14-Magnetic Circuits

ECEGR 3500

Electrical Energy Systems

Professor Henry Louie

Overview

- Introduction
- Ferromagnetic Materials
- Magnetic Circuit Assumptions
- Observations of Magnetic Circuits
- Magnetic Circuit Analysis

Introduction

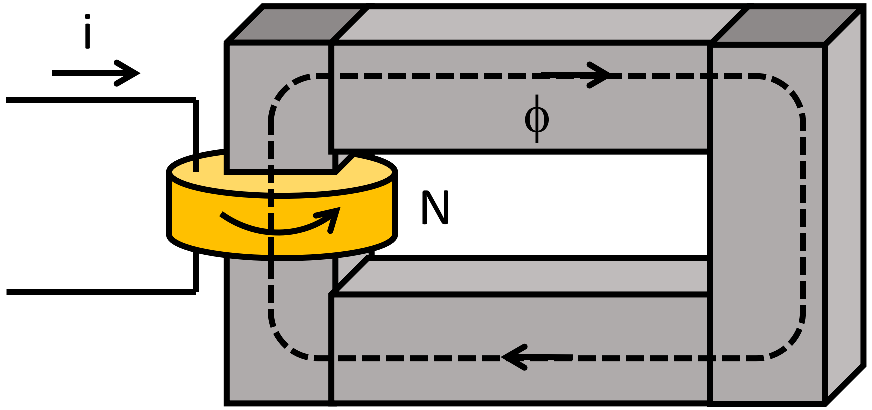
- Faraday's Law is the basis for transformer operation
- Application of Faraday's Law requires knowledge of flux density
- How can these quantities be computed for a given physical arrangement?

Ferromagnetic Materials

- Experience a strong attractive force to an applied magnetic field
  - Force can be several thousand times stronger than that in paramagnetic materials
  - Resulting magnetic field may be stronger than the applied field
- Ferromagnetic materials include
    - Iron, cobalt, nickel
- Relative permeability may be several thousand (compared with 1.0 for free space)

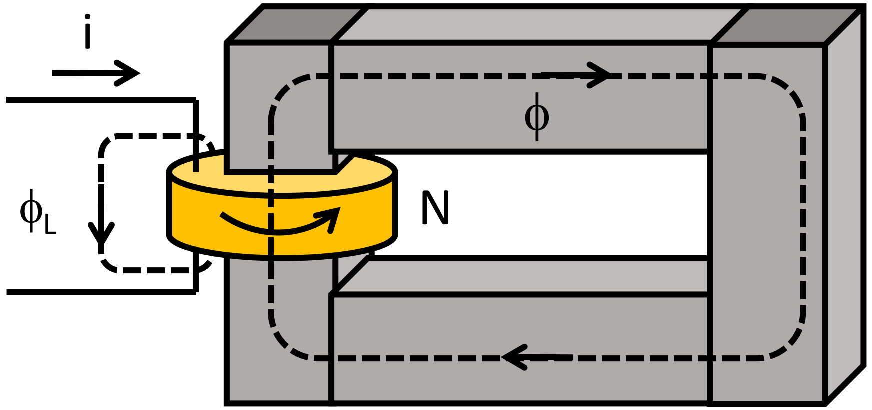
Magnetic Circuit Assumptions

- Let magnetic flux be set up by a coil of wire with dc current, i
- Current establishes flux ϕ in the core
- Let there be N turns of wire on the coil



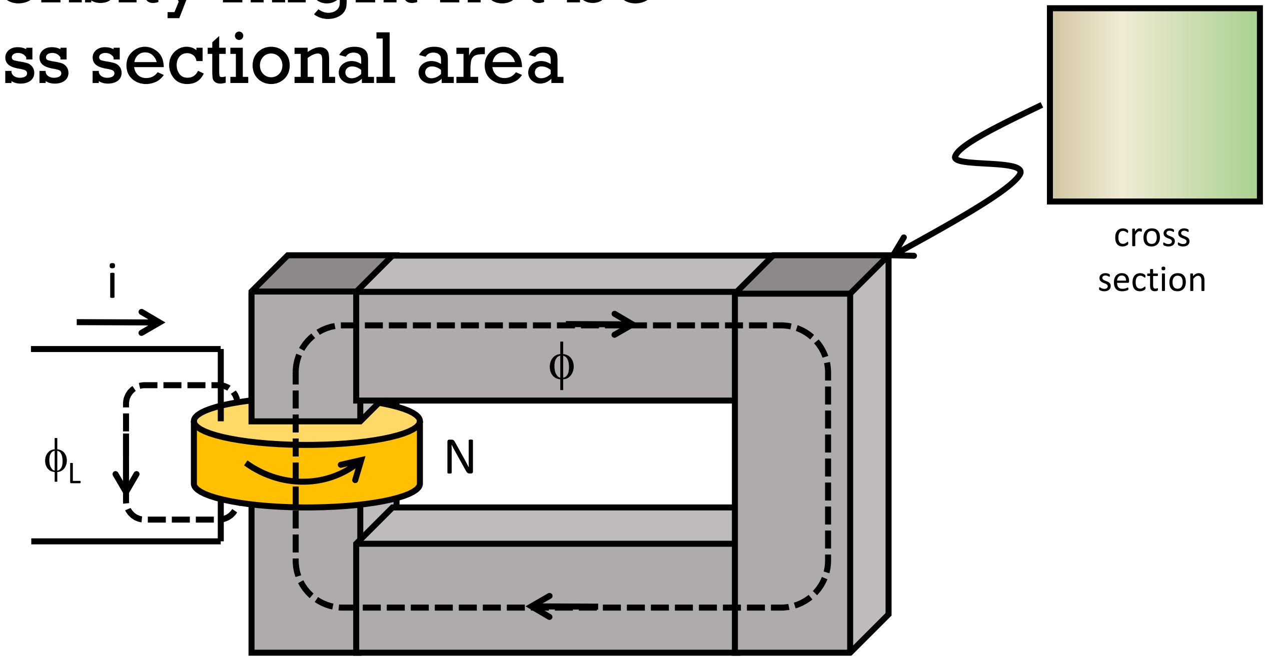
Magnetic Circuit Elements

- Some flux does not pass through the core
  - leakage flux: ϕ_L
  - small compared to ϕ
- Leakage flux can be reasonably ignored



Magnetic Circuit Elements

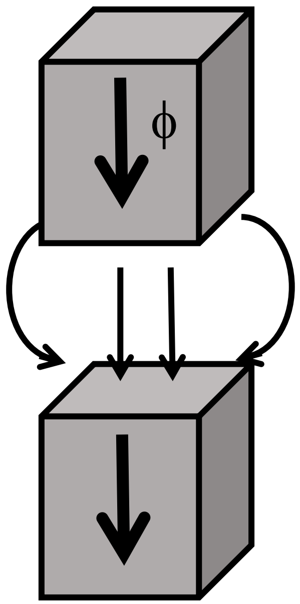
Magnetic flux density might not be uniform in a cross sectional area



