

# 09-PV Array Basics

*Off-Grid Electrical Systems in Developing Countries*  
Chapter 7.1–7.5

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## Learning Outcomes

At the end of this lecture, you will be able to:

- ✓ understand the significance of PV in off-grid systems
- ✓ define *irradiance* and *insolation*
- ✓ develop and analyze a circuit model of an ideal PV cell
- ✓ describe and analyze the I-V curve of a PV cell and how it relates to the power produced by the cell

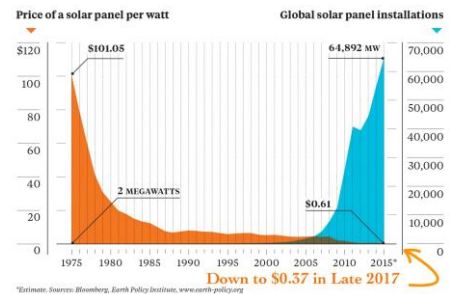
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## Photovoltaic (PV) Arrays

- PV arrays are commonly used in off-grid systems
- Default choice of energy conversion technology
  - Adequate sunlight in most energy-impooverished regions
  - Low data collection requirements
  - Low and falling prices

### Solar on Fire

As prices have dropped, installations have skyrocketed.



Note: prices in SSA tend to be higher

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## The Solar Resource

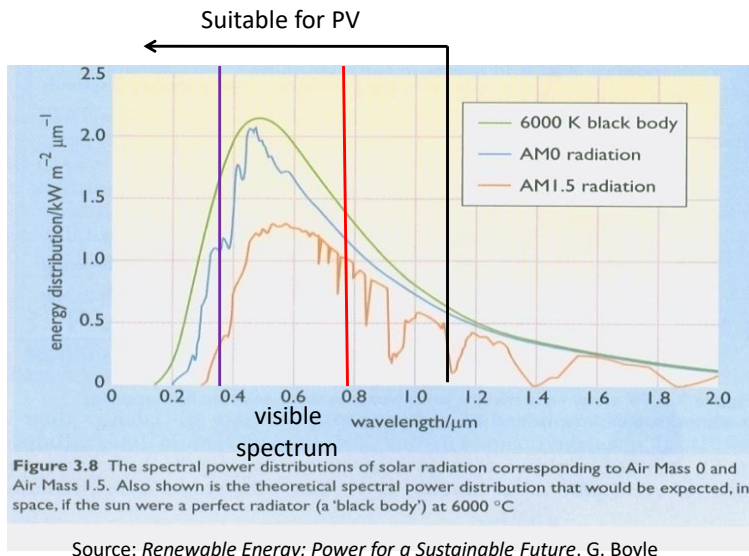
- Energy from the sun is transferred to the earth through electromagnetic radiation
  - $3.8 \times 10^{24}$  J annually (10,000 mankind's usage)
- Radiation from the sun has a wide spectral distribution, some of which is within the visible range



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## Irradiance and Insolation

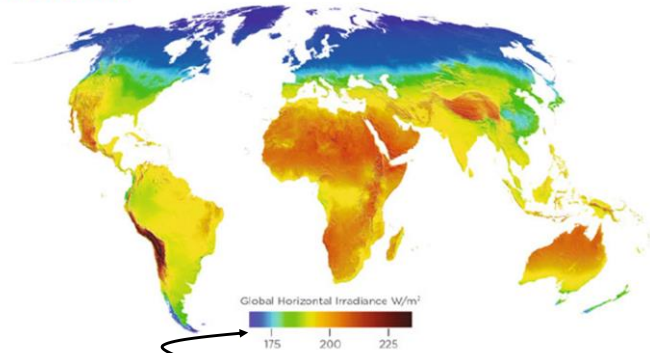
- Irradiance ( $G$ ): density of the power produced by sunlight ( $\text{W}/\text{m}^2$ )
- Insolation (not “insulation”) ( $I$ ): energy provided by sunlight per square meter of area over a period of time, usually per day ( $\text{kWh}/\text{m}^2/\text{day}$ )

