

13-Transformer EMF

Text: Chapter 11.1

ECEGR 3500

Electrical Energy Systems

Professor Henry Louie

→ Overview

- Introduction
- Notation, Variables and Units
- Maxwell's Equations
- Faraday's Law

→ Introduction

- We know that transformers convert electrical power from one voltage level to another, but how do they work?
- Fundamental operating principle is derived from one of Maxwell's equations (Faraday's Law)

→ Notation, Variables and Units

- **Bold:** vector quantity
- **E :** electric field intensity (V/m)
- **B :** magnetic flux density (Wb/m²) or Tesla (T)
- **Φ :** magnetic flux, (Wb)
- **H :** magnetic field intensity, (A/m)
- **J :** volume current density (C/m²)
- **ρ :** volume charge density (C/m³)

→ Maxwell's Equations

- Named for James Clerk Maxwell (1831-1879)
- Describe the nature of electromagnetic fields
- Set of four equations
 - Ampere's Law
 - Faraday's Law
 - Gauss's Law (flux)
 - Gauss's Law (charge)



→ Maxwell's Equations

Faraday's Law and Ampere's Law are especially important for electrical machines, so we focus on them

→ Ampere's Law

- Current through a conductor will produce a circular magnetic field around it
 - Line integral of the magnetic field around the conductor equals the current through it

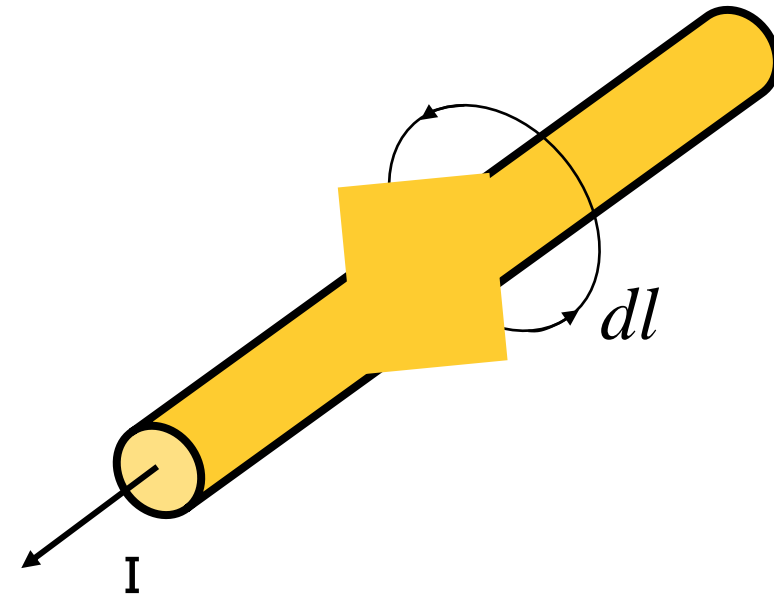
- Mathematically

$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}$$

or

$$\oint_C \mathbf{H} \cdot d\mathbf{l} = \oint_S \mathbf{J} \cdot d\mathbf{s} + \int_S \frac{\partial \mathbf{D}}{\partial t} \cdot d\mathbf{s}$$

