mathrm{~Wh} / \mathrm{m}^{2}
$$

## Irradiation

- GHI values are usually given as time-average values
- 10 minutes
- l Hour
- l Day
- l Month
- To compute the clearness index, average values of $G_{0}$ must be used


## Irradiation

- Let $\bar{G}_{\mathrm{b}}$ be the hourly averaged value of $\mathrm{G}_{0}$ (average irradiance on an extraterrestrial horizontal surface $\beta=0$ )
$\bar{G}_{0}=G_{0 n} \int_{\omega}^{o+15^{\circ}} \cos (\delta) \cos (\phi) \cos (\omega)+\sin (\delta) \sin (\phi) d \omega$
$\bar{G}_{0}=G_{0 n} \int_{\omega}^{a+15^{\circ}} \cos (\delta) \cos (\phi) \cos (\omega) d \omega+G_{0 n} \int_{\omega}^{\omega+15^{5}} \sin (\delta) \sin (\phi) d \omega$
$\bar{G}_{0}=\frac{12}{\pi} G_{0 n}\left[\cos (\delta) \cos (\phi)\left(\sin \left(\omega+15^{\circ}\right)-\sin (\omega)\right)+\frac{\pi 15^{\circ}}{180^{\circ}} \sin (\delta) \sin (\phi)\right]$
$K_{T}=\frac{\bar{G}_{G H}}{\bar{G}_{0}}$
- Clearness index using hourly-averaged values is


## Irradiance Algorithm

- Computer programs such as Homer and RETScreen compute irradiance for a surface under specified conditions
- Algorithms differ in detail, but in approach they are very similar
- Once irradiance is known, power output of any arrangement of PV panels can be computed



## Air Mass

- Effects of absorption depend on the mass of air the radiation travels through
- The mass of air that solar radiation travels before reaching the surface varies with zenith angle
- This mass is smallest at solar noon with the sun directly overhead



## Atmospheric Absorption

- Air Mass Ratio (AM) = PC/PB
- for $\theta_{z}<70^{\circ}$, this approximates to $A M \approx \frac{1}{\cos \theta_{z}}$
- AM0 is at the top of the atmosphere
- AM is sometimes divided by direct irradiance (AMD XX) and global irradiance (AMG XX) at air mass XX
- AMG $1.5\left(\theta_{z}=48^{\circ}\right)$ is the PV industry standard for the spectrum distribution, with $1000 \mathrm{~W} / \mathrm{m}^{2}$ irradiance
- How common is $\mathbf{G}=1000 \mathrm{~W} / \mathrm{m}^{2}$ ?