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# Solar Resource

VAISALA



Units are average irradiance

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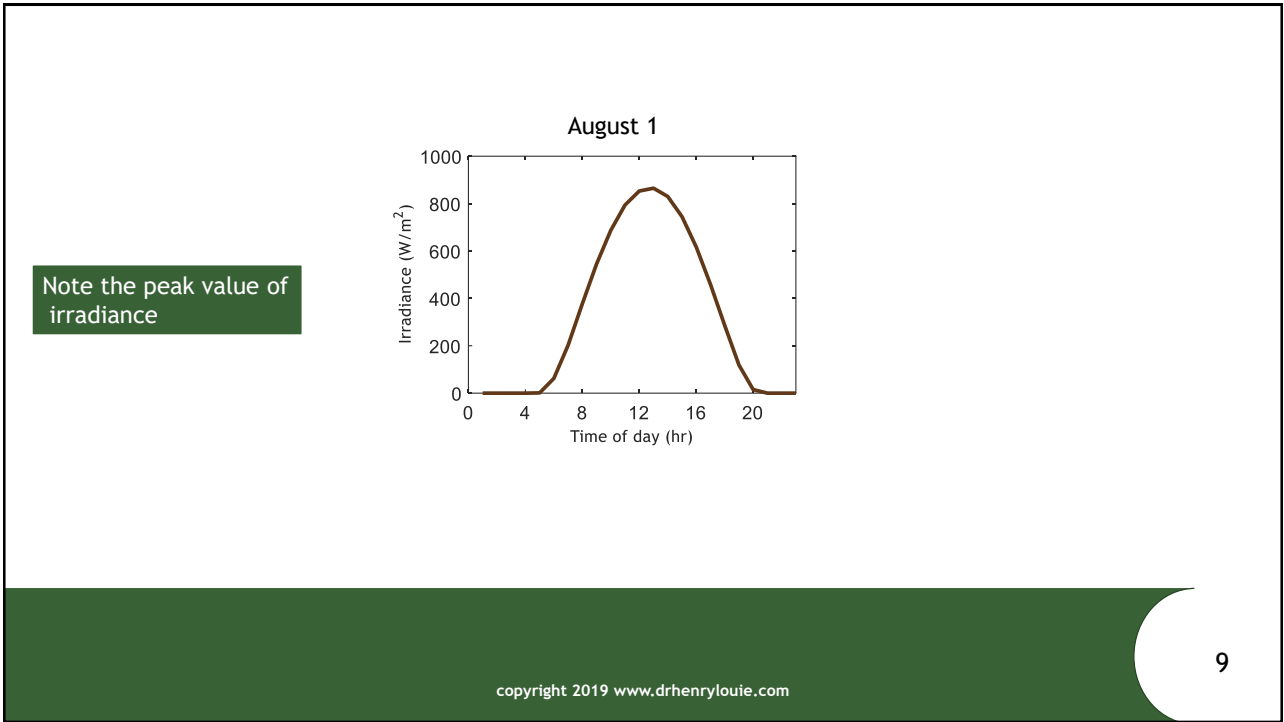
# Typical Values

- Irradiance of  $1000 \text{ W/m}^2$  is approximately the sunlight a horizontal surface receives on a clear sunny day in the mid-continental U.S.
- Typical insolation: 3.5 to 6 kWh/m<sup>2</sup>/day

