

11-Conductor Sizing

ECEGR 3500

Electrical Energy Systems

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→ Overview

- Conductor Sizing
- Voltage Limits
- Thermal Limits
- Influence of Distribution System Design

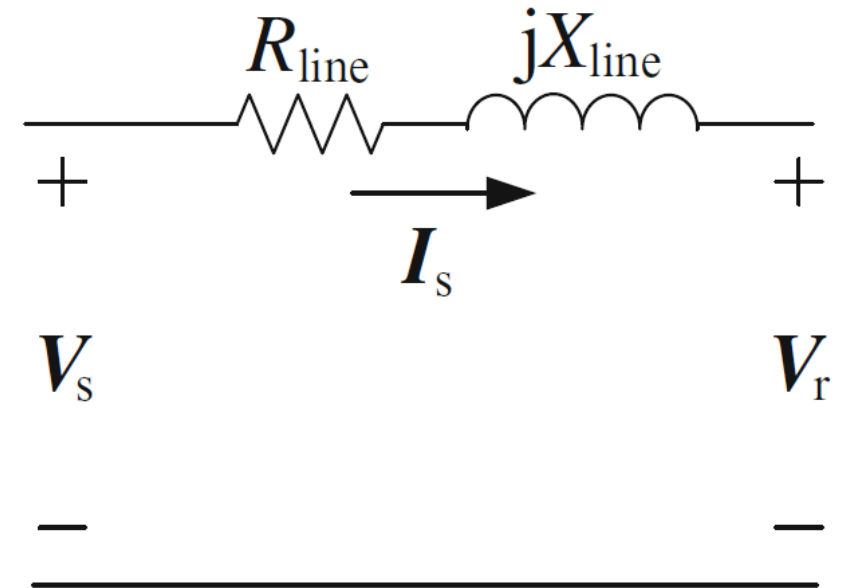
→ Conductor Sizing

- An important consideration in energy system design is conductor selection
- Considerations:
 - Insulation type
 - Appropriate for environment (indoor, outdoor, buried, corrosive, shielded, etc.)
 - Conductor cross-sectional area
 - Voltage drop
 - Heat dissipation (thermal limit)

Conductor Model

- A conductor can be modeled as a series impedance between a sending end and a receiving end
 - V_s : sending-end voltage, V
 - V_r : receiving-end voltage, V
 - I_s : conductor current, A
 - R_{line} : conductor resistance, Ω
 - X_{line} : conductor inductive reactance, Ω

$$Z_{\text{line}} = R_{\text{line}} + jX_{\text{line}}$$



Conductors longer than ~50km have shunt capacitance (to ground) that should be included in the model

→ Conductor Resistance

- AC resistance of conductor depends on:

- ℓ : length, m
- ρ : resistivity, Ohm-m
- A_{line} : cross-sectional area, m²
- s : skin effect (unitless)

$$R_{\text{line}} = s\rho \frac{\ell}{A_{\text{line}}}$$

- Skin effect is frequency dependent, and increases resistance by ~1-4%

→ Inductive Reactance

- Inductance of a line depends on:
 - Length of line
 - Physical distance separating the phases (or hot and neutral in single phase)
 - Effective radius of the conductors (bundling)
- We will not derive how the inductance is computed, but note:

$$X_{\text{line}} = \omega L$$

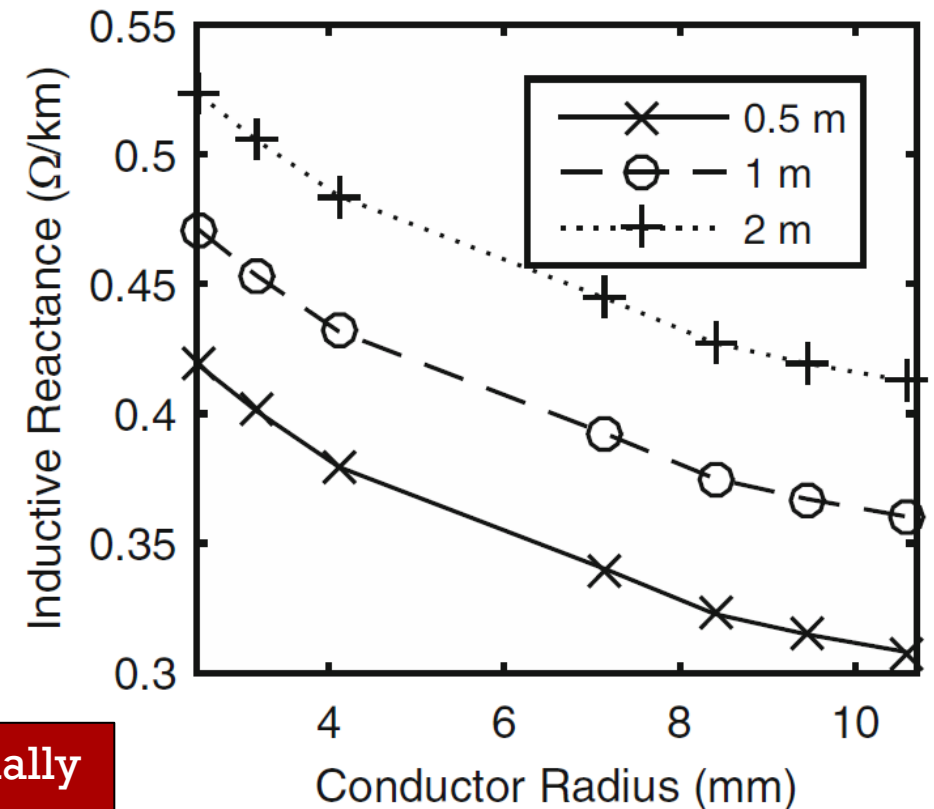
- ω : frequency, rad/s
- L : inductance, H

Inductive Reactance

- As conductors are placed farther apart, the inductive reactance increases (non-linearly)
- As cross-sectional area increases, the inductive reactance decreases

To minimize conductor inductance, place conductors close together and use larger conductors

Low-voltage conductors in a circuit (like in a house) are usually placed close enough that we can ignore inductance



→ Smaller Conductors vs. Larger Conductors

- Trade-off of conductor cross-sectional area:
 - Material and installation costs increase with cross-sectional area
 - Larger cross-sectional area reduces losses (less heat) and reduces voltage drop
- General approach: select the smallest conductor size that results in an acceptable voltage drop (from sending to receiving end) AND does not exceed thermal limits (does not overheat)

→ Voltage Drop

- Voltage drop: magnitude of the voltage difference between sending and receiving ends of the conductor
- Components like induction motors and lighting ballasts can overheat if voltage is too low
- Acceptable amount depends on application and standards followed, examples:
 - 5-10% for higher voltage distribution lines
 - 3-5% for lower voltage in buildings
- Voltage drop depends on the length of the line and the current magnitude

→ Voltage Drop

- Voltage reach: maximum length that a conductor can be without violating a voltage drop limit for a given amount of current
- Example: if at a current of 180A the voltage along a distribution line decreases by 2.5% per km, then the voltage reach using a limit of 10% is 4 km

