

Grounding

Text: Chapter 9.2-9.3

ECEGR 3500

Electrical Energy Systems

Professor Henry Louie

» Overview

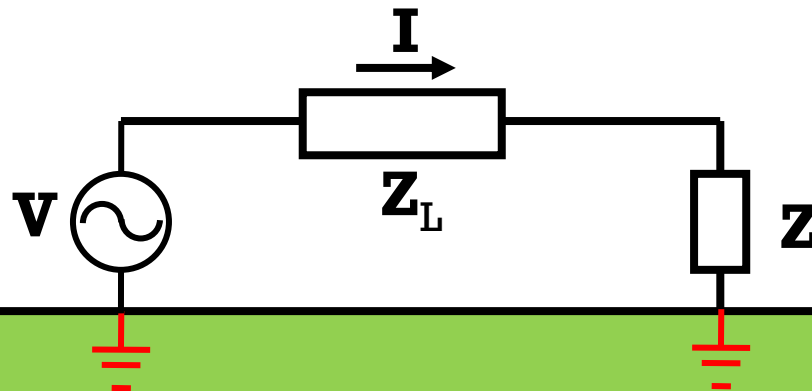
- Ground Resistance
- Ground Resistance of Objects
- Measuring Ground Resistance
- Ground Resistance of People
- Touch Potential
- Step Potential

→ Question

- Why are electrical engineers so obsessed with ground?

→ Ground Resistance

- Consider a circuit whose source and load are grounded (literally connected to the soil)
- Current will flow, even though there is no dedicated conductor

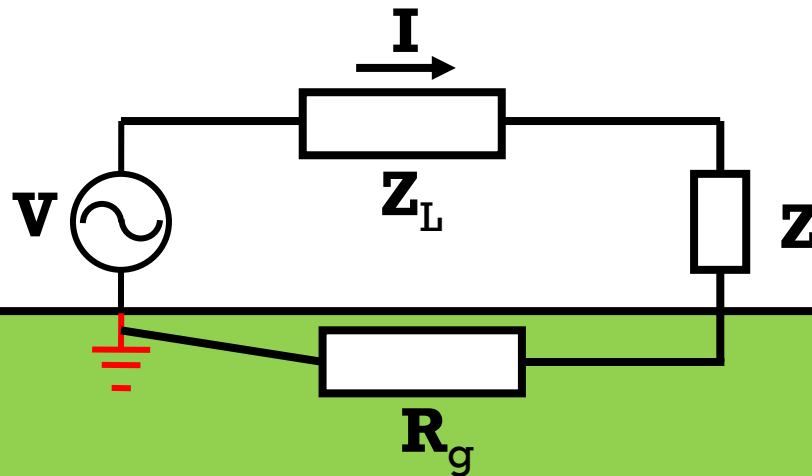


→ Ground Resistance

- However, we must account for the resistance through the ground between the two points

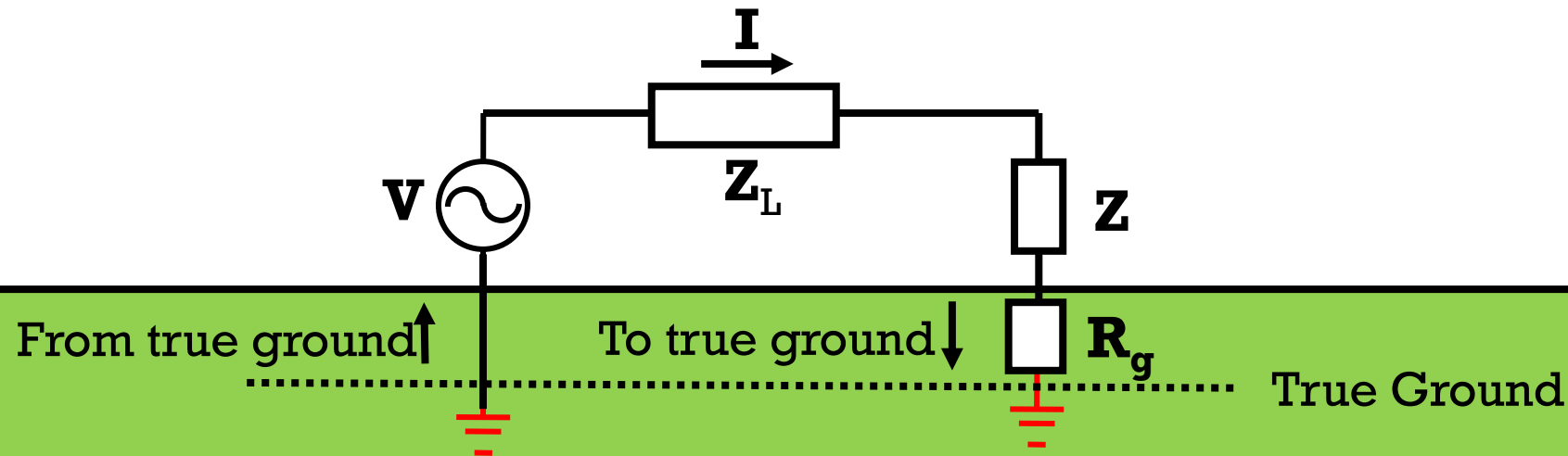
$$\mathbf{V} = \mathbf{I}(\mathbf{Z}_L + \mathbf{Z} + \mathbf{R}_g)$$

Sometimes \mathbf{R}_g is neglected, but it is usually important



Ground Resistance

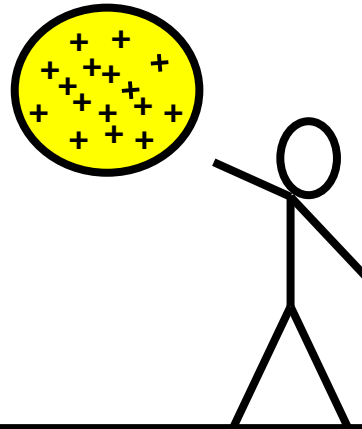
- Another way of thinking of it is that the source is perfectly connected to “true ground”, and the ground resistance is the resistance to true ground (true ground is deep within the earth)



→ Ground Resistance

- True ground can infinitely source or sink current without changing its potential

When you touch a charged
conductive object, where does the charge go?



Ground Resistance of Objects

» Ground Resistance

- The ground resistance of an object depends on several factors:
 - material of the object
 - shape of the object
 - placement of the object
 - soil condition

→ Ground Resistance of Objects

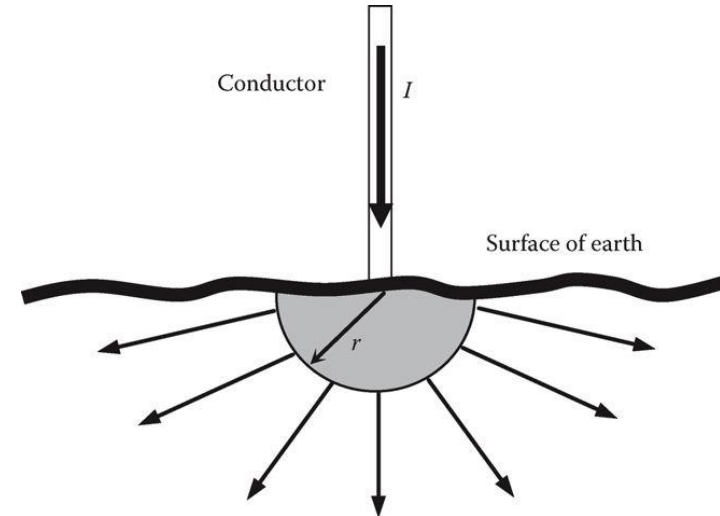
- Consider a hemisphere object buried in soil whose resistivity ρ is uniform
- Any current flowing through the object will have the density

$$J = \frac{I}{2\pi r^2}$$

where

J : current density, in A/m^2

r : hemisphere radius, in m



→ Ground Resistance of Objects

- For the soil surrounding the object:

$$J(x) = \frac{I}{2\pi x^2} \quad \text{for } x \geq r$$

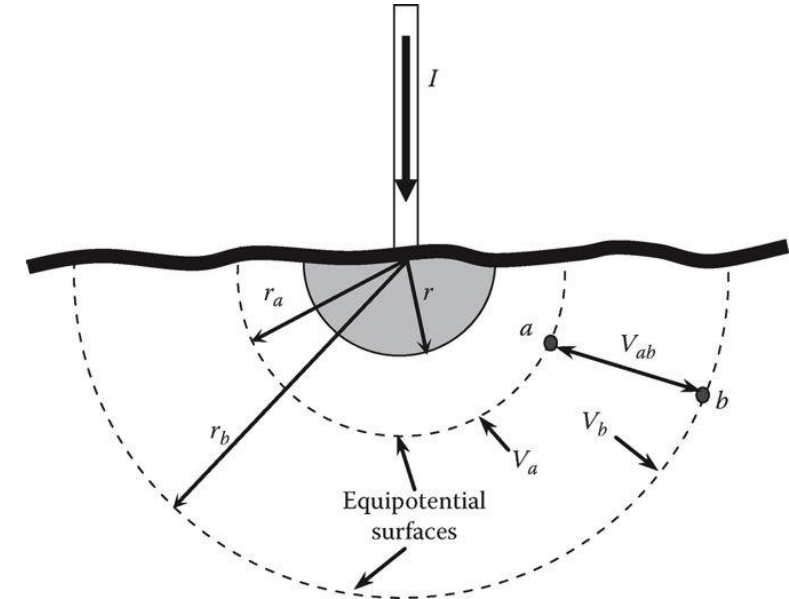
- x is the distance from the center, in m