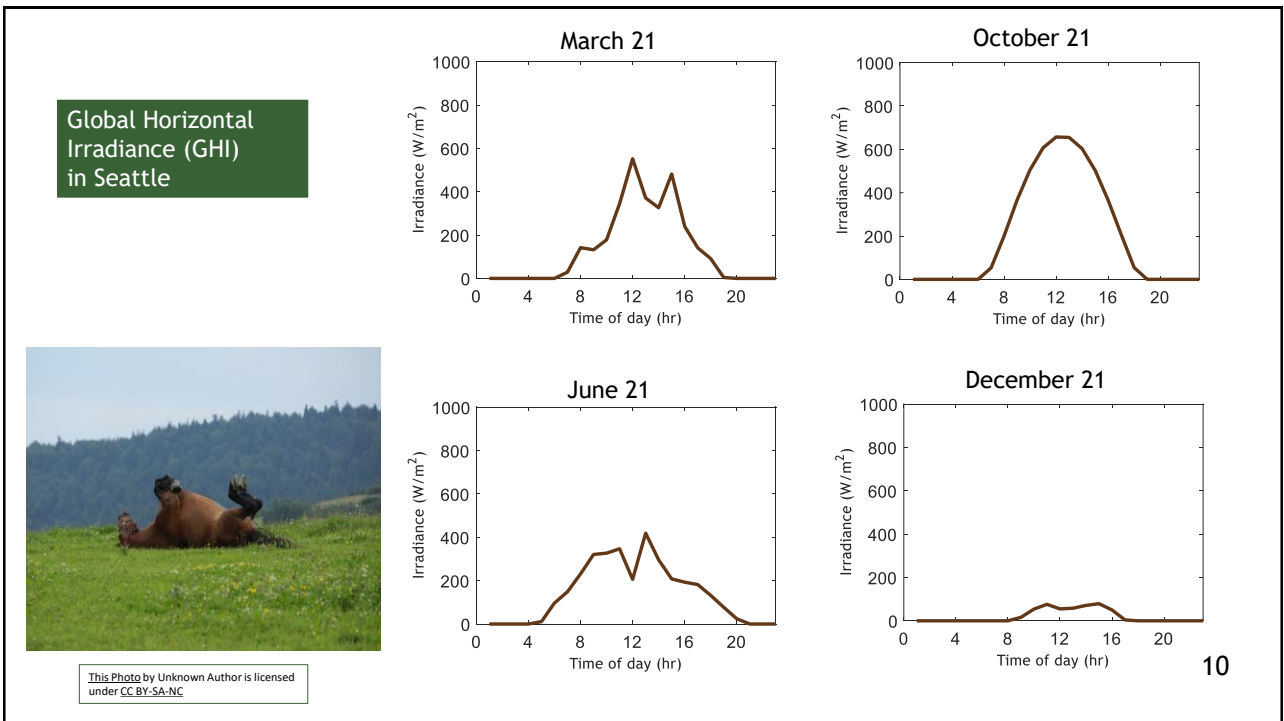
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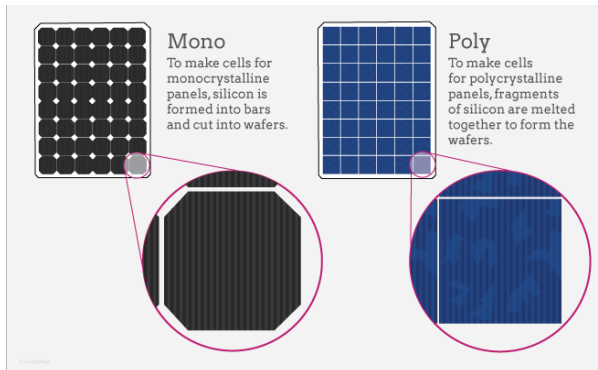


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## Physical Description



There are two common types of PV cells: mono-crystalline and poly-crystalline

A typical 0.156m x 0.156m (6.1in x 6.1 in) cell produces 2 to 4 W

source <https://www.civicsolar.com/support/installer/articles/monocrystalline-cells-vs-polycrystalline-cells-whats-difference>

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## How Polycrystalline PV Cell are Manufactured

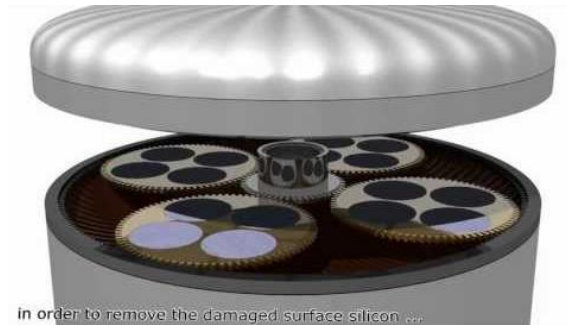


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## How Monocrystalline PV Cell are Manufactured



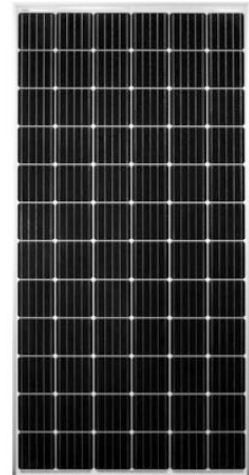
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## PV Modules