Magnetic Circuit Elements

- Fringing occurs in air gaps
- Flux density decreases (cross sectional area increases)

Rest of magnetic circuit not shown



Summary: Magnetic Circuit Assumptions

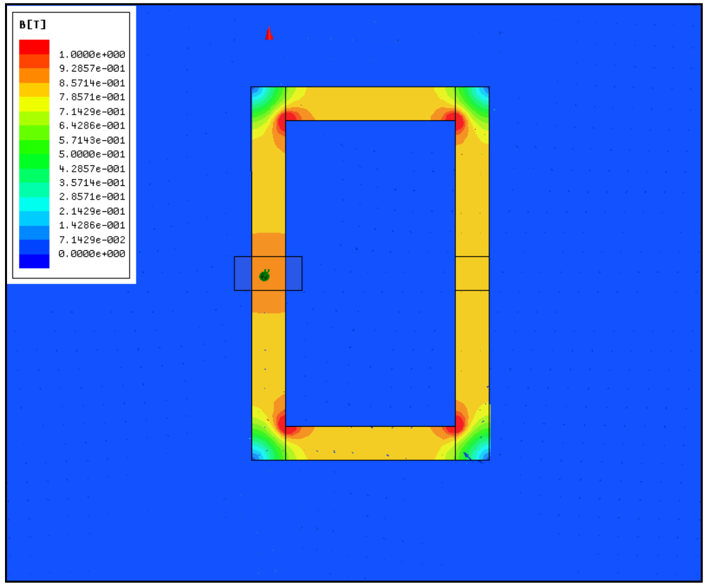
- Magnetic flux flows entirely through the magnetic material (no leakage)
- Magnet flux density is uniform throughout the cross section the material
- Fringing across air-gaps is negligible

Magnetic Circuit Observations

Flux density (flux) is less in a circuit with an air gap

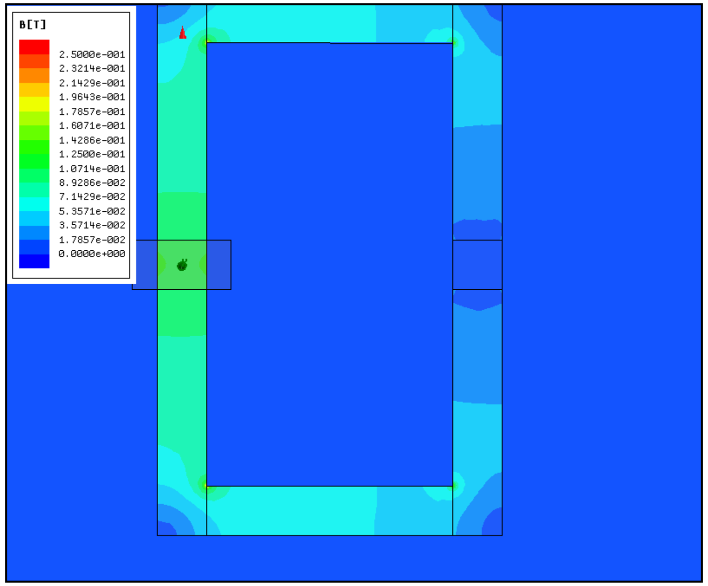
Without air gap

Red: 1.0



With air gap

Red: 2.5



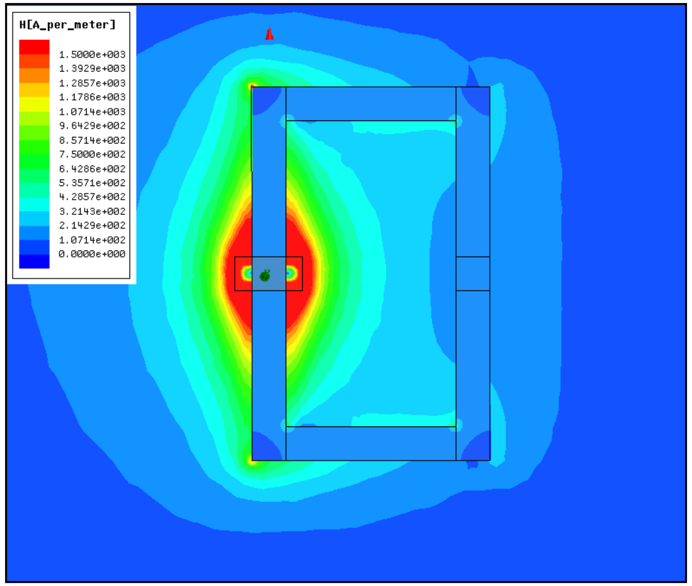
Note: circuits have same dimensions, but the zoom is different.

Magnetic Circuit Observations

Field intensity is low in iron (high permeability) and high in air gap (low permeability)