- Since current is flowing through resistance (the soil), we expect there to be a voltage difference in the soil

- From physics: $E(x) = \rho J(x)$ for $x \geq r$

E : electric field intensity (V/m)

ρ : resistivity (Ohm-m)



Ground Resistance of Objects

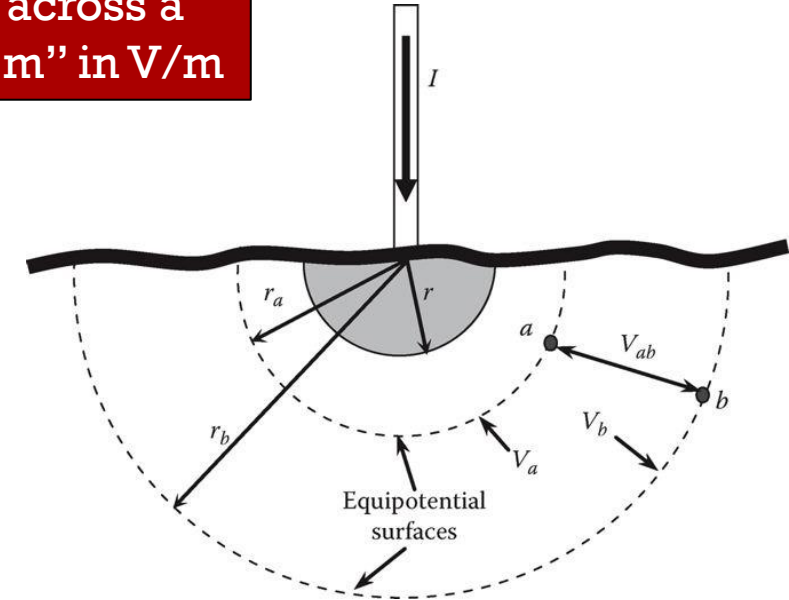
- The voltage between two distances

$$V_{ab} = \int_a^b E(x)dx = \int_a^b \rho J(x)dx = \frac{\rho I}{2\pi} \left(\frac{1}{r_a} - \frac{1}{r_b} \right)$$

You need to integrate across a length to cancel the “m” in V/m

- Since true ground is very far from the surface, let r_b go to infinity, and applying Ohm's Law

$$R_g = \frac{\rho}{2\pi r}$$



» Exercise (Example 9.3)

A hemisphere 2 m in diameter is buried in wet-organic soil (resistivity of 10 Ohm-m). Compute the ground resistance of the hemisphere. Compute the resistance between the hemisphere and a point 10m away from it.

Exercise (Example 9.3)

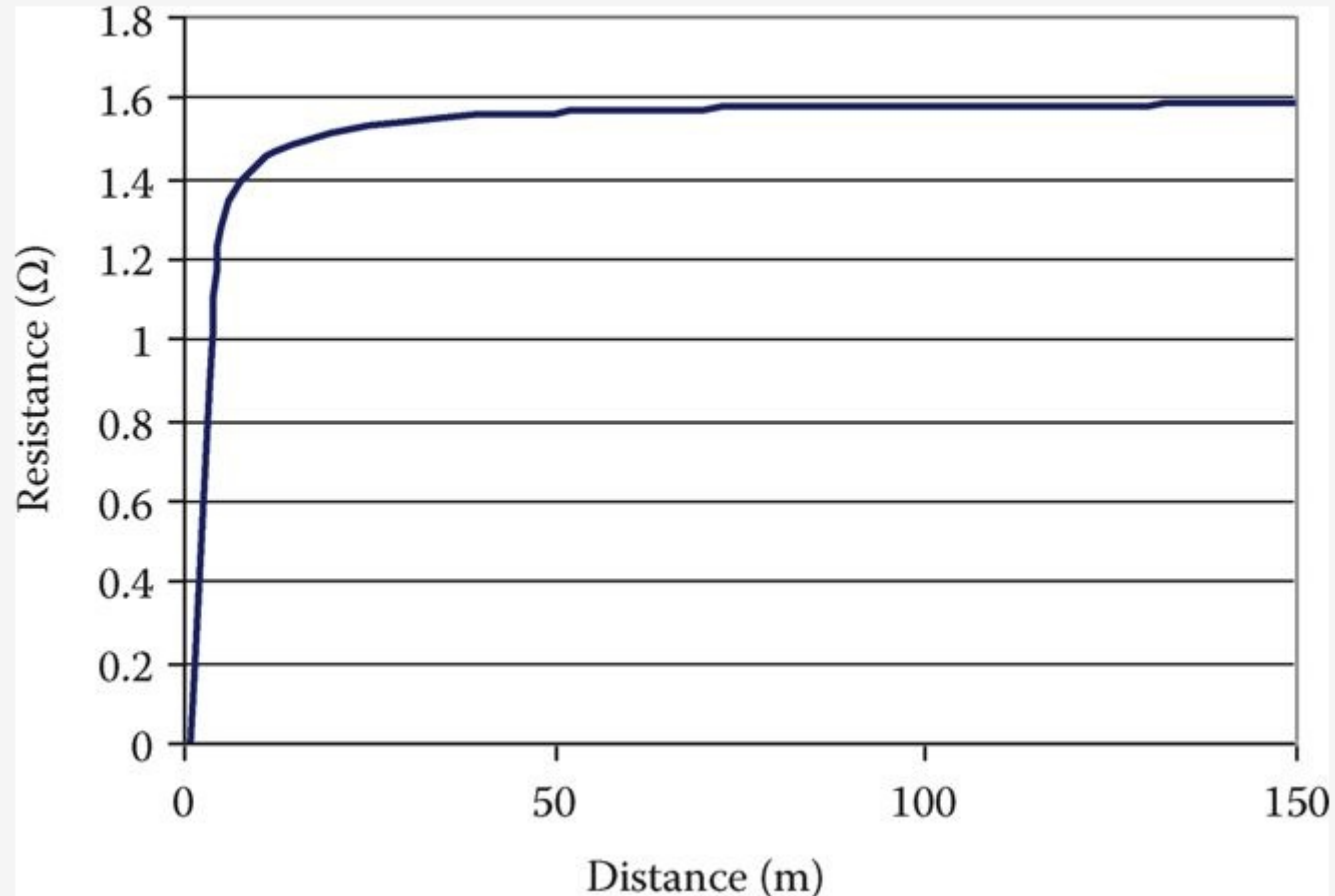
A hemisphere 2 m in diameter is buried in wet-organic soil. Compute the ground resistance of the hemisphere. Compute the resistance between the hemisphere and a point 10m away from it.

Answer.

$$R_g = \frac{\rho}{2\pi r} = \frac{10}{2\pi \times 1} = 1.6\Omega$$
$$R_{ab} = \frac{V_{ab}}{I} = \frac{\rho}{2\pi} \left[\frac{1}{r_a} - \frac{1}{r_b} \right] = \frac{10}{2\pi} \left[\frac{1}{1} - \frac{1}{10} \right] = 1.4\Omega$$

→ Ground Resistance of a Hemisphere

FIGURE 9.8 Ground resistance between the hemisphere and points at various distances.



Note that the ground resistance begins to plateau very close to the electrode, with little variation as distance increases.