- PV modules are made from several cells that are connected together (in series)
- Largest modules are 300 to 400W and contain 72 series-connected cells
  - Weight ~24 kg
  - Area ~2 m<sup>2</sup>



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## Basic Principles

- Electricity is produced via the *photovoltaic* effect
- Power produced by PV module is approximately proportional to incident irradiance
- Power is inherently DC

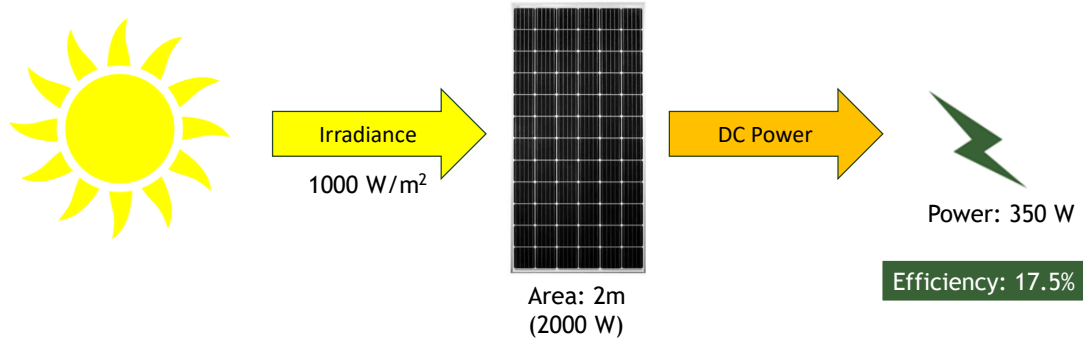
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## Example Efficiency



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## Basic Principles

- PV cell is a p-n junction (diode)
- Recall p-n junctions are made from doped silicon crystals, which have internal built-in electric fields
- Photons from sunlight excite electrons into the conduction band, built-in field separates charge and voltage appears

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# Unilluminated PV Cell

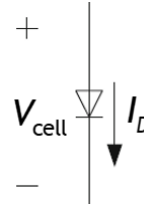
A PV cell in the dark (unilluminated) operates as a diode

$$I_D = I_0 (e^{V_D/V_T} - 1)$$

$$V_T = \frac{nkT}{q}$$

Note error in book (7.2)

$V_T$ : 25.8mV at room temperature  
 $I_0$ : usually small (e.g.  $10^{-9}$  A)



$k$ : Boltzmann's constant ( $1.38 \times 10^{-23}$  J/K)  
 $n$ : ideality factor (unitless), is equal to 1 for an ideal diode  
 $T$ : temperature (K)  
 $q$ : charge  $1.602 \times 10^{-19}$  (C)  
 $I_0$ : reverse bias saturation current (A)

## Exercise

What voltage appears across an open-circuited PV cell when it is unilluminated? Let  $V_t = 25.8$  mV and  $I_0 = 10^{-9}$  A.

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What voltage appears across an open-circuited PV cell when it is unilluminated? Let  $V_t = 25.8$  mV and  $I_0 = 10^{-9}$  A.

$$I_D = I_0 \left( e^{V_D/V_T} - 1 \right)$$

$$\frac{I_D}{I_0} = \left( e^{V_D/V_T} - 1 \right)$$

$$\frac{I_D}{I_0} + 1 = e^{V_D/V_T}$$

$$\ln \left( \frac{I_D}{I_0} + 1 \right) = \frac{V_D}{V_T}$$

$$\ln \left( \frac{I_D}{I_0} + 1 \right) V_T = V_D$$

$$\ln \left( \frac{0}{I_0} + 1 \right) V_T = V_D$$

$$0 \times V_T = V_D$$

$$V_D = 0V$$

No voltage appears.