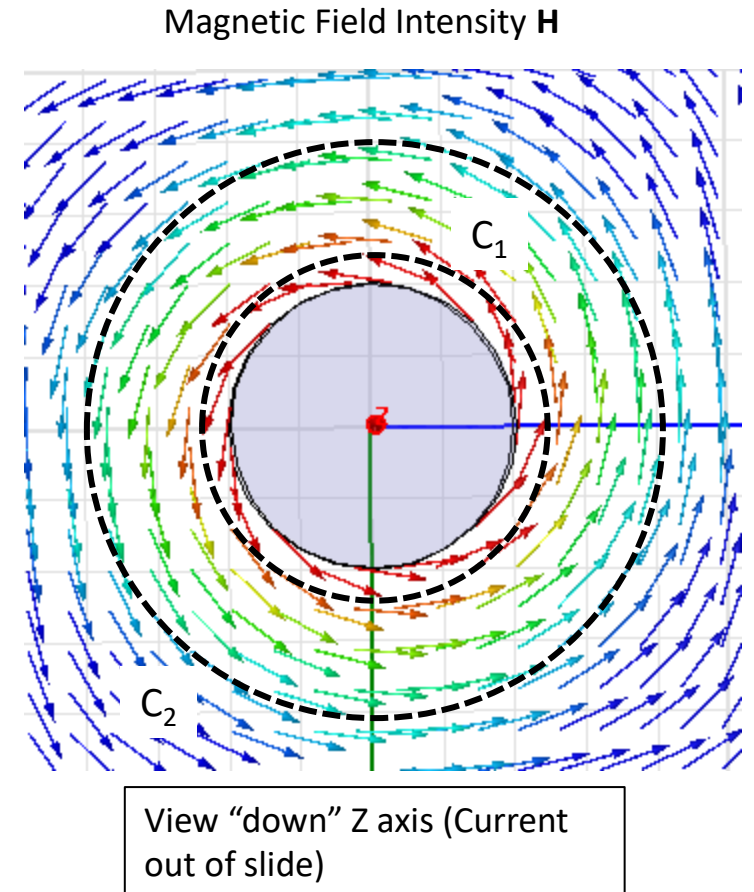
curl



→ Ampere's Law

- Direction of \mathbf{H} field is CCW
 - Right Hand Rule
- Field strength decreases as path length increases
 - Field becomes weaker as distance increases

$$\oint_{C_1} \mathbf{H} \cdot d\mathbf{l} = \oint_{C_2} \mathbf{H} \cdot d\mathbf{l} = \text{constant}$$



→ Magnetic Flux Density

- Related to ***H*** is the Magnetic Flux Density (***B***)

$$\mathbf{B} = \mu_0(\mathbf{H} + \mathbf{M}) \longleftarrow \text{We will assume that } \mathbf{M} = 0$$

- where:

- ***B***: magnetic flux density (Wb/m²)
- ***M***: magnetization field (A/m)
- μ_0 : permeability of free space $4\pi \times 10^{-7}$ (H/m)
- μ_r : relative permeability

A material is magnetized if ***M*** \neq 0
Note: ***H*** and ***M*** may have different signs
(directions)

→ Magnetic Flux Density

- Inside linear isotropic homogeneous (LIH) medium

$$\mathbf{B} = \mu_0(\mathbf{H} + \mathbf{M})$$

$$\mathbf{M} = \chi\mathbf{H}$$

$$\mathbf{B} = \mu_0(1 + \chi)\mathbf{H}$$

$$\mathbf{B} = \mu_0\mu_r\mathbf{H} = \mu\mathbf{H}$$

Note: permanent magnets are not LIH

- where:

- χ : magnetic susceptibility of the medium
- μ_r : relative permeability

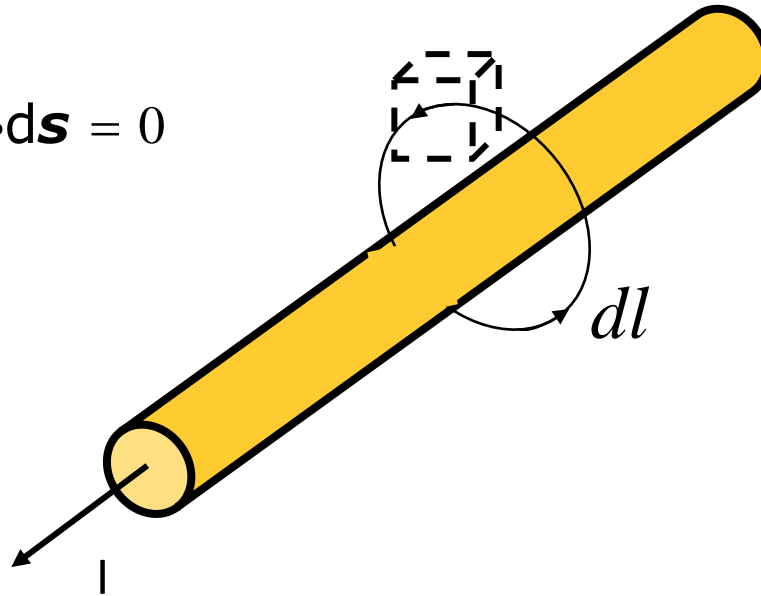
Note: magnetic susceptibility can be positive or negative