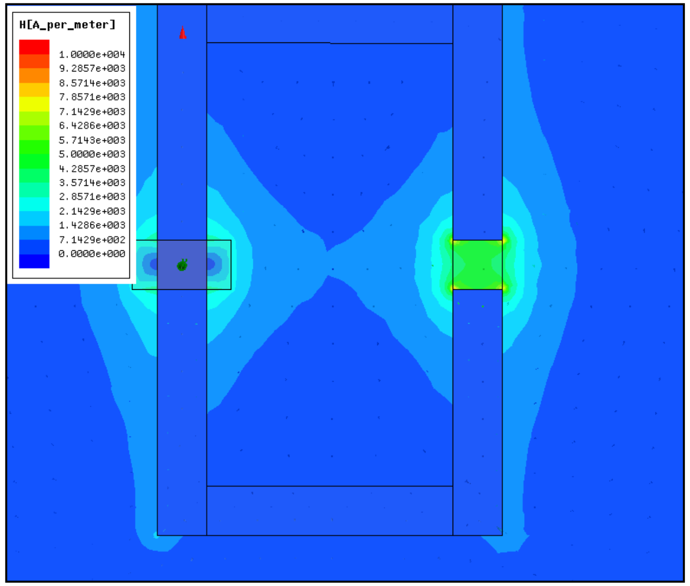
Without air gap

Red: 1500



With air gap

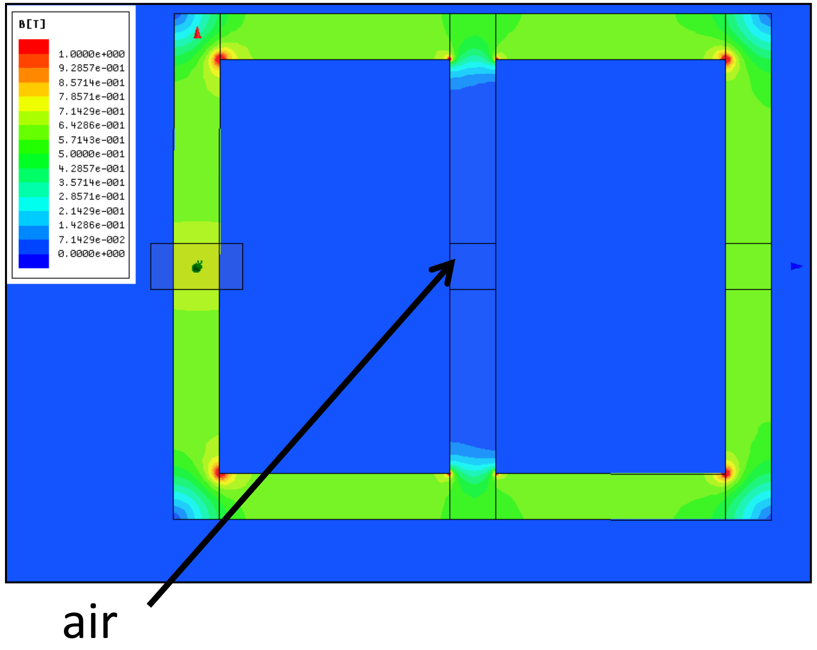
Red: 10000



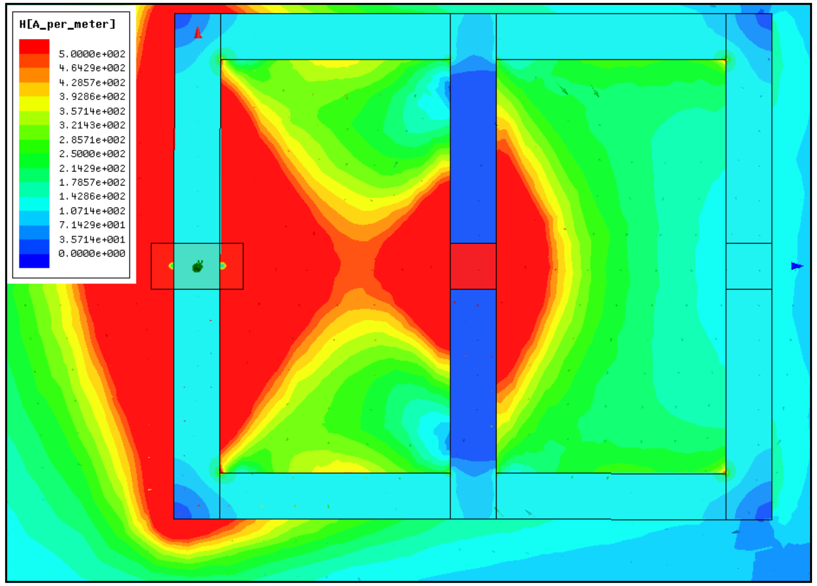
Note: circuits have same dimensions, zoom is different.

Magnetic Circuit Observations

B field

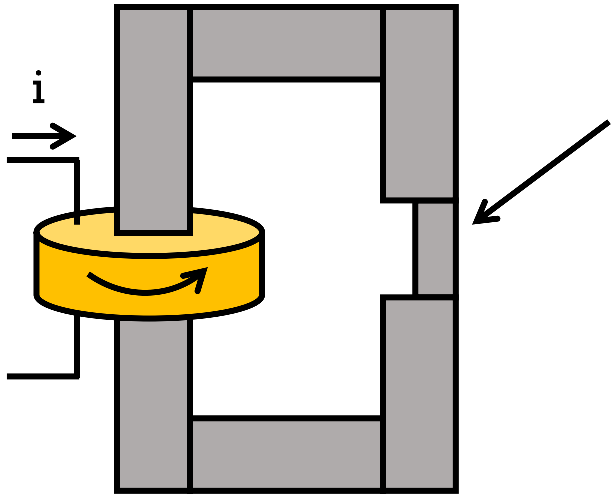


H field



Also note: flux "prefers" path of high permeability.

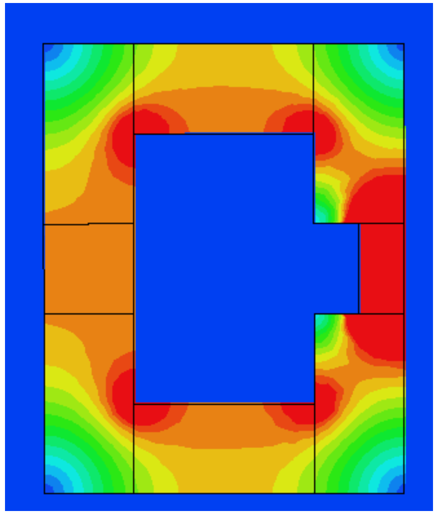
Magnetic Circuit Observations



How does the flux density in this segment compare to the others?

Assume uniform depth

Magnetic Circuit Observations



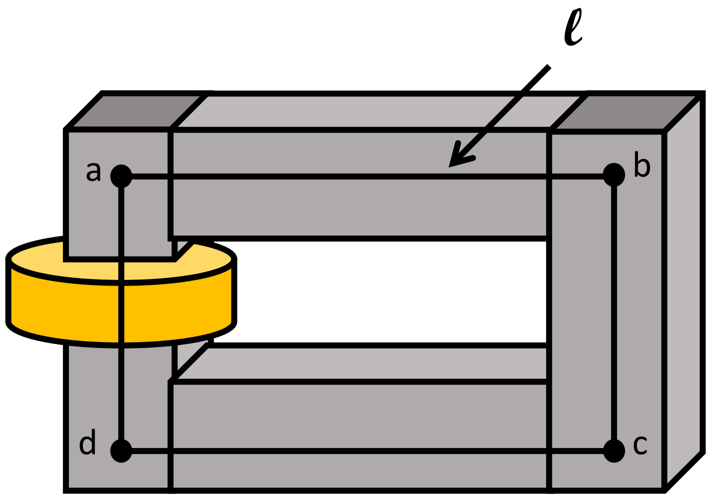
Magnetic Circuit Analysis

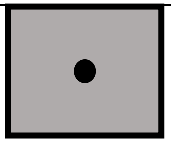
- We are often interested in designing magnetic circuits that maximize the flux through a winding or across an air gap
- The flux through a portion of a magnetic circuit depends on:
  - The source magnetic material
  - The physical properties of the magnetic circuit
    - Size (length, cross-sectional area)
    - Material