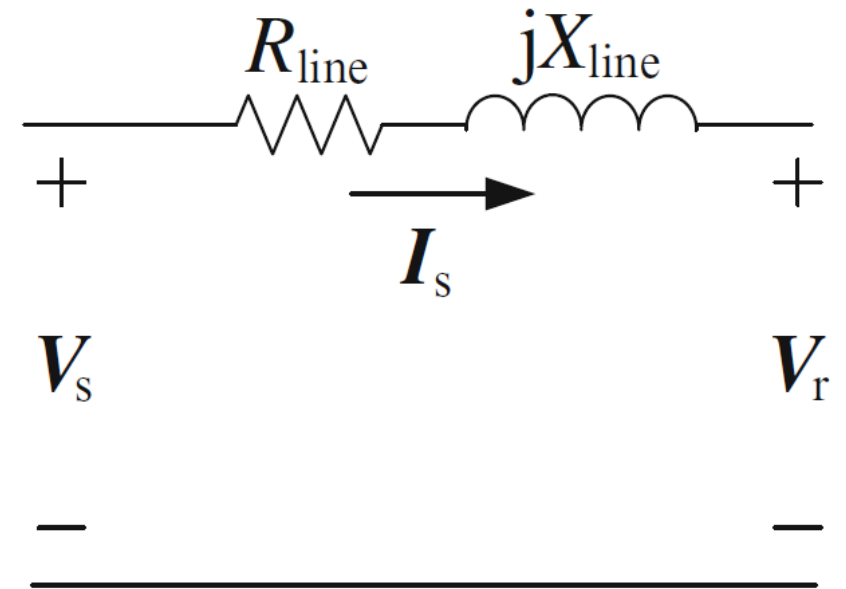
→ Voltage Drop

- Voltage drop associated with a conductor is

$$V_{\text{drop}} = |V_s - V_r| = |I_s| \times |Z_{\text{line}}|$$

- Expressed as percent

$$V_{\text{drop},\%} = \frac{V_{\text{drop}}}{|V_s|} \times 100$$



→ Exercise

The conductors used in a three-phase 33 kV distribution line have an impedance of $Z_{\text{line}} = 1.75 + j1.0 \Omega$. The magnitude of the line current is 115 A. Compute the voltage drop as a percent.

Exercise

The conductors used in a three-phase 33 kV distribution line have an impedance of $Z_{\text{line}} = 1.75 + j1.0 \Omega$. The magnitude of the line current is 115 A. Compute the voltage drop as a percent.

$$V_{\phi} = |V_s| = \frac{33,000}{\sqrt{3}} = 19052.6 \text{ V}$$

Remember, the voltage given is assumed to be line-line

$$V_{\text{drop}} = |I_s| \times |Z_{\text{line}}| = 115 \times |1.75 + j1.0| = 231.79 \text{ V}$$

$$V_{\text{drop},\%} = \frac{V_{\text{drop}}}{|V_s|} \times 100 = \frac{231.79}{19052.6} \times 100 = 1.22\%$$

→ What if the voltage drop is too large?

- Use conductors with larger cross-sectional area to reduce the resistance and inductive reactance
- Decrease the separation distance between the conductors to reduce the inductive reactance
- Increase the nominal voltage of the distribution line to reduce the current for a given amount of power
- Include voltage boosting equipment such as transformers or capacitor banks.

→ Thermal Limit

- Current carried by a conductor is limited by its thermal characteristics
- Heat is generated by the current in the conductor due to the resistance
- Too much heat increases the conductor temperature, potentially causing:
 - conductor to weaken
 - conductor to sag
 - insulation to degrade/fail
- Unlike voltage drop, the thermal limit does not depend on conductor length (why?)

→ Thermal Limit

- Power loss along a line:

$$P_{\text{loss}} = |I_s|^2 \times R_{\text{line}}$$

Note: power loss does not depend on inductance of the line

- To reduce power loss (and heat generated):
 - decrease current (this usually is not an option)
 - reduce resistance (larger cross-sectional area, use less resistive material)

» Thermal Limit

- The amount of continuous current that can flow through the conductor without overheating (exceeding its temperature rating) is called the “ampacity” of the conductor
- Ampacity depends on:
 - Heat generated by the conductor
 - Ambient temperature
 - Heat from the sun (if outdoors)
 - Cooling from wind (if outdoors)
 - Heat from other nearby conductors (especially in conduit and raceways)

» Thermal Limits

- Ampacity of a (outdoor) conductor can vary throughout the year as the ambient temperature changes
- Short overloads can be permitted on higher voltage transmission and distribution lines

» Exercise

A conductor that satisfies a voltage drop limit will always satisfy the thermal limit

True

False

→ Exercise

A conductor that satisfies a voltage drop limit will always satisfy the thermal limit

True

False

For example: a short conductor may satisfy the voltage limit, but not the thermal limit

→ Example

Consider a three-phase distribution line that is 25 km long. The line serves a load of 4.145 MW at a power factor of 0.85. The receiving-end voltage is 22 kV. The voltage drop limit is 10%. The conductors are separated by 2m.

Select the conductor with the smallest cross-sectional area that satisfies the thermal and voltage limits from the table on the following slide



	Size (mm ²)	Ampacity (A)	Resistance (Ohm/km)	Reactance (0.5 m spacing) (Ohms/km)	Reactance (1 m spacing) (Ohms/km)	Reactance (2 m spacing) (Ohms/km)
A	13.3	95	2.200	0.419	0.471	0.523
B	21.1	125	1.384	0.401	0.453	0.506
C	33.6	165	0.869	0.379	0.432	0.484
D	107.2	325	0.273	0.340	0.392	0.445
E	135.2	415	0.218	0.323	0.375	0.427
F	201.4	525	0.147	0.318	0.367	0.423
G	241.7	590	0.122	0.308	0.360	0.413