→ Gauss's Law for Magnetic Fields

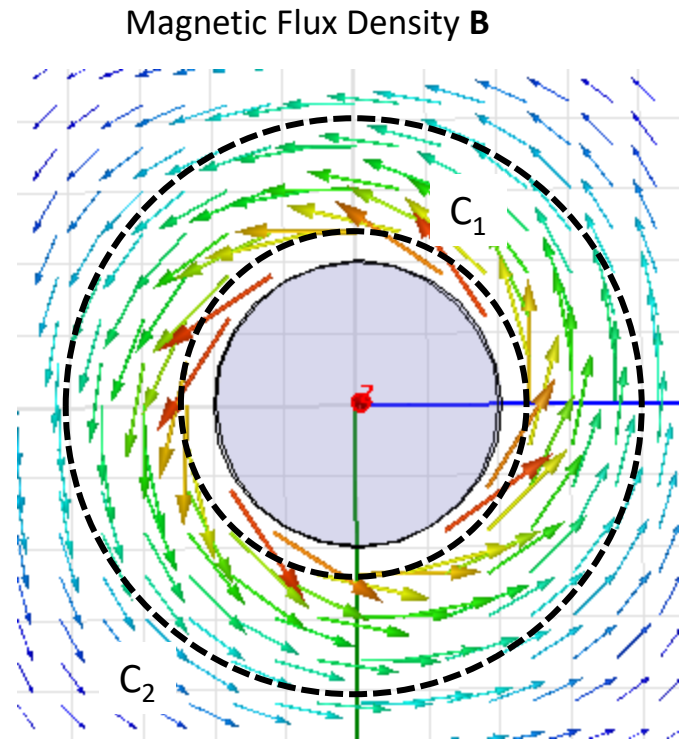
- The magnetic flux entering a volume is equal to the magnetic flux leaving it (it is continuous)
- Mathematically

$$\nabla \cdot \mathbf{B} = 0 \quad \text{or} \quad \oint_S \mathbf{B} \cdot d\mathbf{s} = 0$$

(divergence is 0)



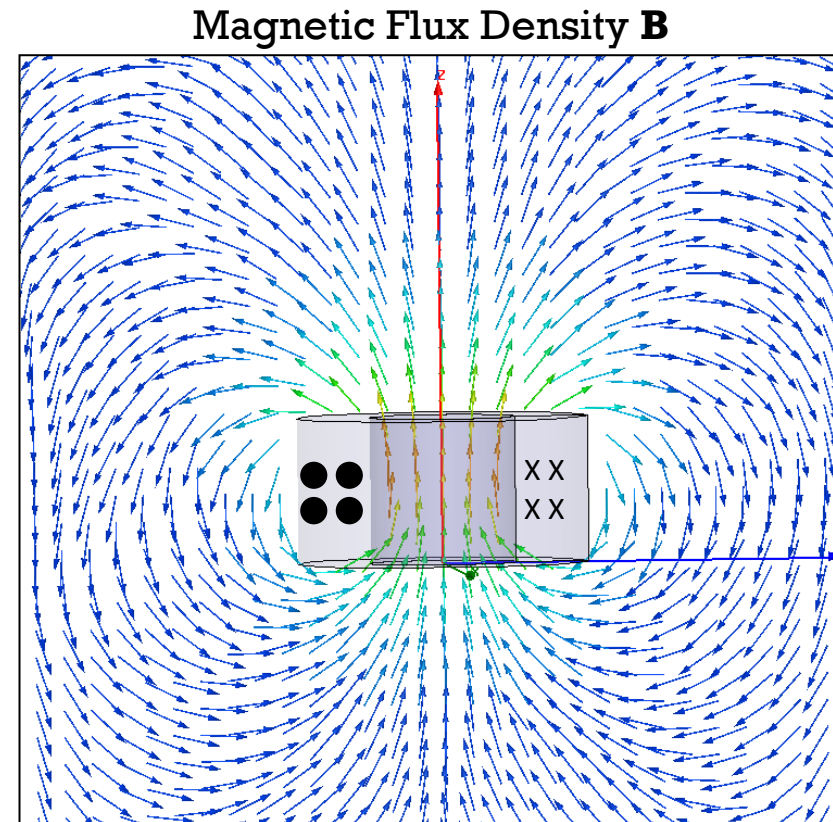
→ Magnetic Flux Density



View "down" Z axis (Current out of slide)

- Direction of \mathbf{B} field is inline with \mathbf{H} field ($\mathbf{M} = 0$ in a vacuum)
- Field strength decreases as path length increases
 - Field becomes weaker as distance increases

→ Magnetic Fields Around a Solenoid



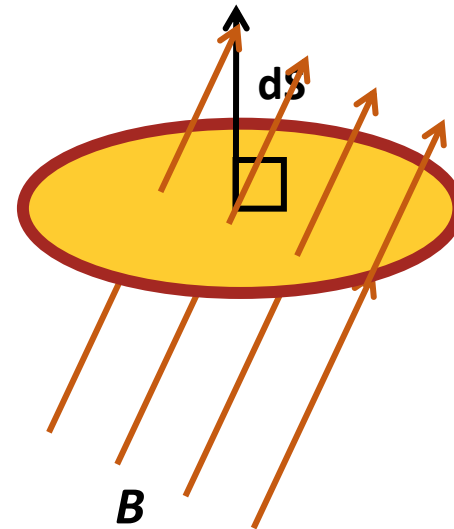
→ Magnetic Flux

- The total magnetic flux, Φ , passing through a surface is:

$$\Phi = \int_s \mathbf{B} \cdot d\mathbf{s}$$

- where:

Φ : magnetic flux (Wb)



→ Faraday's Law

- The change in magnetic flux through a closed path induces a voltage
 - Known as induced electromotive force (emf)

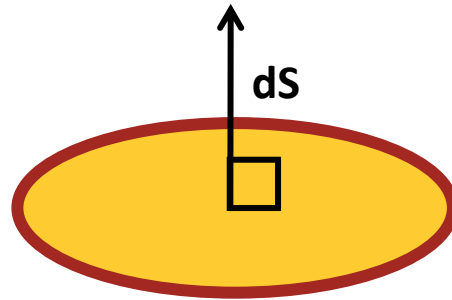
- Mathematically

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \quad \text{or} \quad \oint_C \mathbf{E} \cdot d\boldsymbol{\ell} = -\int_S \frac{\partial \mathbf{B}}{\partial t} \cdot d\mathbf{s}$$

- Understanding Faraday's Law is critical in understanding transformers and electric machines

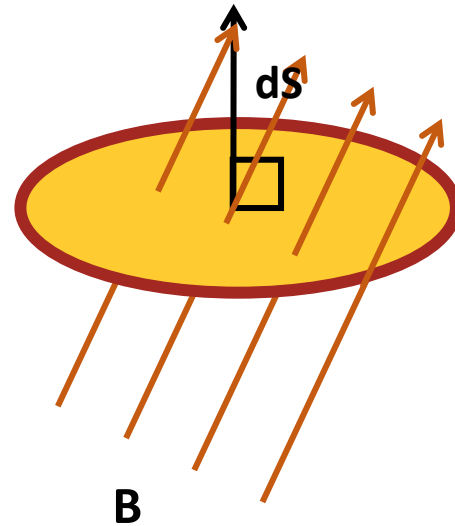
→ Faraday's Law

Consider a closed loop of wire in a time-varying magnetic field with differential surface $d\mathcal{S}$



Faraday's Law

The total magnetic flux, Φ , passing through the loop is: $\Phi = \int_S \mathbf{B} \cdot d\mathbf{s}$



→ Faraday's Law

Voltage is the electric field multiplied by distance so that

$$\oint_C \mathbf{E} \cdot d\mathbf{l} = - \int_s \frac{\partial \mathbf{B}}{\partial t} \cdot d\mathbf{s}$$

$$e = - \int_s \frac{\partial \mathbf{B}}{\partial t} \cdot d\mathbf{s} = - \frac{d\Phi}{dt}$$

e: induced electromotive force (emf), (V)

→ Exercise

Which of the following are ways to increase the voltage induced in a coil?

- A) increase the area enclosed by the coil
- B) reduce the resistance of the coil
- C) increase the time rate of change of the flux density

→ Exercise

Which of the following are ways to increase the voltage induced in a coil?

A) increase the area enclosed by the coil

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C) increase the time rate of change of the flux density

$$\Phi = \int_s \mathbf{B} \cdot d\mathbf{s}$$

$$e = -\frac{d\Phi}{dt}$$

→ Faraday's Law of Induction

- Actually, the magnetic flux may be constant in time and we can still induce voltage
 - It is the change in flux that matters
- If the coil is stationary and the magnetic flux changes, then \mathcal{E} is known as transformer emf
- If the coil is moving and the magnetic flux is constant, then \mathcal{E} is known as motional emf

Important to understand this!

→ Faraday's Law

- A conductor moving in an area of constant flux will have voltage induced in it according to:

$$e = \oint (\mathbf{u} \times \mathbf{B}) \cdot d\boldsymbol{\ell}$$

- \mathbf{u} : velocity of the conductor (m/s)
- Motional emf is due to a force acting on the free electrons in the conductor that moves them to one side or another
 - More on this force later
- Total induced emf is the sum of transformer and motional emf

Summary

- Current gives rise to \mathbf{H} and \mathbf{B} fields, which circulate around the conductor in accordance with the Right Hand Rule
- $|\mathbf{H}|$ decreases as distance from conductor increases
- $|\mathbf{B}|$ is related to $|\mathbf{H}|$ by characteristics of the medium
- Change in flux induces a voltage. The change can have mechanical origins