Magnetic Circuit Analysis

Let ℓ be the mean length of the magnetic path (m)

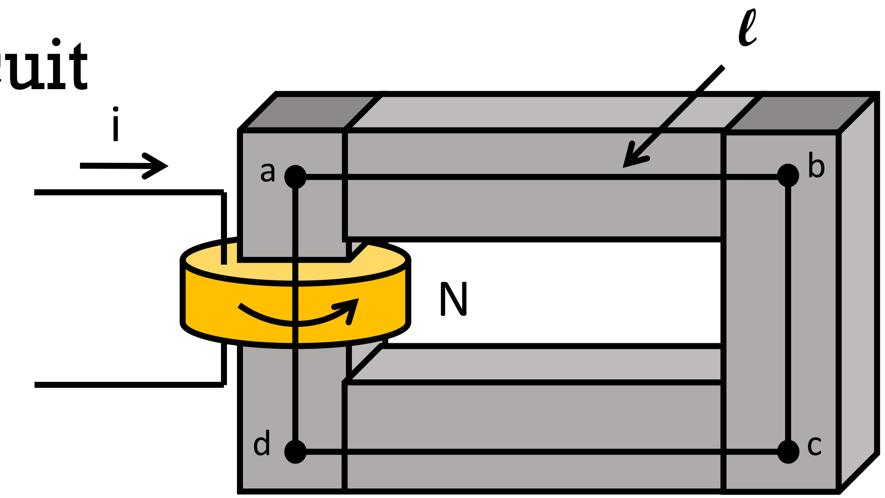
$$\ell = \ell_{ab} + \ell_{bc} + \ell_{cd} + \ell_{da}$$





Magnetomotive Force (mmf)

- The current enclosed by the closed path through the core is (Ampere's Law) $\oint_C \mathbf{H} \cdot d\ell = Ni = \mathcal{F}$
- where
  - $\mathcal{F}$: magnetomotive force (A-t)
- mmf is analogous to voltage in a circuit



Flux in Magnetic Circuits

- Assuming that **H** is uniform in the material, then

$$H\ell = Ni$$

- The magnetic flux density in the material is also uniform and

$$B = \mu H = \frac{\mu Ni}{\ell}$$

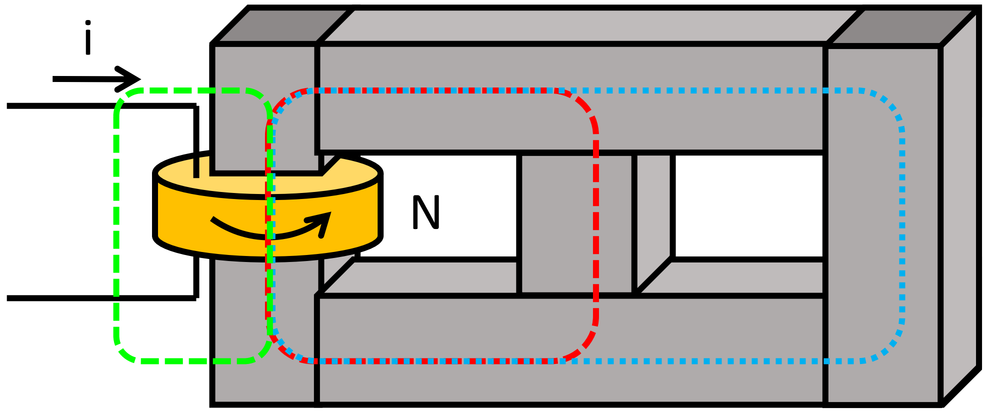
- The flux is:

$$\phi = BA = \frac{\mu NiA}{\ell}$$

  - A: cross sectional area of the material (m^2)

Exercise

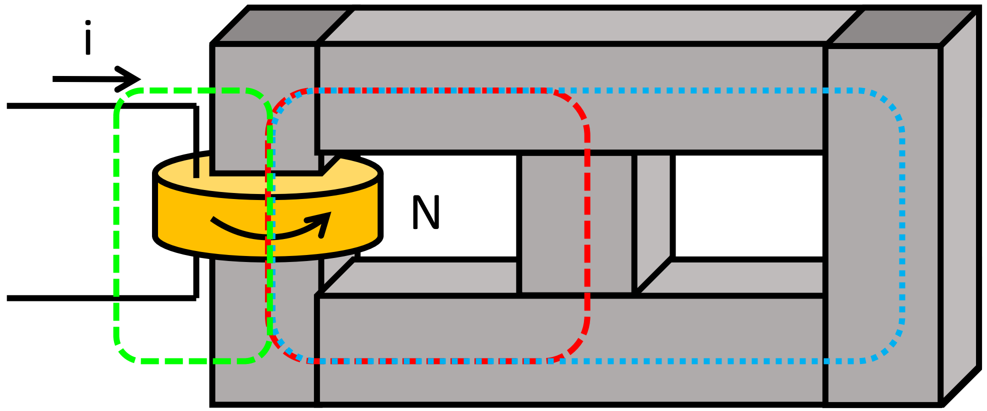
Which path (red, blue or green) results in the greatest mmf?



Cross section is uniform

Exercise

Which path (red, blue or green) results in the greatest mmf?



The mmf is the same. The same number of Ampere-turns is enclosed by each loop (Ni).

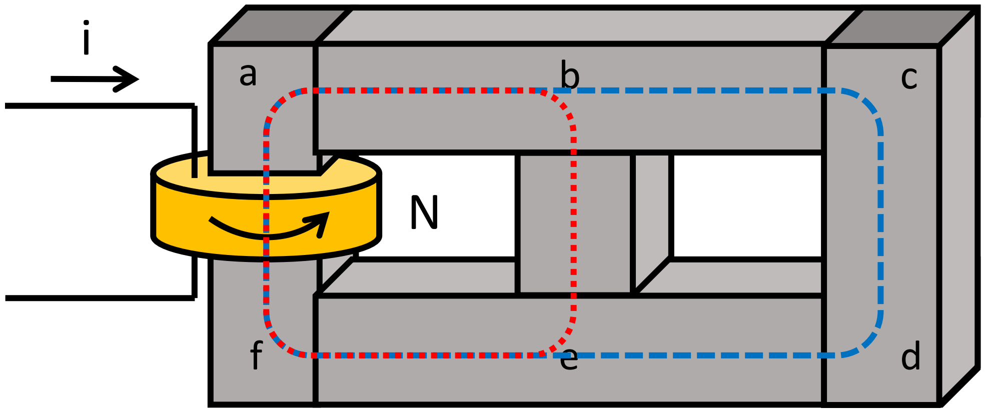
$$\oint_C \mathbf{H} \cdot d\ell = Ni = \mathcal{F}$$