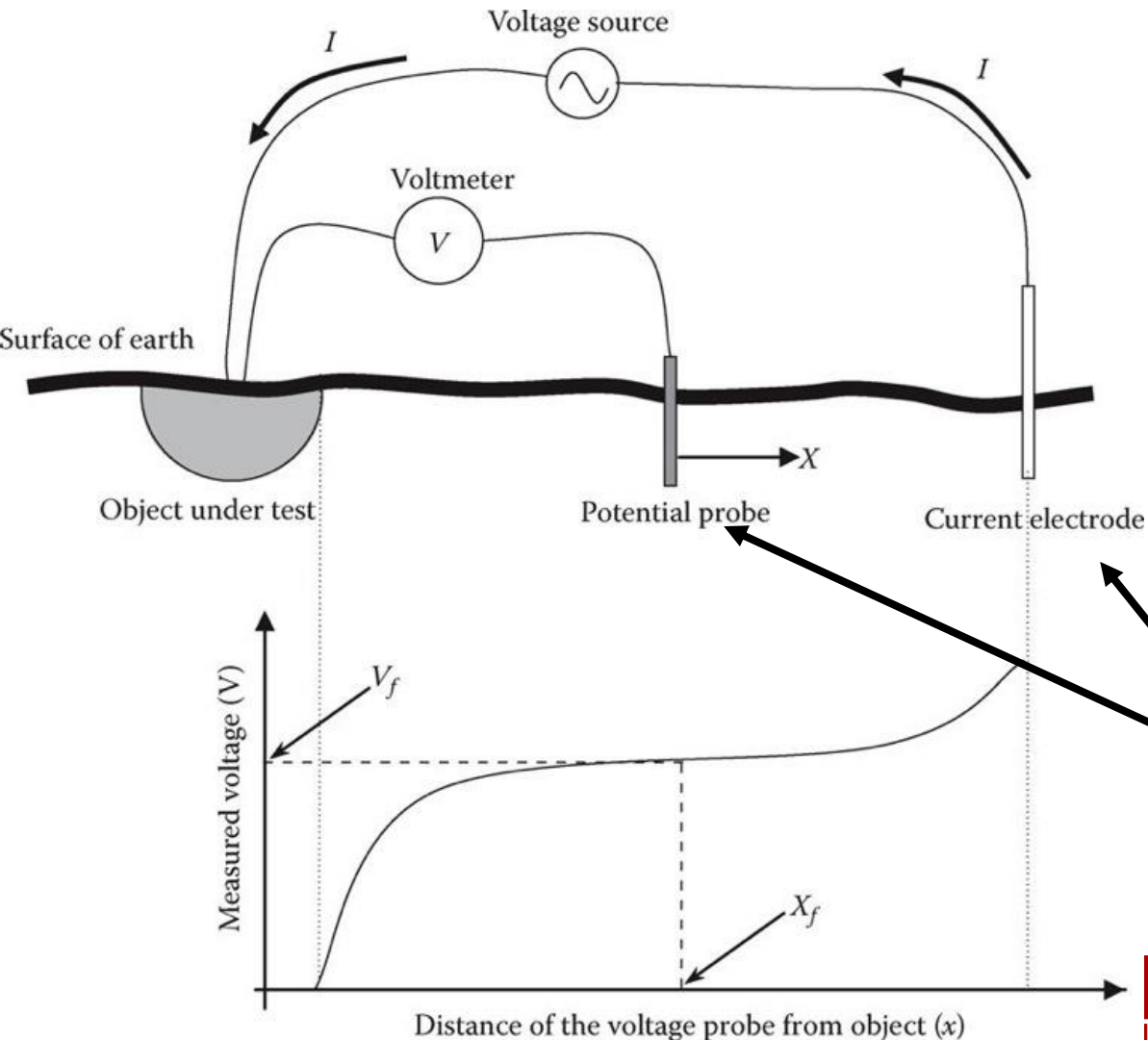
Resistance to true ground is when distance goes to infinity

Measuring Ground Resistance

→ Measuring Ground Resistance

- In new electrical systems, the resistance to ground of the grounding system (ground rods, mats, etc.) is typically measured to see if it is compliant with electrical standards
- Several methods are possible (see IEEE Standard 81 for more details)
 - Fall of Potential
 - Two-Point Method
 - Four-Point Method
 - Slope Method

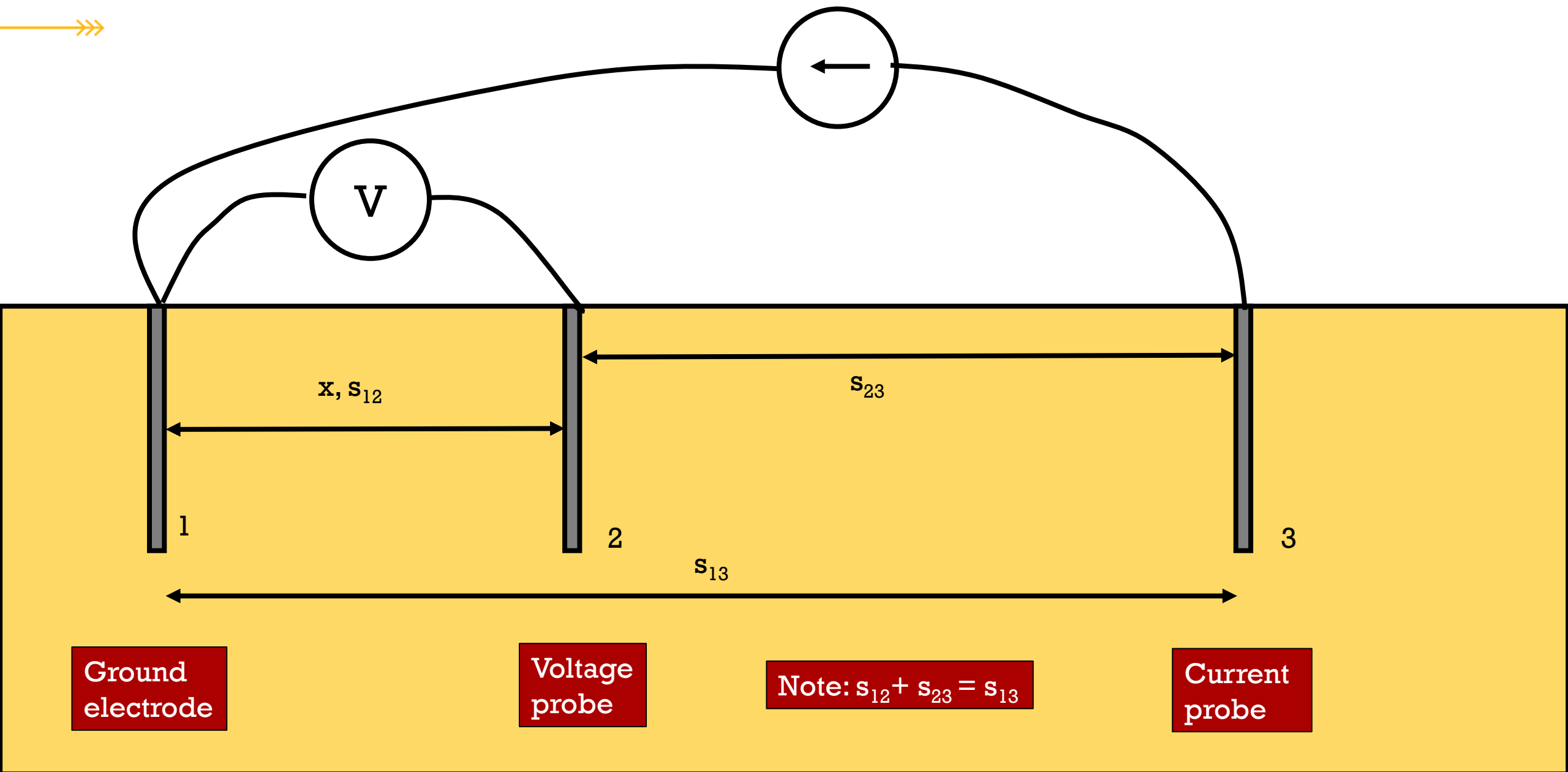
→ Fall of Potential Method



Current I is injected via an external voltage source between ground electrode and current electrode. The voltage (V) between the ground electrode and a moveable voltage probe (potential probe) is measured across various distances.

Resistance is computed as V/I .

Temporary probes/electrodes, used only for the test.



→ Fall of Potential Method [optional content]

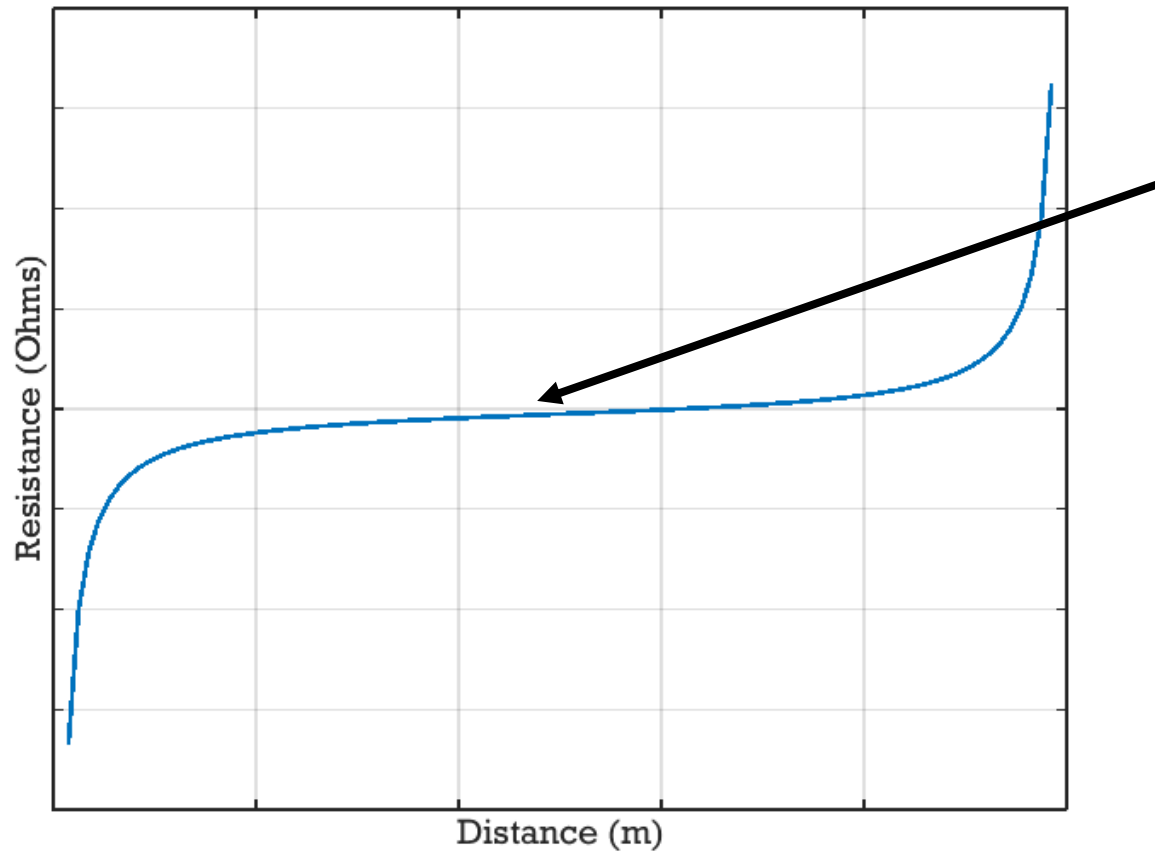
- It can be shown that as the distance of the voltage probe from the ground electrode increases, the measured resistance is equal to:

$$R_{\text{gnd}}(\mathbf{x}) = \frac{\rho}{2\pi} \left(\frac{1}{r} - \frac{1}{x} - \frac{1}{s_{13} - r} + \frac{1}{s_{23}} \right)$$

Note: $x + s_{23} = s_{13}$

- r : radius of the ground electrode, m
- x : separation distance between ground electrode and voltage probe, m
- s_{13} : separation distance between ground electrode and current probe, m
- s_{23} : separation distance between voltage probe and current probe, m

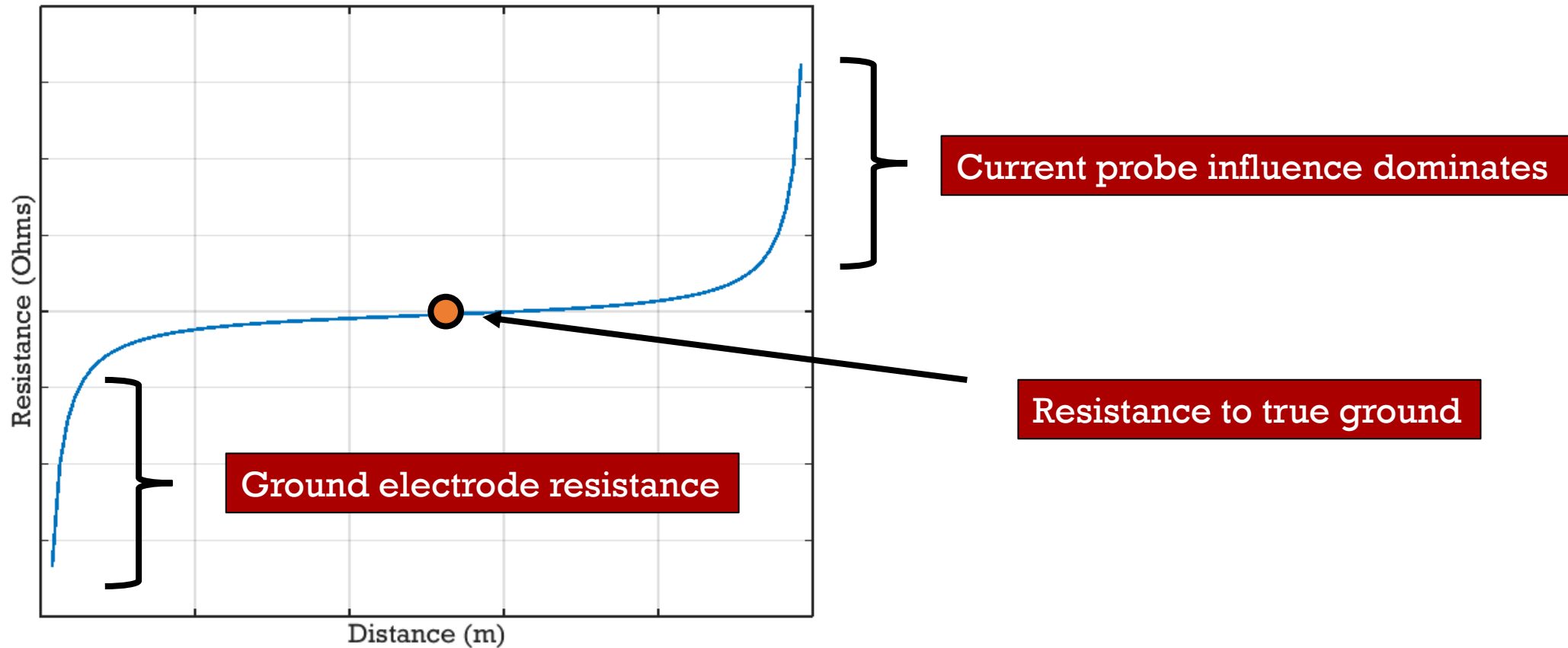
→ Fall of Potential Method [optional content]

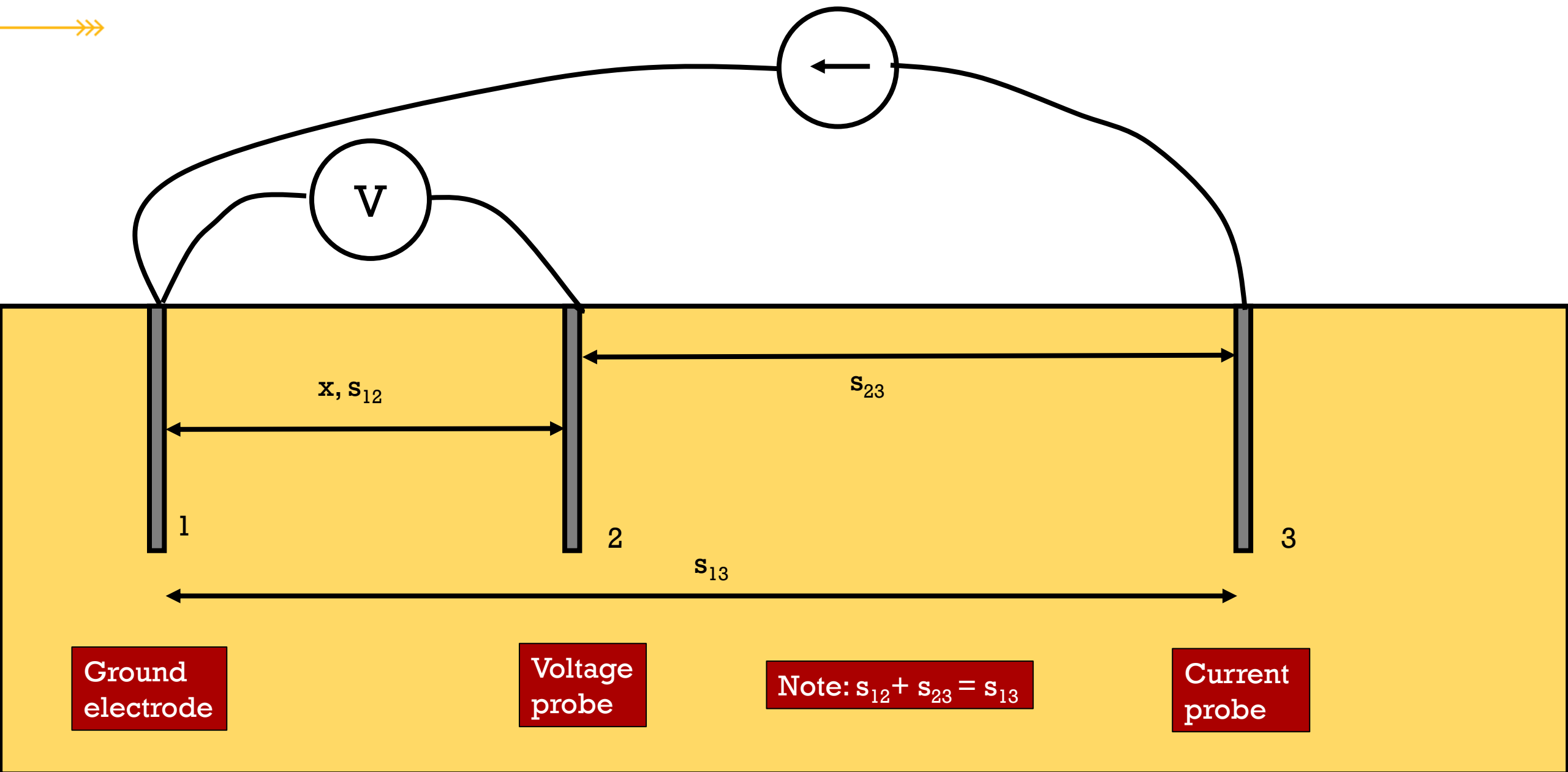


Resistance to true ground is found
at when x is 62% of the s_{13}

See: E. B. Curtis, "Some Of the
fundamental aspects of
ground resistance measurements,"
AIEE, Nov. 1958.