Example

- Consider a three-phase distribution line that is 25 km long. The line serves a load of 4.145 MW at a power factor of 0.85. The receiving-end voltage is 22 kV. The voltage drop limit is 10%. The conductors are separated by 2m.
- First, find the line current magnitude

$$S = \frac{P}{3} \times \frac{1}{PF} = \frac{4.145}{3} \times \frac{1}{0.85} = 1.625 \text{ MVA}$$

Remember, the power given is the total power

$$I_s = \frac{S}{|V_r|} = \frac{1.625 \text{ MVA}}{22 \text{ kV} / \sqrt{3}} = 127.96 \text{ A}$$

Example

Line current is 127.96A

	Size (mm ²)	Ampacity (A)	Resistance (Ohm/km)	Reactance (0.5 m spacing) (Ohms/km)	Reactance (1 m spacing) (Ohms/km)	Reactance (2 m spacing) (Ohms/km)
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A and B, do not meet ampacity (thermal) limit, so consider conductor C

Example

- Conductor C satisfies thermal limit, but what about voltage drop?

$$Z_{\text{line}} = (0.869 + j0.484) \times 25 = 21.73 + j12.10$$

Recall the line is 25 km long

	Size (mm ²)	Ampacity (A)	Resistance (Ohm/km)	Reactance (0.5 m spacing) (Ohms/km)	Reactance (1 m spacing) (Ohms/km)	Reactance (2 m spacing) (Ohms/km)
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Example

- Check the voltage drop next

$$V_{\text{drop}} = |\mathbf{I}_s| \times |Z_{\text{line}}| = 127.96 \times |21.73 + j12.10| = 3.182 \text{ kV}$$

$$|\mathbf{V}_s| = |\mathbf{V}_r| + V_{\text{drop}} = 15.884 \text{ kV}$$

$$V_{\text{drop},\%} = \frac{V_{\text{drop}}}{|\mathbf{V}_s|} \times 100 = \frac{3.182 \text{ kV}}{15.884 \text{ kV}} \times 100 = 20.03\%$$

This exceeds the 10% limit. Although conductor C satisfies the thermal limit, the line is too long to use C, and so we must consider the next larger conductor size

Exercise

- Check to see if conductor D can be used

Line current is 127.96A

	Size (mm ²)	Ampacity (A)	Resistance (Ohm/km)	Reactance (0.5 m spacing) (Ohms/km)	Reactance (1 m spacing) (Ohms/km)	Reactance (2 m spacing) (Ohms/km)
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Exercise

$$Z_{\text{line}} = (0.273 + j0.445) \times 25 = 6.825 + j11.125 \Omega$$

$$V_{\text{drop}} = |\mathbf{I}_s| \times |Z_{\text{line}}| = 127.96 \times |6.825 + j11.125| = 1.67 \text{ kV}$$

$$|\mathbf{V}_s| = |\mathbf{V}_r| + V_{\text{drop}} = 14.372 \text{ kV}$$

$$V_{\text{drop},\%} = \frac{V_{\text{drop}}}{|\mathbf{V}_s|} \times 100 = \frac{1.67 \text{ kV}}{14.372 \text{ kV}} \times 100 = 11.62\%$$