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## Illuminated PV Cell

Effect of photons exciting electrons is modeled as “illumination current”

- always positive
- approx. proportional to incident irradiance, cell area, and cell efficiency
- typical range: 0.5 to 9A

$$I_{\text{cell}} = I_G - I_0 \left( e^{V_{\text{cell}}/V_T} - 1 \right)$$

$$I_{\text{cell}} = I_G - I_D$$

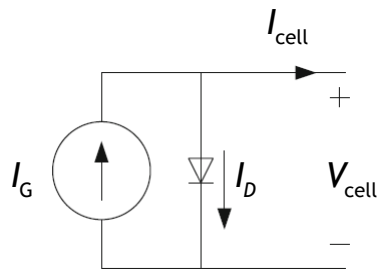
$I_G$ : illumination current (A)  
 $I_{\text{cell}}$ : current out of PV cell (A)  
 $V_{\text{cell}}$ : PV cell voltage (V)

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## Ideal Circuit Model



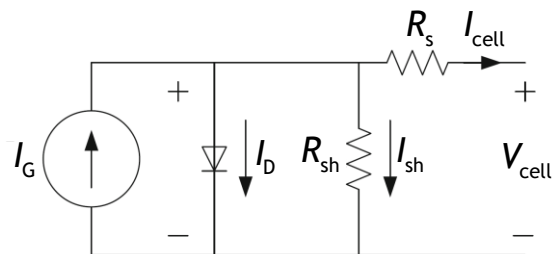
Assume PV cell is ideal unless stated otherwise

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## Circuit Model with Losses (non-ideal)



See text, Chapter 7.5.1

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## Exercise

Which variable should be set to zero when the PV cell is operated under open-circuit conditions?

- A.  $I_G$
- B.  $I_0$
- C.  $I_{\text{cell}}$
- D.  $V_T$

$$I_{\text{cell}} = I_G - I_0 \left( e^{V_{\text{cell}}/V_T} - 1 \right)$$

## Exercise

Which variable should be set to zero when the PV cell is operated under open-circuit conditions?

- A.  $I_G$
- B.  $I_0$
- C.  $I_{\text{cell}}$
- D.  $V_T$

$$I_{\text{cell}} = I_G - I_0 \left( e^{V_{\text{cell}}/V_T} - 1 \right)$$

## PV Cell Open-Circuit Voltage

Open-circuit voltage of a PV cell is computed from

$$I_{\text{cell}} = I_G - I_0 \left( e^{V_{\text{cell}}/V_T} - 1 \right)$$

$$\frac{I_G - I_{\text{cell}}}{I_0} + 1 = e^{V_{\text{cell}}/V_T}$$

$$V_T \ln \left( \frac{I_G - I_{\text{cell}}}{I_0} + 1 \right) = V_{\text{cell}}$$

$$V_{\text{cell,OC}} = V_T \ln \left( \frac{I_G}{I_0} + 1 \right)$$

Typical open-circuit voltage is between 0.6 and 0.7V

## Exercise

Which of these affects the open-circuit voltage of a PV cell?

- A. Temperature
- B. Ideality factor
- C. Irradiance
- D. Reverse saturation current?

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## Short-Circuit Current

Under short-circuit conditions  $V_{\text{cell}} = 0V$

$$I_{\text{cell}} = I_G - I_0 \left( e^{V_{\text{cell}}/V_T} - 1 \right)$$

$$I_{\text{SC}} = I_G - I_0 \left( e^{0/V_T} - 1 \right)$$

$$I_{\text{SC}} = I_G - I_0 (1 - 1)$$

$$I_{\text{SC}} = I_G$$

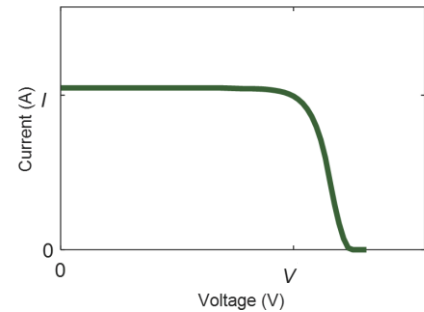
The short-circuit current is equal to the illumination current. The illumination current can be determined by shorting the PV cell and measuring the current (assuming the PV cell is ideal)