→ Fall of Potential Method [optional content]





→ Fall of Potential Method [optional content]

- The voltage measured by the voltmeter is the potential difference between the surface of the ground electrode and the voltage probe caused by the current dissipating through the earth from the ground electrode
- From before, we have:

$$V_{12,A} = \frac{\rho I}{2\pi} \left(\frac{1}{r} - \frac{1}{s_{12}} \right)$$

Recall from before:

$$V_{ab} = \frac{\rho I}{2\pi} \left(\frac{1}{r_a} - \frac{1}{r_b} \right)$$

Recall: s_{12} is the distance between the ground electrode and the voltage probe, and r is the radius of the ground electrode

→ Fall of Potential Method [optional content]

- But we also have current flowing to the current probe
- Using a similar equation, the voltage between 1 and 2 due to the second current probe is:

$$V_{12,C} = \frac{\rho I}{2\pi} \left(\frac{1}{s_{23}} - \frac{1}{s_{13} - r} \right)$$

Recall from before:

$$V_{ab} = \frac{\rho I}{2\pi} \left(\frac{1}{r_a} - \frac{1}{r_b} \right)$$

Recall: s_{23} is the distance between the voltage probe and current probe, and s_{13} is the distance between the ground electrode and current probe

» Superposition [optional content]

- Now use superposition to combine the voltages:

$$V_{12} = V_{12,A} + V_{12,C} = \frac{\rho I}{2\pi} \left(\frac{1}{r} - \frac{1}{s_{12}} \right) + \frac{\rho I}{2\pi} \left(\frac{1}{s_{23}} - \frac{1}{s_{13} - r} \right)$$

$$V_{12} = \frac{\rho I}{2\pi} \left(\frac{1}{r} - \frac{1}{s_{12}} + \frac{1}{s_{23}} - \frac{1}{s_{13} - r} \right)$$

$$R = \frac{\rho}{2\pi} \left(\frac{1}{r} - \frac{1}{s_{12}} + \frac{1}{s_{23}} - \frac{1}{s_{13} - r} \right)$$

Measured Resistance to True Ground [optional content]

- Recall that the resistance to true ground is: $R_g = \frac{\rho}{2\pi r}$

$$R = \frac{\rho}{2\pi} \left(\frac{1}{r} - \frac{1}{s_{12}} + \frac{1}{s_{23}} - \frac{1}{s_{13} - r} \right)$$

Select s_{12} so that these terms cancel each other out.
Can be solved algebraically

- Assuming $r \ll s_{13}$, then when $s_{12} = 0.62s_{13}$, $R = R_g$. Hence, You should measure the resistance at 62% of the distance between the ground electrode and the current probe

Example [optional content]

- Let the distance between the ground electrode and current probe be 100m, and let $r = 0.01\text{m}$
- Then:
 - $s_{13} = 100\text{m}$
 - $s_{12} = 0.62 \times 100 = 62\text{m}$
 - $s_{23} = s_{13} - s_{12} = 100 - 62 = 38\text{m}$

$$\begin{aligned} R &= \frac{\rho}{2\pi} \left(\frac{1}{r} - \frac{1}{s_{12}} + \frac{1}{s_{23}} - \frac{1}{s_{13} - r} \right) = \frac{\rho}{2\pi} \left(\frac{1}{r} - \frac{1}{62} + \frac{1}{38} - \frac{1}{100 - 0.01} \right) \\ &= \frac{\rho}{2\pi} \left(\frac{1}{r} - 0.161 + 0.263 - 0.100 \right) \approx \frac{\rho}{2\pi} \left(\frac{1}{r} \right) \end{aligned}$$

→ Ground Resistance of Objects

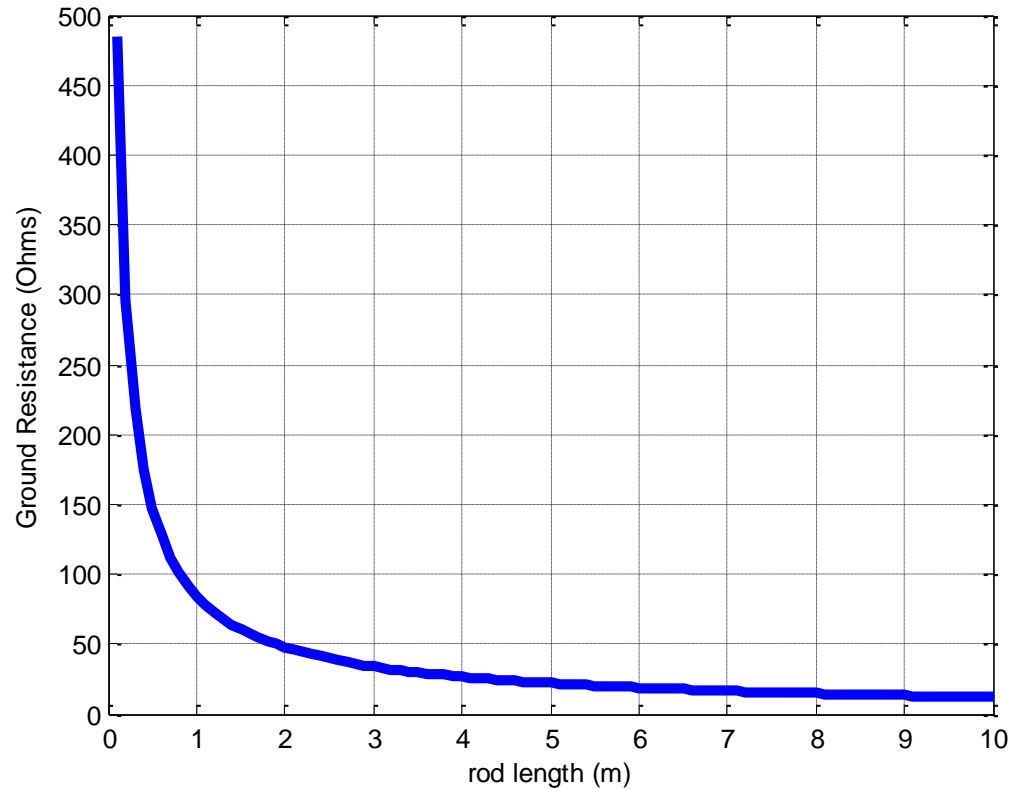
Object	Ground Resistance	Parameters
Rod	$\frac{\rho}{2\pi l} \ln\left(\frac{2l+r}{r}\right)$	l is the length of the rod r is the radius of the rod
Circular plate (disk) at the surface	$\frac{\rho}{4r}$	r is the radius of the disk
Buried wire	$\frac{\rho}{2\pi l} \left(\ln\left(\frac{l}{r}\right) + \ln\left(\frac{l}{2d}\right) \right)$	l is the length of the wire r is the radius of the wire d is the depth at which the wire is buried

» Exercise

The ground resistance of a rod is found by the equation below. If the rod length (ℓ) increases, what happens to the ground resistance?

$$R_g = \frac{\rho}{2\pi\ell} \ln\left(\frac{2\ell + r}{r}\right)$$

→ Ground Resistance of a Rod



$r = 0.01 \text{ m}$
 $\rho = 100 \text{ Ohms-m}$

→ Soil Resistivity

- Soil resistivity widely varies

	Soil Composition			
	Wet Organic	Moist	Dry	Bedrock
Resistivity ρ (Ω -m)	10	100	1,000	10,000