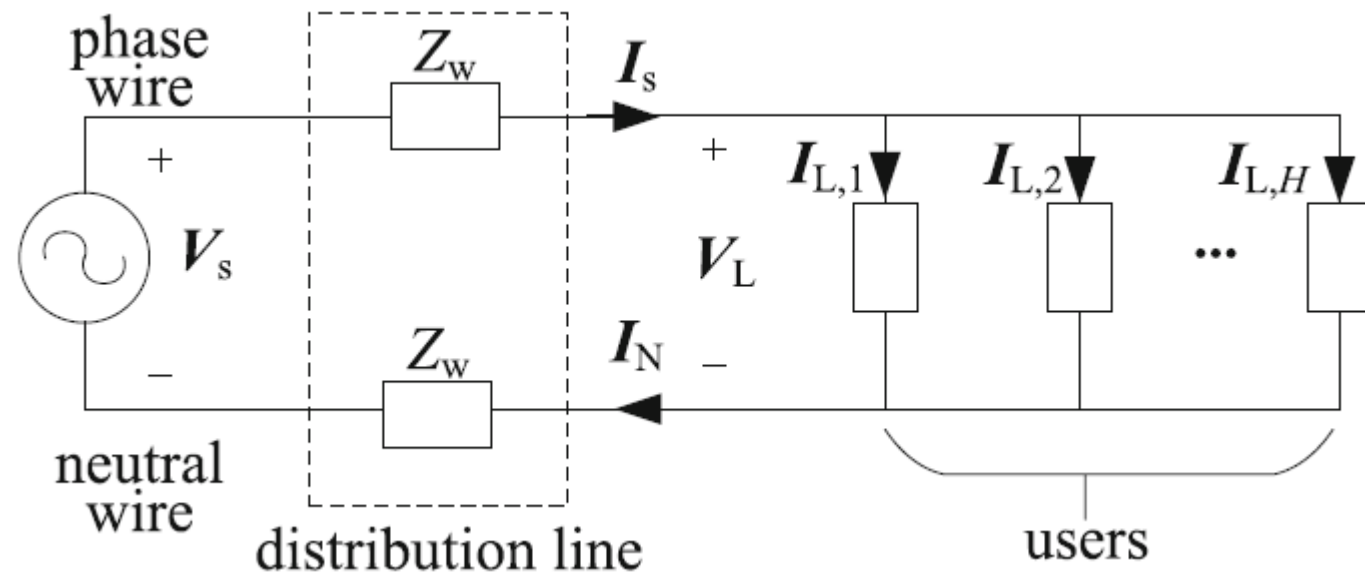
This exceeds the 10% limit. Although conductor D satisfies the thermal limit, the line is too long to use D, and so we must consider the next larger conductor size. We repeat these steps and eventually find that conductor G is needed.

→ Line Configuration

- Previous discussion considered three-phase lines
- Since no current is on neutral (under balanced conditions), the voltage drop associated with the neutral can be ignored
- In other situations, the impedance of the neutral conductor must be included in Z_{line}

Single Phase, Two-Wire

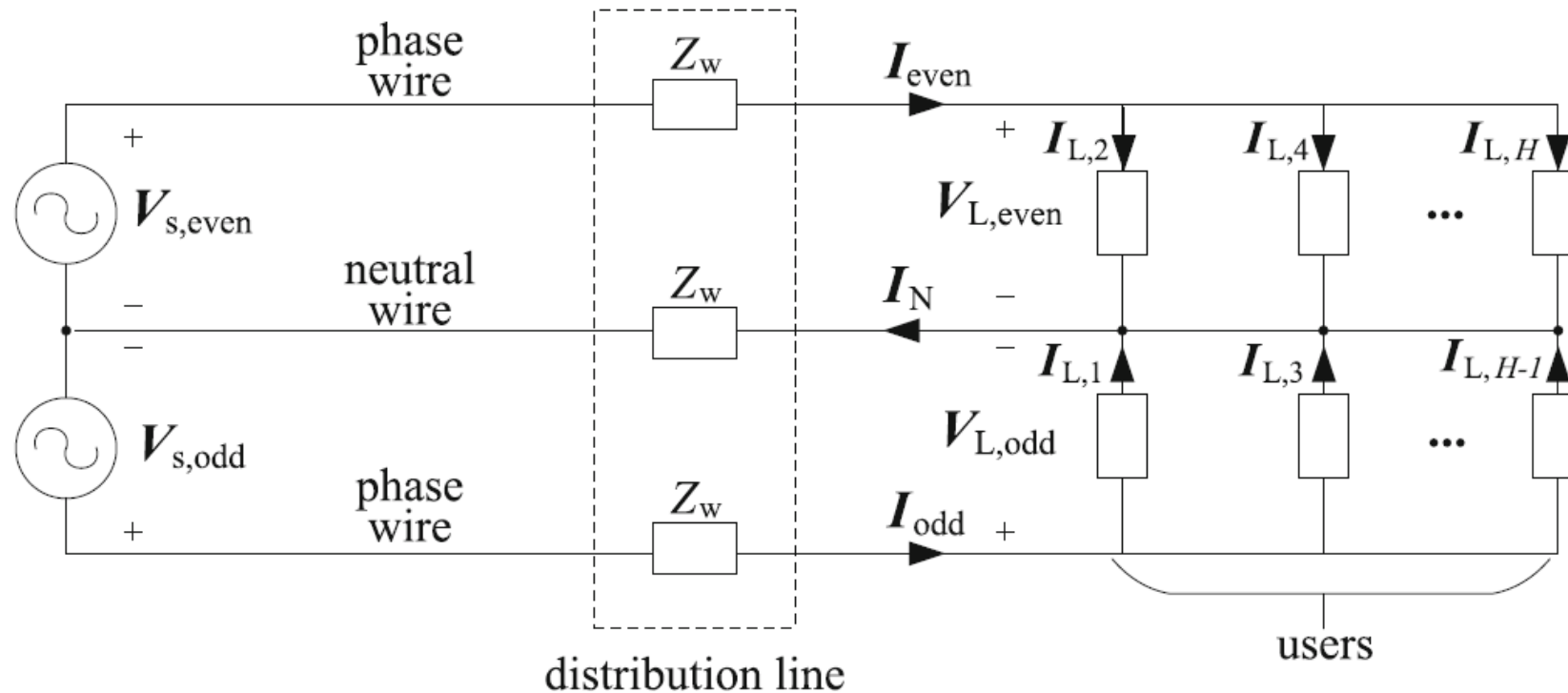
- Z_W : impedance of the distribution line wire (conductor)
- H : number of homes
- I_s : phase conductor current, A
- I_N : neutral current, A
- $I_{L,k}$: current through the k th user load, A