Magnetic Circuit Analysis

mmf is the same, no matter which path is used

$$Ni = H_{fa}\ell_{fa} + H_{ab}\ell_{ab} + H_{be}\ell_{be} + H_{ef}\ell_{ef}$$

$$= H_{fa}\ell_{fa} + H_{ab}\ell_{ab} + H_{bc}\ell_{bc} + H_{cd}\ell_{cd} + H_{de}\ell_{de} + H_{ef}\ell_{ef}$$



Cross section is uniform

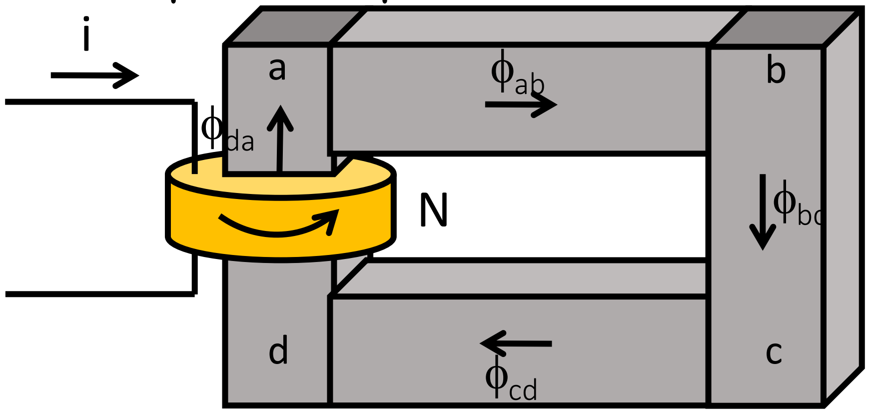
Magnetic Circuit Analysis

$$Ni = H_{da}\ell_{da} + H_{ab}\ell_{ab} + H_{bc}\ell_{bc} + H_{cd}\ell_{cd}$$

$$\mathcal{F} = \frac{B_{da}}{\mu}\ell_{da} + \frac{B_{ab}}{\mu}\ell_{ab} + \frac{B_{bc}}{\mu}\ell_{bc} + \frac{B_{cd}}{\mu}\ell_{cd}$$ using: $H = B/\mu$

$$\mathcal{F} = \left(\frac{B_{da}}{\mu}\ell_{da} + \frac{B_{ab}}{\mu}\ell_{ab} + \frac{B_{bc}}{\mu}\ell_{bc} + \frac{B_{cd}}{\mu}\ell_{cd}\right)\frac{A}{A}$$

$$\mathcal{F} = \phi_{da}\frac{\ell_{da}}{A\mu} + \phi_{ab}\frac{\ell_{ab}}{A\mu} + \phi_{bc}\frac{\ell_{bc}}{A\mu} + \phi_{cd}\frac{\ell_{cd}}{A\mu}$$ using: $\phi = BA$



Cross section is uniform

Magnetic Circuit Analysis

$$\mathcal{F} = \phi_{da}\frac{\ell_{da}}{A\mu} + \phi_{ab}\frac{\ell_{ab}}{A\mu} + \phi_{bc}\frac{\ell_{bc}}{A\mu} + \phi_{cd}\frac{\ell_{cd}}{A\mu}$$

$$\phi = \phi_{da} = \phi_{ab} = \phi_{bc} = \phi_{cd}$$

Since flux entering a boundary = flux leaving the boundary

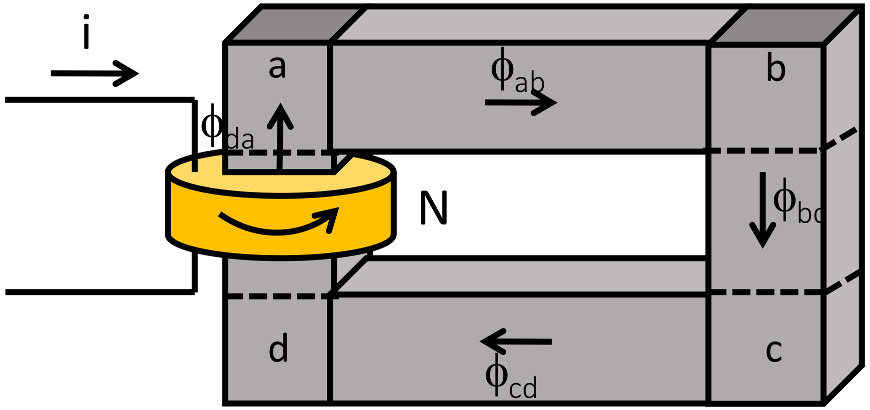
$$\oint_s \mathbf{B \cdot ds} = 0$$

$$\mathcal{F} = \phi\left(\frac{\ell_{da}}{A\mu} + \frac{\ell_{ab}}{A\mu} + \frac{\ell_{bc}}{A\mu} + \frac{\ell_{cd}}{A\mu}\right)$$

$$\mathcal{F} = \phi(\Re_{da} + \Re_{ab} + \Re_{bc} + \Re_{cd})$$

$$\Re \triangleq \frac{\ell}{\mu A}$$

$\mathcal{R}$: reluctance (A-t/Wb)



Cross section is uniform

Magnetic Circuit Analysis

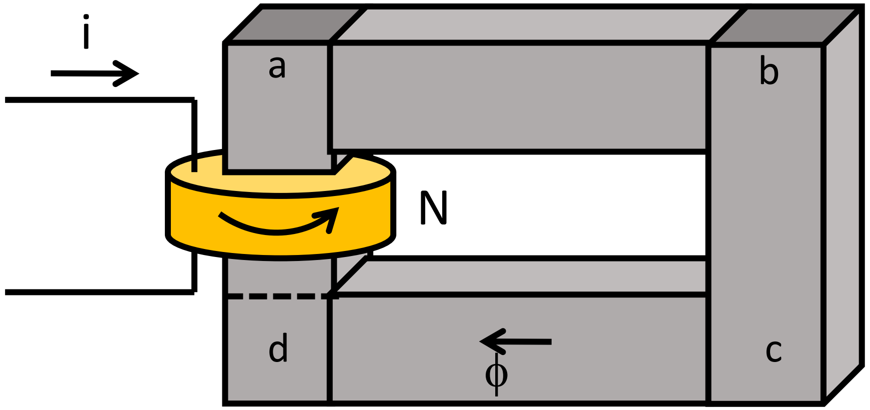
$$\mathcal{F} = \phi(\Re_{da} + \Re_{ab} + \Re_{bc} + \Re_{cd})$$