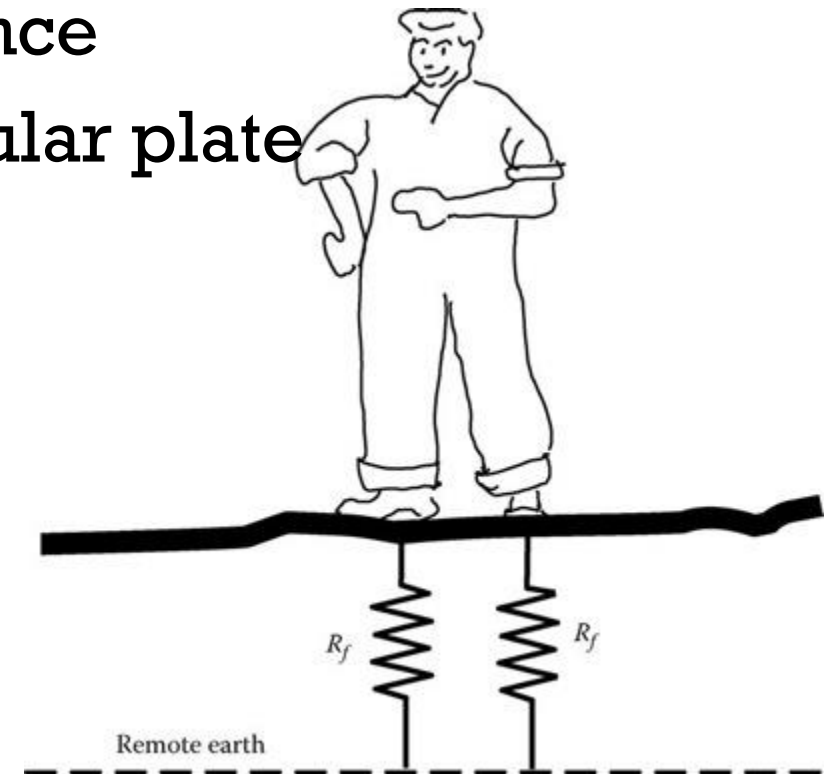
Ground Resistance of People

→ Ground Resistance of People

- Each foot of a person has a ground resistance
- Approximate the bottom of a foot as a circular plate

$$R_f = \frac{\rho}{4r}$$

From a previous table



➤➤ Ground Resistance of People

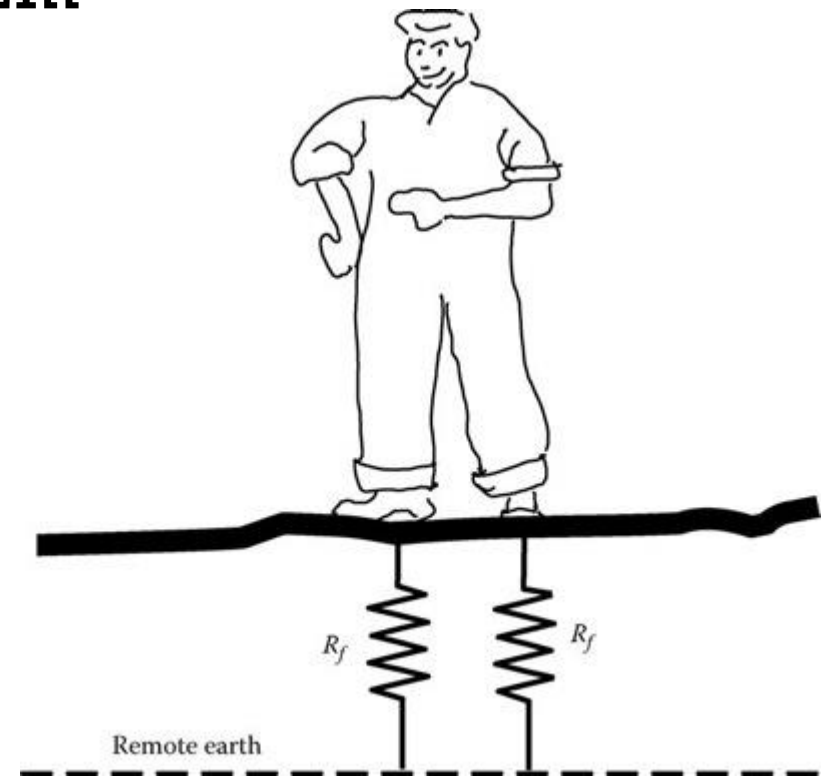
- Assuming an average footprint of 0.02m^2

$$A = \pi r^2$$

$$r = \sqrt{\frac{A}{\pi}}$$

$$R_f = \frac{\rho}{4\sqrt{\frac{A}{\pi}}}$$

$$R_f = \frac{\rho}{4\sqrt{\frac{0.02}{\pi}}} \approx 3\rho$$



→ Ground Resistance of People

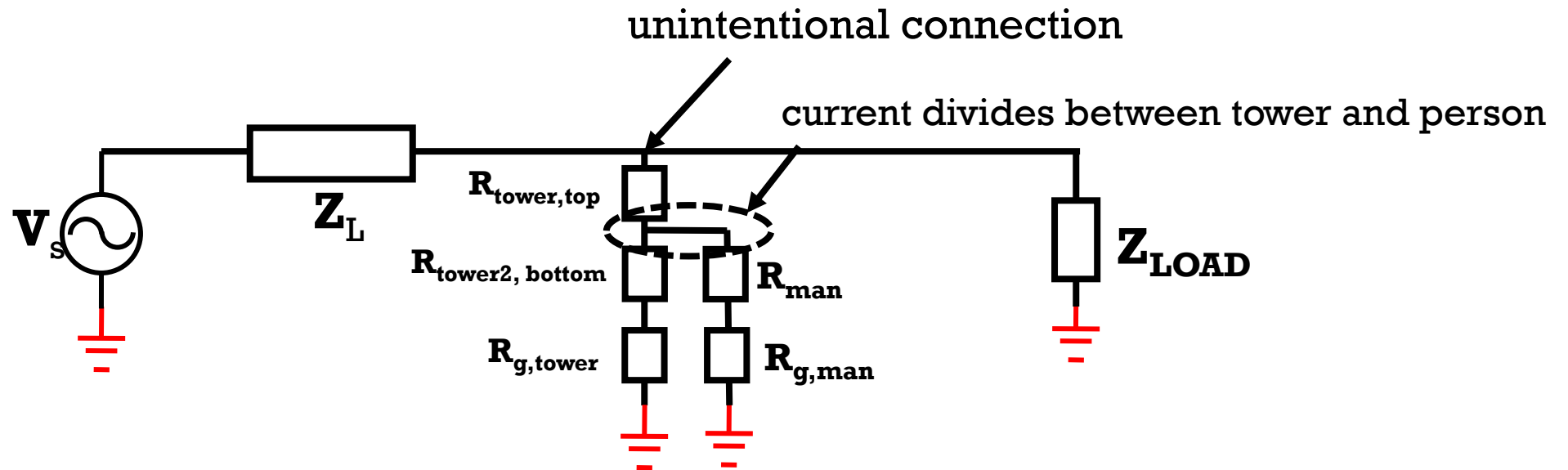
- For a person standing, their ground resistance is the parallel combination of the ground resistance of each foot

$$R_g = R_f || R_f = 0.5R_f \approx 1.5\rho$$

Touch Potential

Touch Potential

- Touch potentials occur when current flows through an object to ground
- A grounded person touches the object, and the current will divide, some of it flowing through the person, possibly causing harm

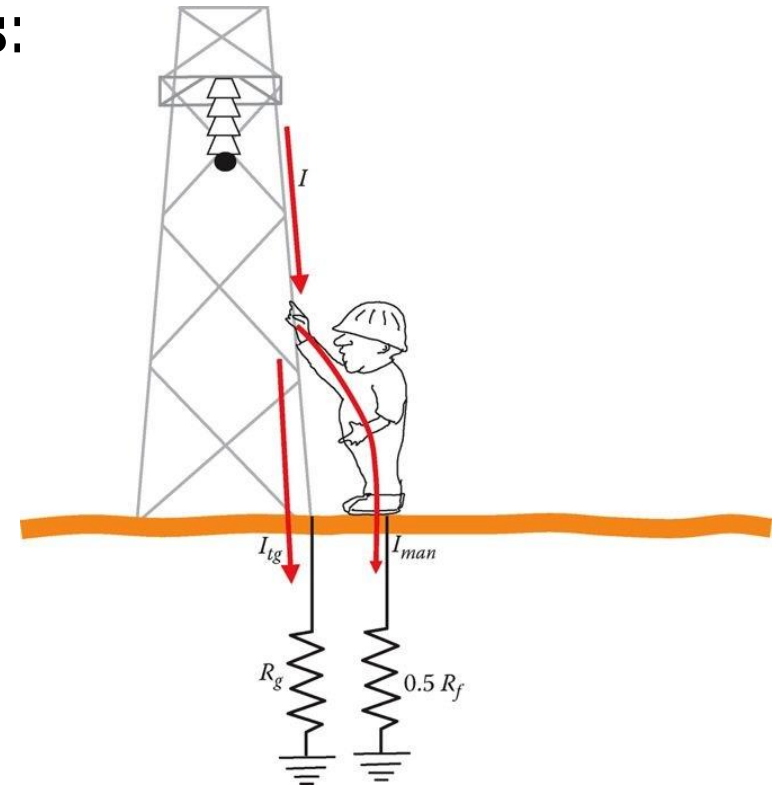
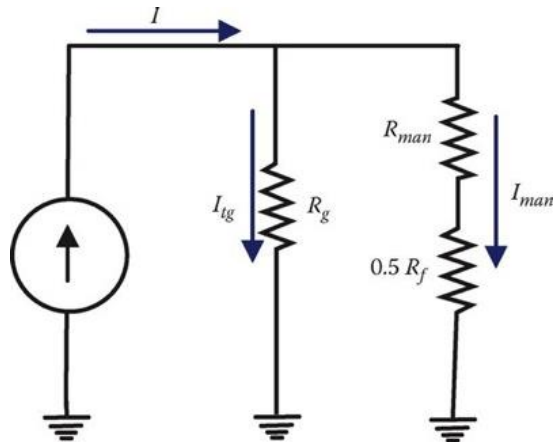


