# 07-Three-Phase Analysis 

Text: Chapter 8.3<br>ECEGR 3500<br>Electrical Energy Systems<br>Professor Henry Louie

## Overview

- Three-Phase Power
- Three-Phase Analysis
- Per-Phase Analysis
- Practical Considerations


## Questions

- How does three phase power compare to single phase power?
- What is the "neutral" connection in a three-phase system?


## Conventions \& Assumptions

- Voltage: line-to-line in rms
- Current: line in rms
- Current direction: source to load
- Balanced three phase


## Three-Phase Power

Benefits of three-phase power:

- efficient use of conductors over three, single phases
- rotating field is needed for some loads (e.g. three phase motors)
- effective for power transfer
- per-phase analysis can be used in many cases
- constant power delivery to three phase loads


## Three-Phase Power

Regardless of connection (wye or delta), the power of one phase of a three-phase system is:

$$
\begin{array}{cc}
P_{\mathrm{ph}}=V_{\mathrm{ph}} I_{\mathrm{ph}} \cos \theta & \begin{array}{l}
\text { Recall that } V_{\mathrm{ph}} \text { and } I_{\mathrm{ph}} \text { are the magnitudes } \\
\text { of the phase voltage and phase current, an } \\
\theta=\theta_{v}-\theta \text { (ancle of phase voltace - ancle o }
\end{array} \\
Q_{\mathrm{ph}}=V_{\mathrm{ph}} I_{\mathrm{ph}} \sin \theta & \begin{array}{l}
\text { (and }
\end{array}
\end{array}
$$

$$
\boldsymbol{S}_{\mathrm{ph}}=P_{\mathrm{ph}}+j Q_{\mathrm{ph}}
$$



Recall that line-line voltage is equal to the phase voltage in delta-connected systems

## Three-Phase Power

- The three-phase power $(P, Q, S)$ is the sum of the power from the individual phases
- By symmetry, we can show that the power associated with each phase is equal, therefore:

$$
\begin{aligned}
& P=3 P_{\mathrm{ph}}=3 P_{\mathrm{ph}}=3 V_{\mathrm{ph}} I_{\mathrm{ph}} \cos \theta \\
& Q=3 Q_{\mathrm{ph}}=3 Q_{\mathrm{ph}}=3 V_{\mathrm{ph}} I_{\mathrm{ph}} \sin \theta \\
& \boldsymbol{S}=3 \boldsymbol{S}_{\mathrm{ph}}=3 V_{\mathrm{ph}} I_{\mathrm{ph}} \cos \theta+j 3 V_{\mathrm{ph}} I_{\mathrm{ph}} \sin \theta
\end{aligned}
$$

- By default, when we refer to "power" in a three-phase system, we are referring to the three-phase power NOT the power of an individual phase


## Exercise

A three-phase motor consumes 3000 W and 150 VAR. What is the complex power provided by B-phase?

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A three-phase motor consumes 3000 W and 150 VAR. What is the complex power provided by B-phase?

Answer: $\boldsymbol{S}=1000+\mathrm{j} 50$ VA. The power is evenly divided by each of the phases.

## Alternative Power Expressions

- Consider a wye-connected system:

$$
\begin{aligned}
& P=3 V_{\mathrm{ph}} I_{\mathrm{ph}} \cos \theta=3 V_{\mathrm{ph}} I_{l} \cos \theta \\
& P=3 \frac{V_{\mathrm{nl}}}{\sqrt{3}} I_{l} \cos \theta=(\sqrt{3} \sqrt{3}) \frac{V_{\mathrm{n}}}{\sqrt{3}} I_{l} \cos \theta=\sqrt{3} V_{\mathrm{nl}} I_{l} \cos \theta
\end{aligned}
$$

- For reactive power:

$$
Q=\sqrt{3} V_{\mathrm{H}} I_{1} \sin \theta
$$

But don't be misled, the term $\sqrt{3} V_{11}$ arises from mathematical
simplification only. The voltage
$\sqrt{3} V_{11}$ does not appear in the system

## Alternative Power Expressions

- Consider a delta-connected system:

$$
\begin{aligned}
& P=3 V_{\mathrm{ph}} I_{\mathrm{ph}} \cos \theta=3 V_{11} I_{\mathrm{ph}} \cos \theta \\
& P=3 V_{l l} \frac{I_{l}}{\sqrt{3}} \cos \theta=(\sqrt{3} \sqrt{3}) V_{l l} \frac{I_{l}}{\sqrt{3}} \cos \theta=\sqrt{3} V_{11} I_{l} \cos \theta
\end{aligned}
$$

The end result is the same for delta and wye-connected systems!