$\mathcal{F}$ → voltage

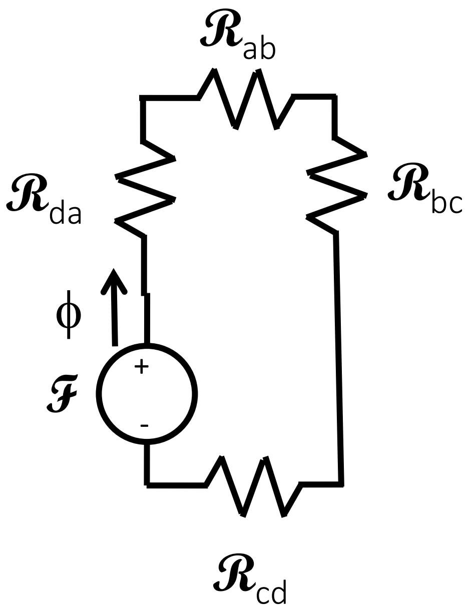
ϕ → current

$(\Re_{da} + \Re_{ab} + \Re_{bc} + \Re_{cd})$ → resistance

Equation can be modeled and solved like a circuit. Important!



Circuit equivalent



Magnetic Circuits

- Note that

$$\phi = \frac{\mathcal{F}}{\Re} = \frac{NiA\mu}{\ell}$$

- For electric circuits

$$i = \frac{V}{R} = \frac{V}{\frac{\ell}{\sigma A}}$$

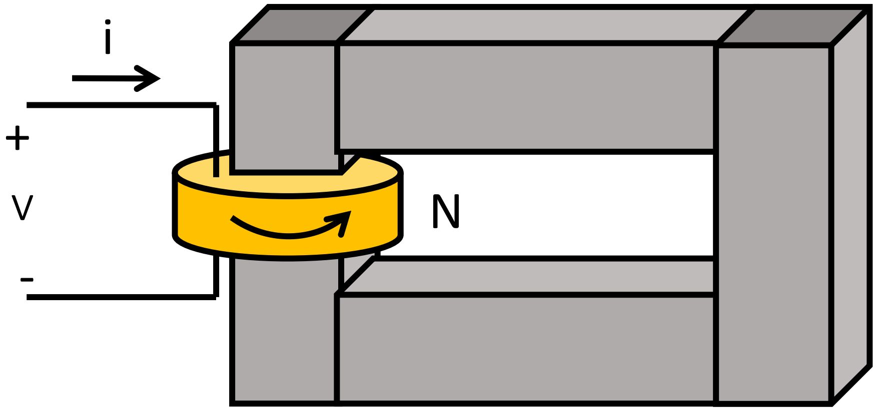
  - σ: conductivity (S/m)

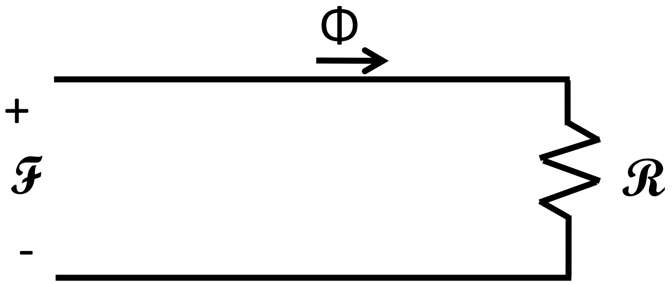
Analogous equations

- Ohm's law for magnetic circuits $\mathcal{F} = \phi\Re$
  - mmf = flux x reluctance
- Checking the units
  - A-t = Wb x (A-t/Wb) = A-t

Magnetic Circuits

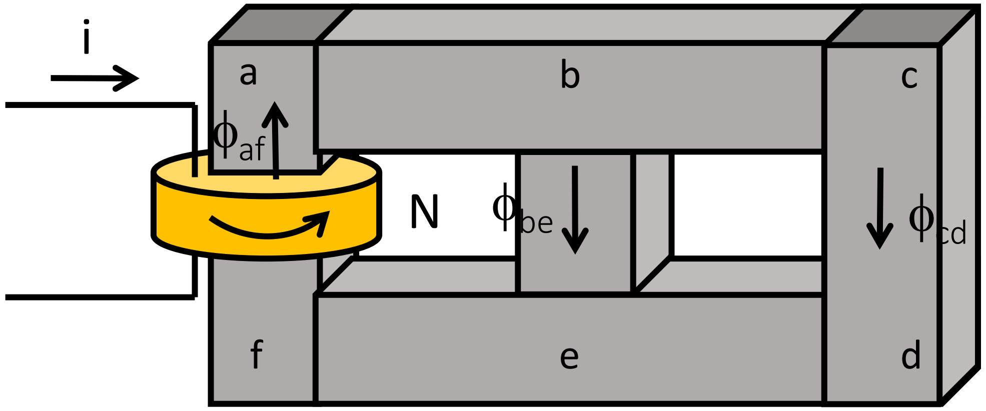
Circuit Quantity	Magnetic Quantity
Voltage, v (volt)	mmf, $\mathcal{F}$ (A-turns)
Current, i (Ampere)	magnetic flux, ϕ, (Wb)
Resistance, R (Ohm)	Reluctance, $\mathcal{R}$, (A-turns/Wb)
Conductivity, σ (S/m)	Permeability, μ (H/m)

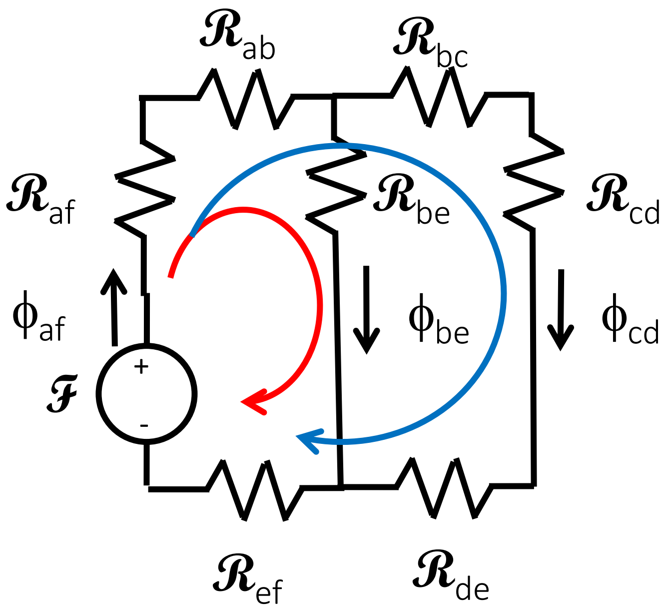




Magnetic Circuit Analysis

KVL and KCL and all other circuit theorems apply to equivalent electric circuit.