Touch Potential

- Current through the man is computed as:

$$I_{man} = I \left(\frac{R_g}{R_g + R_{man} + 0.5R_f} \right)$$

Current divider

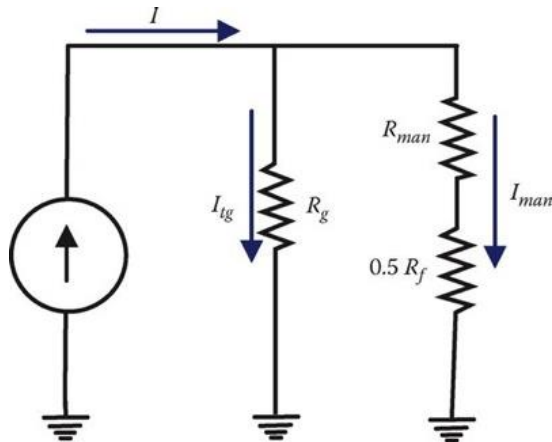


➤ Touch Potential

- Ground Potential Rise (GPR): voltage that the grounded object attains with respect to the remote earth

$$\text{GPR} = I_{tg} R_g$$

Also known as “earth potential rise”



GPR is important in substation design

» Exercise (Example 9.4)

A power line insulator has partially failed and 10 A passes through the tower structure to the ground. Assume that the tower ground is a hemisphere with a radius of 0.5 m, and the soil surrounding the hemisphere is moist.

Compute the voltage of the tower (wrt ground)

Exercise (Example 9.4)

A power line insulator has partially failed and 10 A passes through the tower structure to the ground. Assume that the tower ground is a hemisphere with a radius of 0.5 m, and the soil surrounding the hemisphere is moist.

Compute the voltage of the tower (wrt ground)

Answer. $R_g = \frac{100}{2\pi \times 0.5} = 32\Omega$ Using the resistivity of moist soil

$$GPR = IR_g = 10 \times 32 = 320V$$

» Exercise (Example 9.4)

A power line insulator has partially failed and 10 A passes through the tower structure to the ground. Assume that the tower ground is a hemisphere with a radius of 0.5 m, and the soil surrounding the hemisphere is moist.

Assume that a man with body resistance of 1.0 k Ω touches the tower while standing on ground. Compute the current passing through the man.

Exercise (Example 9.4)

Assume that a man with body resistance of $1.0 \text{ k}\Omega$ touches the tower while standing on ground. Compute the current passing through the man.

$$R_f = 3\rho = 300\Omega$$

$$\mathbf{I}_{\text{man}} = \mathbf{I} \left(\frac{R_g}{R_g + R_{\text{man}} + 0.5R_f} \right) = \mathbf{I}_{\text{man}} = 10 \left(\frac{32}{32 + 1000 + 150} \right) = 270.7\text{mA}$$

We can compute how long the person would survive for using Dalziel's formula.

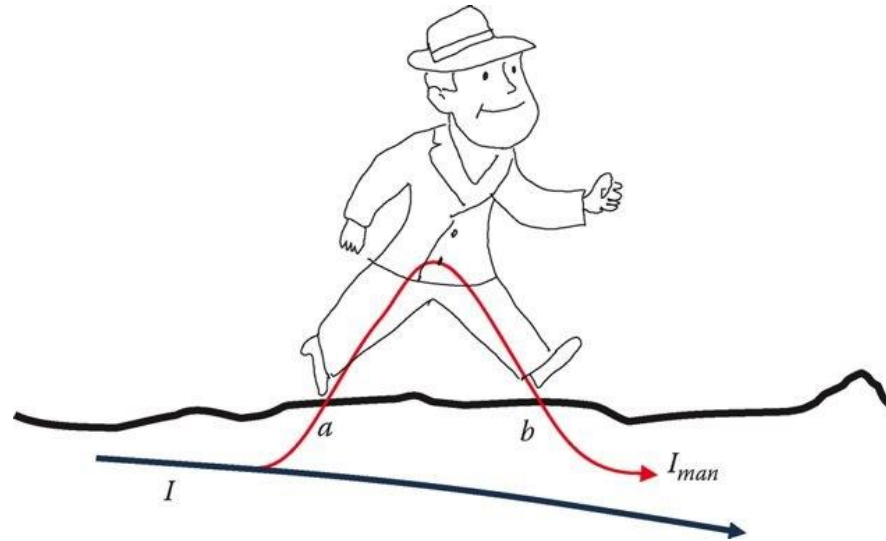
» An Aside...

- On occasion, a copper thief will attempt to remove the ground connection from a transformer or tower
- If there is even a small amount of current flowing to ground, an extremely high voltage can develop once the connection is cut, which can injure/kill the thief

Step Potential

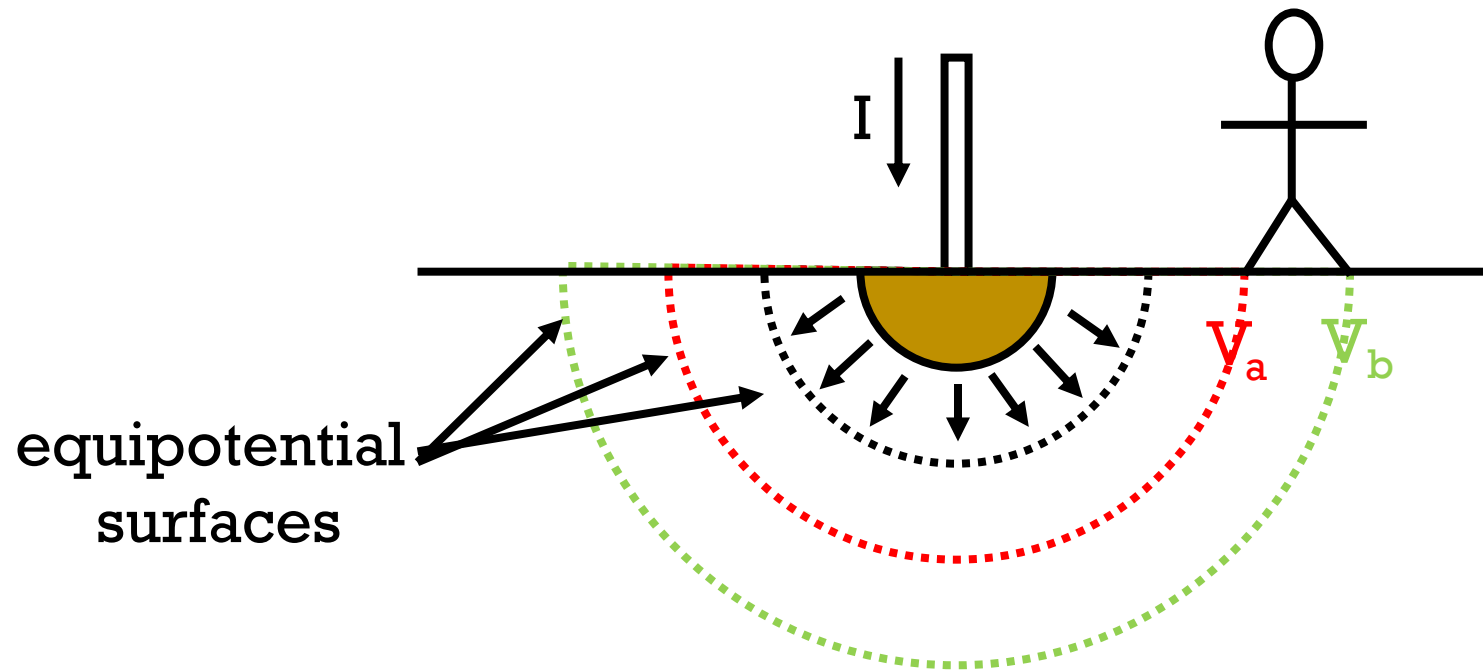
Step Potential

- When large amounts of current flow through the ground, a person walking can be shocked due to the voltage potential between their feet