Assuming all users consume the same current ($I_{L,1} = I_{L,2} = \dots = I_{L,H}$):

$$V_{\text{drop}} = |V_s - V_L| = 2|I_s Z_W| = 2H |I_L| |Z_W|$$

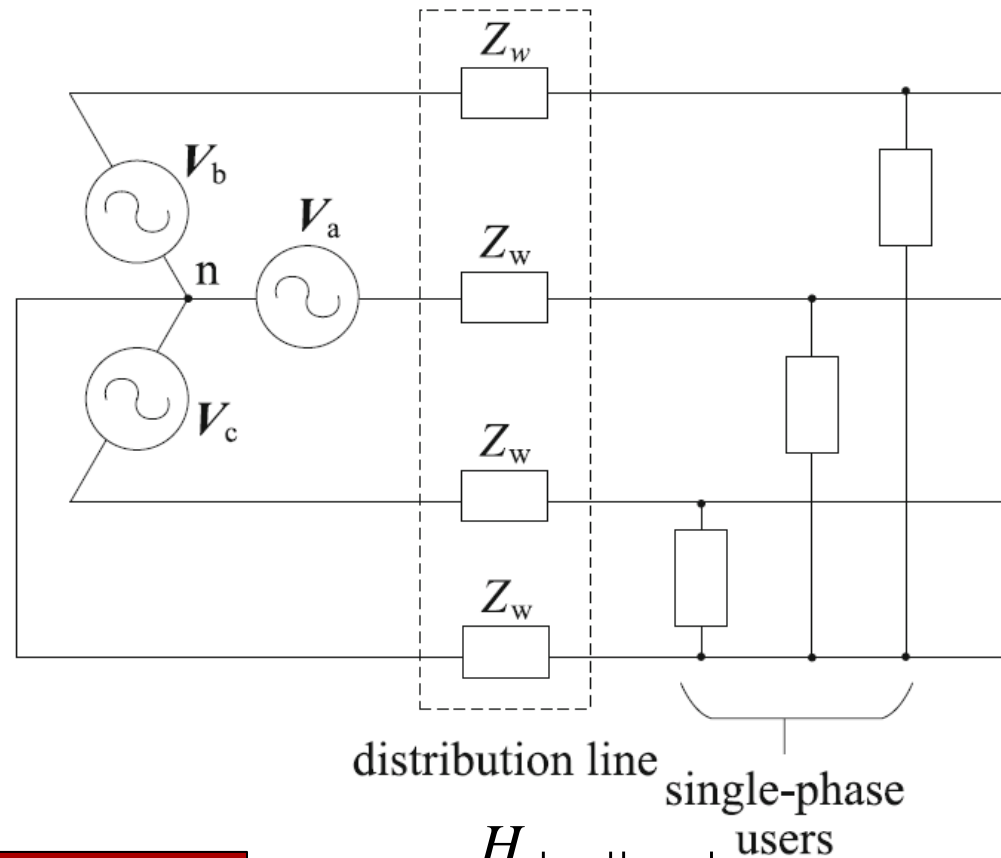
Split-Phase (single phase, three wire)