- For reactive power:

$$
Q=\sqrt{3} V_{11} I_{1} \sin \theta
$$

The end result is the same for delta and wye-connected systems!

These equations are a useful shortcut if the line-line voltage and line current are known

## Example

- Consider the shown three-phase system
- The line-line voltage is 277 V , and the line current is 13 A
- The load has a lagging power factor of 0.85 lagging
- Compute the real, reactive, and complex power consumed by the load



## Example

- First identify the given values



## Example

The power consumed by the load is:
$P=\sqrt{3} V_{11} I_{l} \cos \theta=\sqrt{3} \times 277 \times 13 \times 0.85=5032 \mathrm{~W}$
$\theta=\cos ^{-1}(0.85)=0.55$ radians $=31.5^{\circ}$
$Q=\sqrt{3} V_{11} I_{l} \sin \theta=\sqrt{3} \times 277 \times 13 \times 0.527=3286$ VAR
$S=5302+j 3286$ VA


## Three-Phase Power

## What about instantaneous power?

$$
\begin{gathered}
\mathrm{p}(\mathrm{t})=\mathrm{i}_{\max } \mathrm{v}_{\max } \cos \left(\omega \mathrm{t}+0^{\circ}\right) \cos \left(\omega \mathrm{t}+\theta_{\mathrm{ia}}{ }^{\circ}\right)+ \\
\mathrm{i}_{\max } \mathrm{v}_{\max } \cos \left(\omega \mathrm{t}-120^{\circ}\right) \cos \left(\omega \mathrm{t}+\theta_{\mathrm{ia}}{ }^{\circ}-120^{\circ}\right)+ \\
\mathrm{i}_{\max } \mathrm{v}_{\max } \cos \left(\omega \mathrm{t}+120^{\circ}\right) \cos \left(\omega \mathrm{t}+\theta_{\mathrm{ia}}{ }^{\circ}+120^{\circ}\right) \\
\mathrm{p}(\mathrm{t})=\frac{1}{2} \mathrm{v}_{\max } \mathrm{i}_{\max }\left[\cos \left(-\theta_{\mathrm{ia}}\right)+\cos \left(2 \omega \mathrm{t}+\theta_{\mathrm{ia}}\right)+\right. \\
\cos \left(-\theta_{\mathrm{ia}}\right)+\cos \left(2 \omega \mathrm{t}+\theta_{\mathrm{ia}}-120^{\circ}\right)+ \\
\left.\cos \left(-\theta_{\mathrm{ia}}\right)+\cos \left(2 \omega \mathrm{t}+\theta_{\mathrm{ia}}+120^{\circ}\right)\right]
\end{gathered}
$$

Next use: $\cos (x+y)=\cos (x) \cos (y)-\sin (x) \sin (y)$

## Three-Phase Power

$$
\begin{aligned}
p(t)=\frac{1}{2} v_{\max } & i_{\max }\left[3 \cos \left(-\theta_{\mathrm{ia}}\right)+\cos \left(2 \omega t+\theta_{\mathrm{ia}}\right)\right. \\
& +\cos \left(2 \omega t+\theta_{\mathrm{ia}}\right) \cos \left(-120^{\circ}\right)-\sin \left(2 \omega t+\theta_{\mathrm{ia}}\right) \sin \left(-120^{\circ}\right) \\
& \left.+\cos \left(2 \omega t+\theta_{\mathrm{ia}}\right) \cos \left(120^{\circ}\right)-\sin \left(2 \omega t+\theta_{\mathrm{ia}}\right) \sin \left(120^{\circ}\right)\right]
\end{aligned}
$$

$\mathrm{p}(\mathrm{t})=\frac{1}{2} \mathrm{v}_{\text {max }} \mathrm{i}_{\text {max }}\left[3 \cos \left(-\theta_{\mathrm{ia}}\right)+\cos \left(2 \sin -\theta_{\mathrm{ia}}\right)+\right.$

$\mathrm{p}(\mathrm{t})=\frac{3}{2} \mathrm{v}_{\text {max }} \mathrm{i}_{\text {max }} \cos \left(\theta_{\mathrm{ia}}\right)$ What does this say about the real power?