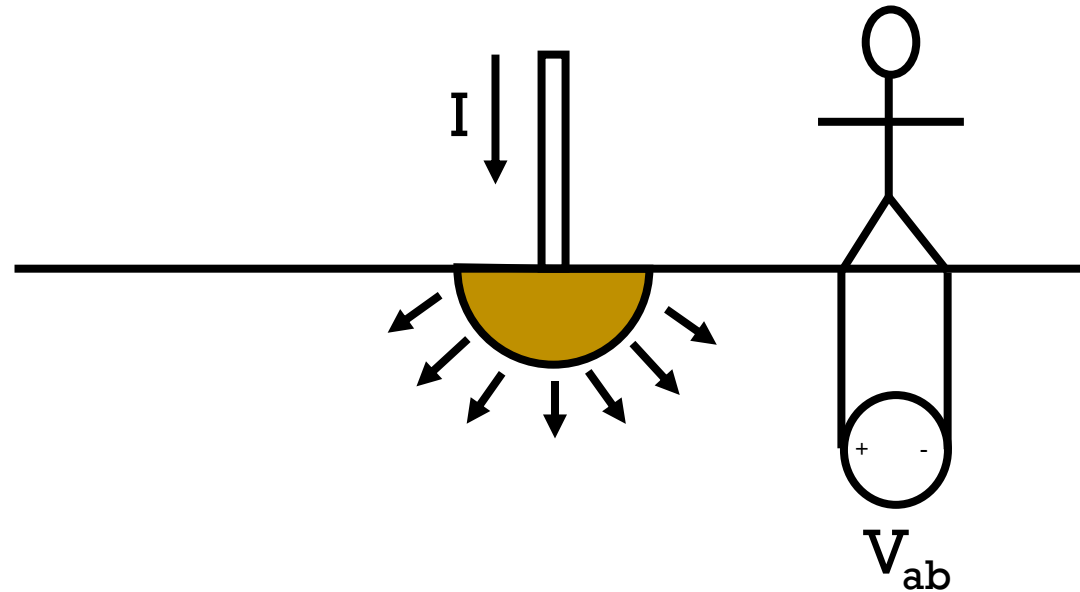
Step Potential Due to Ground Current Through a Hemisphere

There is a voltage difference between the person's feet: V_{ab}



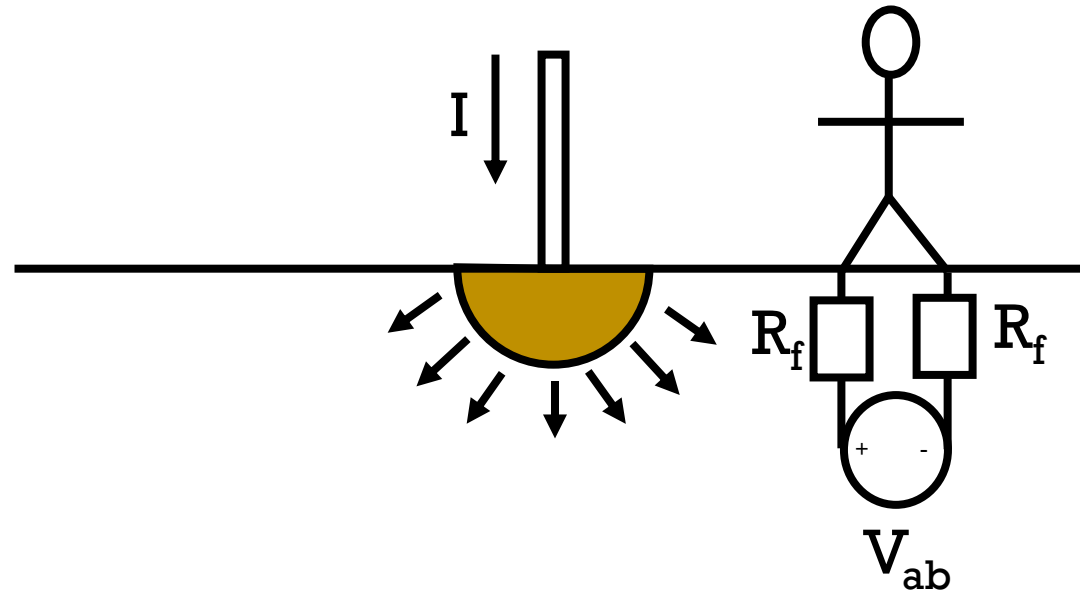
Step Potential Due to Ground Current Through a Hemisphere

Model V_{ab} as a voltage source



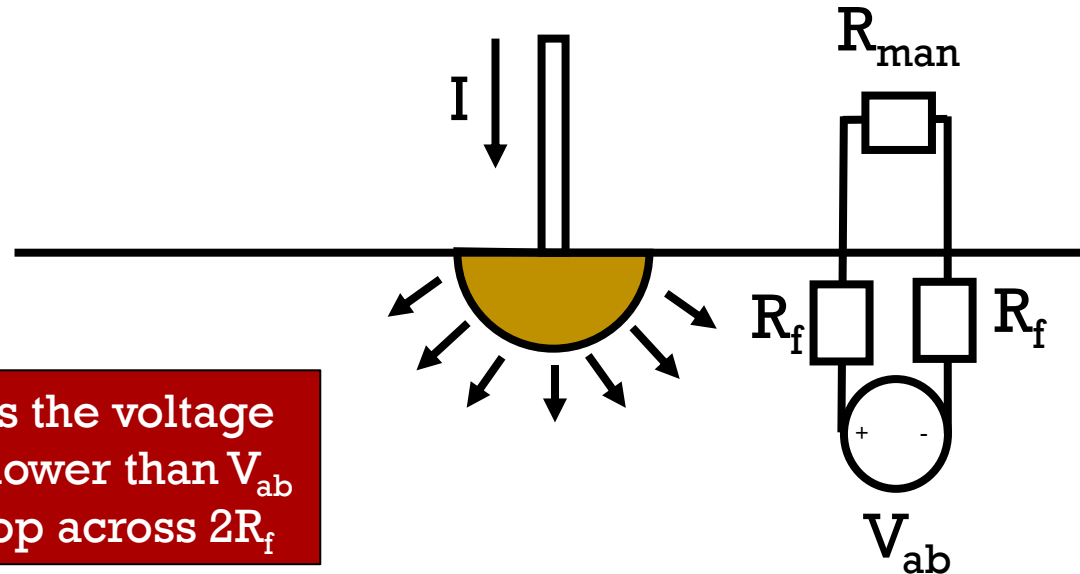
Step Potential Due to Ground Current Through a Hemisphere

Need to include the ground resistance of the person (each foot)



Step Potential Due to Ground Current Through a Hemisphere

Now model the person's leg-to-leg resistance



The “Step Potential” is the voltage across R_{man} , which is lower than V_{ab} due to the voltage drop across $2R_f$

Step Potential

- If the current flows to ground through a hemisphere (previous example), the voltage between the feet is computed as:



$$V_{ab} = \int_a^b E(x) dx = \int_a^b \rho J(x) dx = \frac{\rho I}{2\pi} \left(\frac{1}{r_a} - \frac{1}{r_b} \right)$$

where r_a is set to distance between the center of the hemisphere and the closest foot, and r_b is the distance between the center of the hemisphere and the furthest foot

Step Potential

- Step potential is why if you are near a downed power line or caught outside during a lightning storm, you should use small steps (or shuffle)

■ The proper way to move away from the power line is to **shuffle away with small steps**, keeping your feet together and on the ground at all times. This will minimize the potential for a strong electric shock.




Downed Power Line Safety

Downed power lines can carry an electric current strong enough to cause serious injury or even death. Electricity wants to move from a high voltage zone to a low voltage zone – and it could do that through your body.

Safety Tips

- If you see a downed power line, move **away from it** and anything touching it. The ground around power lines – up to 35 feet away – may be energized.
- You cannot tell whether or not a power line is energized just by looking at it. You should **assume that all downed power lines are live**.
- The proper way to move away from the power line is to **shuffle away with small steps**, keeping your feet together and on the ground at all times. This will minimize the potential for a strong electric shock.
- If you see someone who is in direct or indirect contact with the downed line, **do not touch the person**. You could become the next victim. Call 911 for help.
- Do not attempt to move a downed power line or anything else in contact with it by using an object such as a broom or stick. Even non-conductive materials like wood or cloth, can conduct electricity if even slightly wet.
- Be careful not to **touch or step in water** near where a downed power line is located.
- Do not drive over downed power lines.
- If your car comes in contact with a downed power line while you are inside, **stay in the car**. Honk your horn to summon help, but direct others to stay away from your car.
- If you must leave your car because it is on fire, jump out of the vehicle with both feet together and **avoid contact with both the car and the ground** at the same time. Shuffle away from the car.



» Exercise (Example 9.6)

A short circuit current of 1000 A passes through hemisphere grounding object. A person is walking 5 m away from the center of the hemisphere. Assume that the leg-to-leg body resistance of the person is 1 k Ω , and the soil surrounding the hemisphere is moist. Compute the current through the person and his step potential. Assume the person's stride is 0.6 m.

Exercise (Example 9.6)

A short circuit current of 1000 A passes through hemisphere grounding object. A person is walking 5 m away from the center of the hemisphere. Assume that the leg-to-leg body resistance of the person is 1 k Ω , and the soil surrounding the hemisphere is moist. Compute the current through the person and his step potential. Assume the person's stride is 0.6 m.

Answer.
$$V_{th} = \frac{I\rho}{2\pi} \left(\frac{1}{r_a} - \frac{1}{r_b} \right) = \frac{1000 \times 100}{2\pi} \left(\frac{1}{5} - \frac{1}{5.6} \right) = 341V$$

$$R_f = 3\rho = 300\Omega$$

$$I_{man} = \frac{341}{600 + 1000} = 213.13mA$$

$$V_{step} = I_{man} \times R_{man} = 213.13V$$