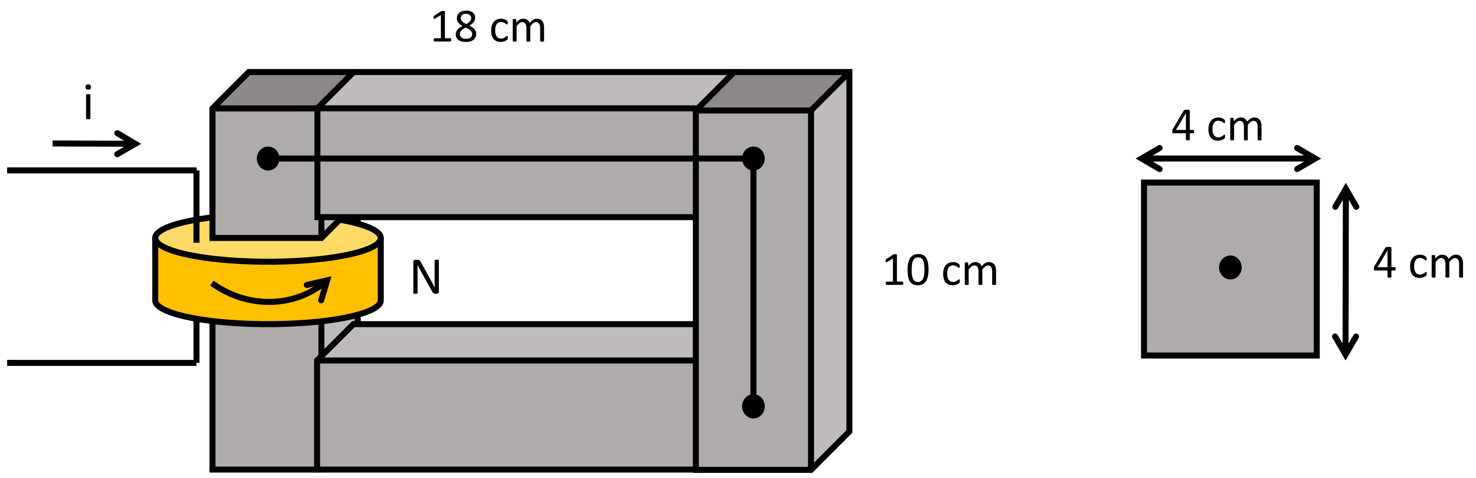


Magnetic Circuit Analysis

- Note: linear circuits assumed in analogy, therefore the magnetic circuit must be linear
  - Linear magnetic circuit = constant permeability
  - Ferromagnetic materials do not have constant permeability (see BH curve)
- Non-linear magnetic circuits can be solved iteratively

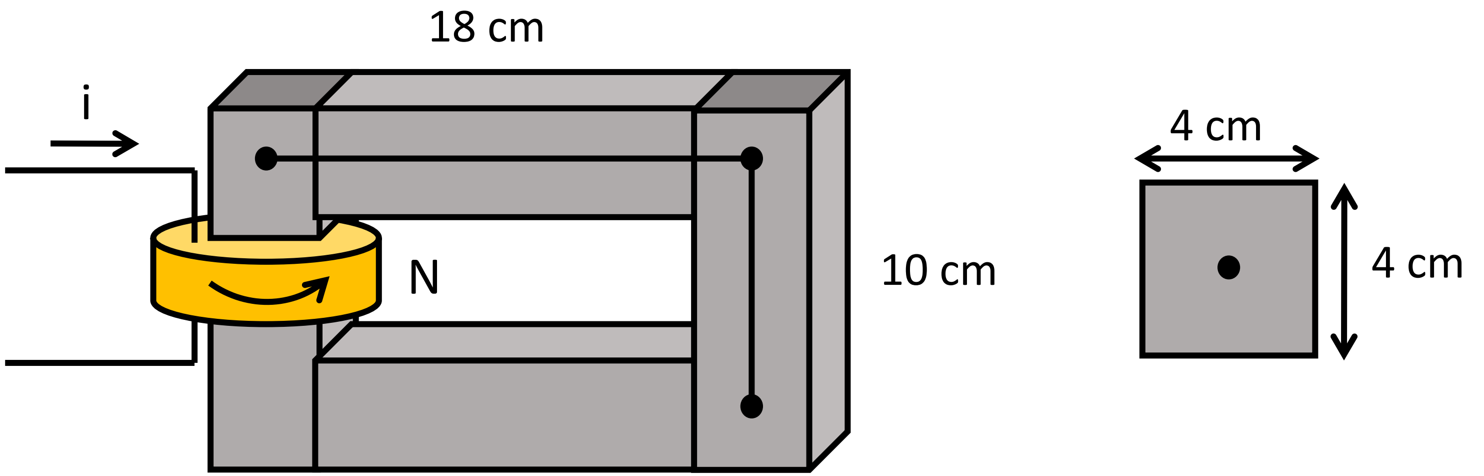
Example

- Compute the flux flowing through the material given:
  - i = 1 A
  - N = 700
  - μ_r = 1000 (assumed to be constant)