## I-V Curve

The current–voltage curve of PV cell can be plotted solving

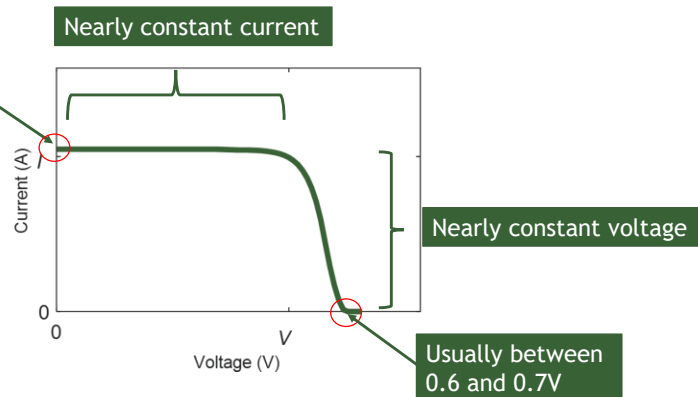
$$I_{\text{cell}} = I_G - I_0 \left( e^{V_{\text{cell}}/V_T} - 1 \right)$$

for different values of  $V_{\text{cell}}$



## I-V Curve

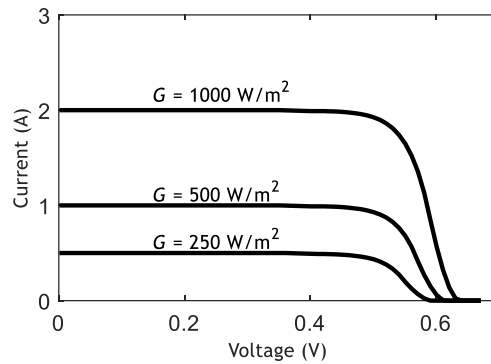
Equal to illumination current





## I-V Curve at Different Irradiance

Recall that illumination current increases with irradiance



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## Maximum Power Point

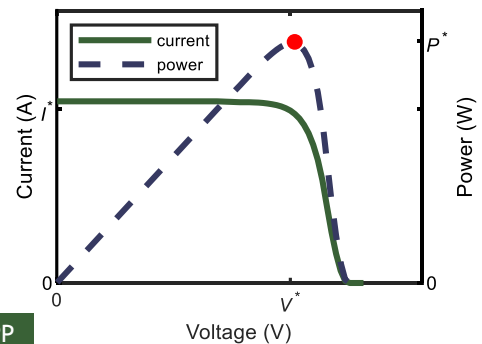
- Power output is found from

$$P_{\text{cell}} = I_{\text{cell}} V_{\text{cell}}$$

- PV arrays have a unique maximum power operating point (maximum power point, MPP)

$$P_{\text{cell}}^* = V_{\text{cell}}^* I_{\text{cell}}^*$$

here the \* indicates that the values correspond to the MPP (not complex conjugate)



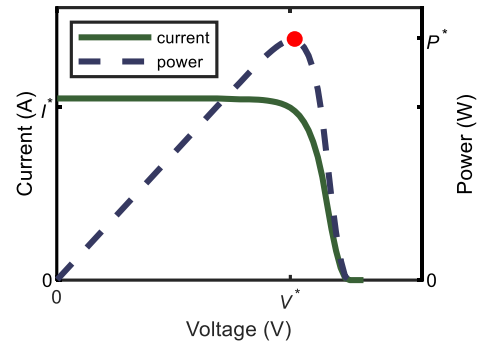
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## Maximum Power Point

- MPP voltage might not correspond to load impedance or battery voltage
- Use a “maximum power point tracker” to ensure PV cell (module) is operating at the maximum power point

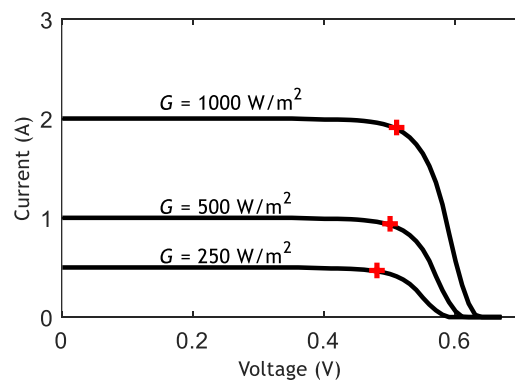


## Maximum Power Point at Different Irradiances

Maximum power point voltage and current depends on the irradiance

$$P_{\text{cell}}^*(G) = V_{\text{cell}}^*(G) I_{\text{cell}}^*(G)$$

Here we show that the voltage, current, and power at the maximum power point are functions of the irradiance




# Contact Information

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