Assuming the load is evenly split

$$V_{\text{drop}} = \frac{H}{2} |I_L| |Z_w|$$

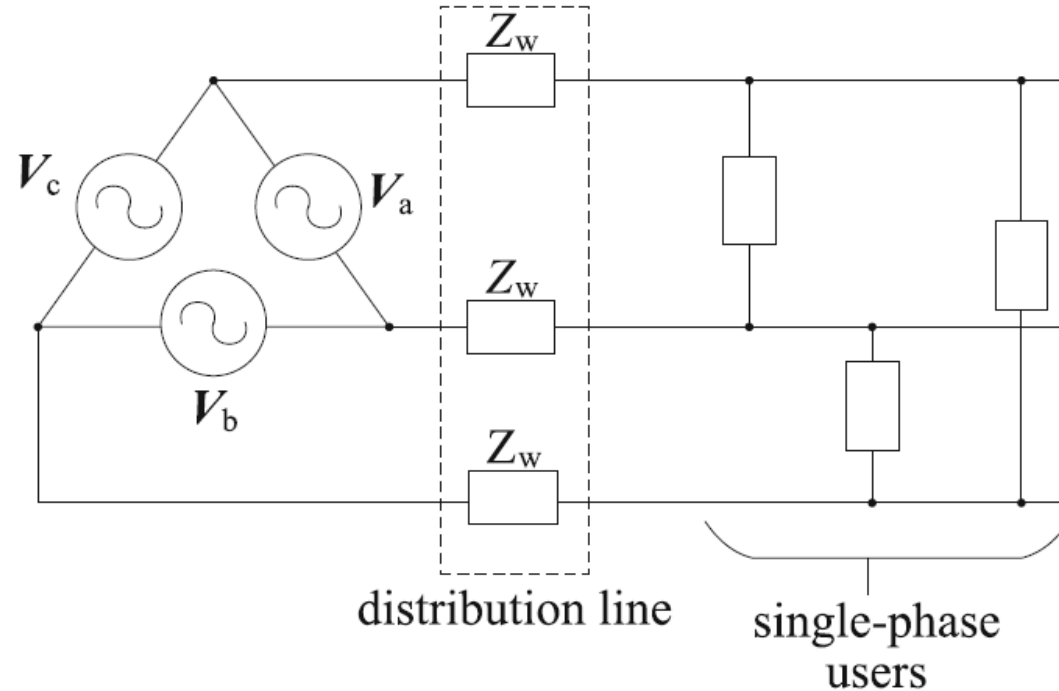
Three-Phase Wye (four-wire)



Assuming the load is balanced

$$V_{\text{drop}} = \frac{H}{3} |I_L| |Z_w|$$

Three-Phase, Delta (three-wire)



$$V_{\text{drop}} = H |I_L| |Z_w|$$

This is different from the wye case because I_L is through the load (phase current), but the current through Z_w is line current (which is greater) than the phase current

→ Distribution Configuration Summary

Configuration	Voltage drop	Power loss
Single-phase, two-wire	$2H I_L Z_w $	$2(H I_L)^2 R_w$
Split-phase (single-phase, three-wire)	$\frac{1}{2}H I_L Z_w $	$\frac{1}{2}(H I_L)^2 R_w$
Three-phase, wye (four-wire)	$\frac{1}{3}H I_L Z_w $	$\frac{1}{3}(H I_L)^2 R_w$
Three-phase, delta (three-wire)	$H I_L Z_w $	$(H I_L)^2 R_w$

→ Distribution Configuration Summary

Configuration	No. of conductors	Area per conductor	Total area	Conductor cost relative to single-phase, two-wire
Single-phase, two-wire	2	1.0	2.0	1.0
Single-phase, three-wire (split-phase)	3	0.250	0.75	0.375
Three-phase, four-wire (wye)	4	0.167	0.667	0.333
Three-phase, three-wire (delta)	3	0.50	1.5	0.75

Three-phase, four-wire is the most efficient distribution configuration

→ Key Points

- Conductors should be sized to not overheat and to avoid excessive voltage drop
- Configuration of the distribution system affects the voltage drop and power loss