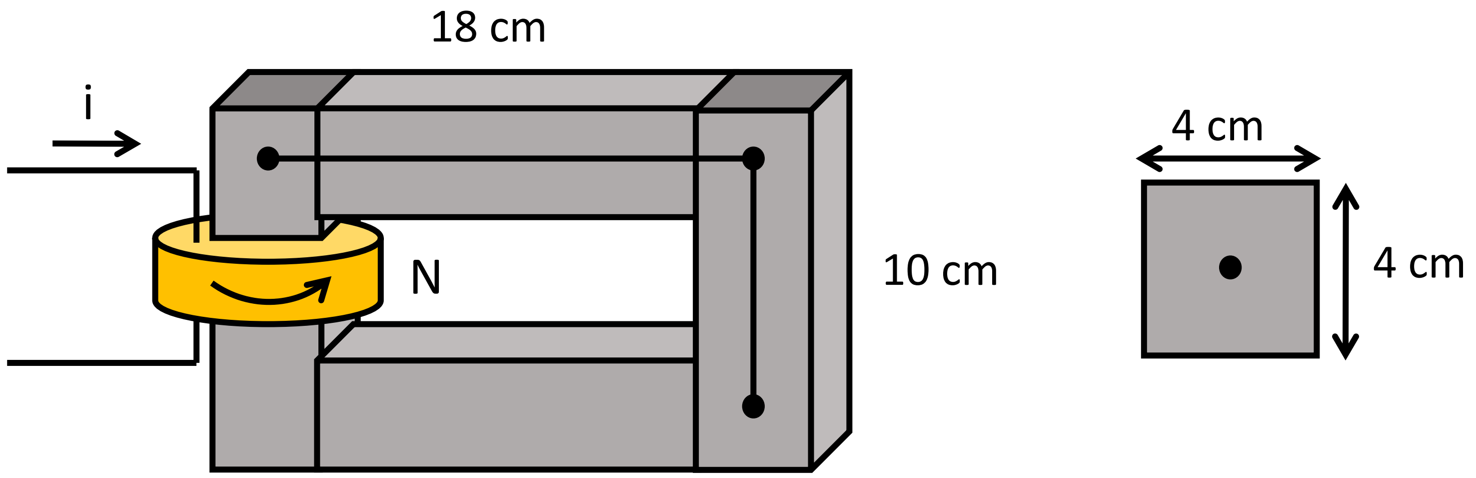
Example

- Want to use: $\mathscr{F} = Ni = \Phi\mathfrak{R}$, $\mathfrak{R} = \dfrac{\ell}{\mu A}$
- First compute ℓ



Example

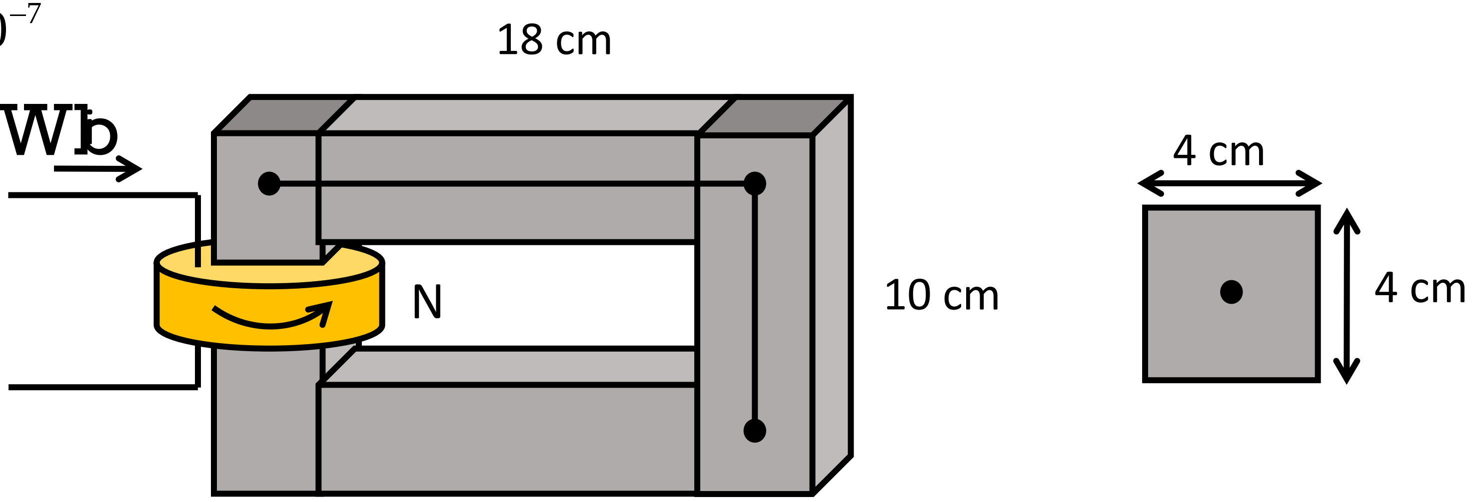
- First compute ℓ
  - $\ell = 18 + 10 + 18 + 10 = 0.56$ m



Example

- Want to use: $\mathscr{F} = Ni = \Phi\Re$, $\Re = \dfrac{0.56}{\mu A}$
- Computing A:

  A = 0.04 x 0.04 = 0.0016m^2
- Computing μ: $1000 \times 4\pi \times 10^{-7}$
- Gives: $\mathscr{R}$ = 278,520 A-t/Wb

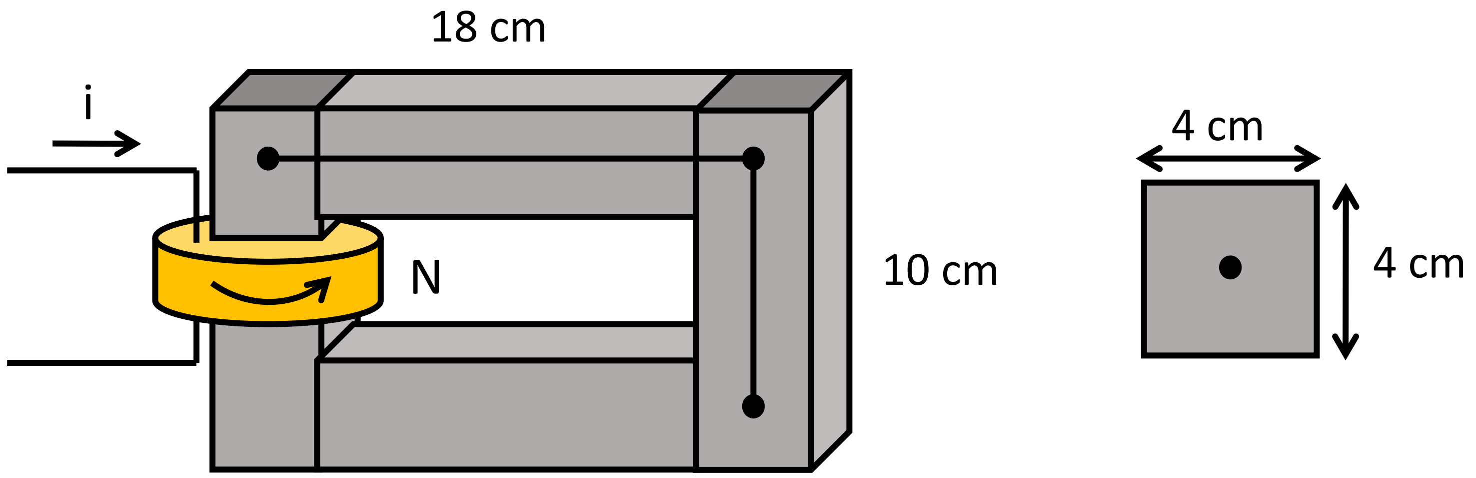


Example

- Solving

$$\mathcal{F} = Ni = \Phi \Re$$

$$\Phi = \frac{Ni}{\Re} = \frac{700 \times 1}{278520} = 0.0025 \text{ Wb}$$



Summary

- Basic assumptions for magnetic circuit analysis:
  - No leakage
  - Uniform flux density
  - No fringing across air-gaps

- Assumptions are reasonable for most magnetic circuits