## Three-Phase Power



## Three-Phase Analysis

- Balanced Three-Phase Theorem
- Assume:
- balanced three-phase system
- all loads and sources are Y-connected (or convert them to Y-connected loads/sources)
- no mutual inductances between phases
then
- all neutrals have the same voltage
- the phases are completely decoupled
- all corresponding network variables occur in balanced sets of the same sequence as the sources


## Per-Phase Analysis

- Consider the following circuit
- Is it balanced?



## Per-Phase Analysis

- To analyze this circuit, we first need to find a per-phase equivalent
- Need to transform the Delta load to Y load



## Per-Phase Analysis

- To analyze this circuit, we first need to find a per-phase equivalent
- Need to transform the Delta load to Y load



## Per-Phase Analysis

- Analyze a single phase (arbitrarily a-phase)
- Analyze this phase to find the current, power, etc



## Numerical Example



## Exercise

Solution outline

- draw per-phase equivalent
- solve circuit for current
- compute single phase power
- translate per-phase values to 3-phase


## Exercise

- Convert $\Delta$ to $\mathrm{Y} \quad Z_{2}=2.82+\mathrm{j} 2.82 \Omega$
- Redraw circuit



## Exercise

- Redraw circuit
- Draw per-phase equivalent



## Exercise

- Draw per-phase equivalent $\quad V_{\mathrm{an}}=120 \angle 0^{\circ} \mathrm{V}$
- Solve for $\boldsymbol{I}_{\mathrm{a}}$ and $\boldsymbol{S}$

$$
\begin{aligned}
& \boldsymbol{I}_{\mathrm{a}}=40.32 \angle-35.68^{\circ} \mathrm{A} \\
& \boldsymbol{S}=3930.2+\mathrm{j} 2822.5 \mathrm{VA}
\end{aligned}
$$



## Exercise

- Shifting and solving

$$
\begin{aligned}
& \boldsymbol{I}_{\mathrm{b}}=40.32 \angle\left(-35.68^{\circ}-120^{\circ}\right)=40.32 \angle\left(-155.68^{\circ}\right) \mathrm{A} \\
& \boldsymbol{I}_{\mathrm{c}}=40.32 \angle\left(-35.68^{\circ}+120^{\circ}\right)=40.32 \angle\left(84.32^{\circ}\right) \mathrm{A} \\
& \boldsymbol{I}_{\mathrm{n}}=0 \mathrm{~A} \\
& \mathbf{S}_{3 \Phi}=3(3930.19+\mathrm{j} 2822.51)=11790.57+\mathrm{j} 8467.53 \mathrm{VA}
\end{aligned}
$$

## Per-Phase Analysis

If delta-connected voltage sources are present, convert them to their line-neutral equivalent and then do per-phase analysis


$$
\begin{aligned}
& \boldsymbol{V}_{\mathrm{an}}=\frac{\boldsymbol{V}_{\mathrm{ab}}}{\sqrt{3}} \angle-30^{\circ} \\
& \boldsymbol{V}_{\mathrm{bn}}=\frac{\boldsymbol{V}_{\mathrm{bc}}}{\sqrt{3}} \angle-30^{\circ} \\
& \boldsymbol{V}_{\mathrm{cn}}=\frac{\boldsymbol{V}_{\mathrm{ca}}}{\sqrt{3}} \angle-30^{\circ}
\end{aligned}
$$

For convenience, we usually re-define the reference angle as the a-phase line-neutral voltage

## Practical Considerations



> Three phase systems and
> components often referred to by
> RMS line-line voltages
> e.g. $500 \mathrm{kV}, 13.6 \mathrm{kV}, 480 \mathrm{~V}$

Three phase systems and
components often referred to by |S|
e.g. 100 MVA transformer

## Practical Considerations

- Neutral usually bonded to ground at service panel (see National Electric Code for details)
- Balanced load assumption can be questionable
- Neutral current nonzero
- Neutral not at ground potential


## Practical Considerations

Ground
(green)


## Summary

- Current (voltages) sum to zero and no current flows on neutral conductor
- Total $S, P, Q=$ three times single phase $S, P, Q$
- Three phase systems analysis: convert into wye connections, solve per-phase

