15-Ideal Transformers

Text 11.1

ECEGR 3500

Electrical Energy Systems

Professor Henry Louie

Overview

- Self Inductance
- Transformer Theory of Operation
- Ideal Single Phase Transformers

Introduction

Introduction

“homemade” Zambian transformer

Introduction

- Transformers are important electrical-electrical energy conversion components
- One important reason we use AC is because we can easily change the voltage levels, which reduces losses
- Transformers enable this conversion of voltage level
  - High efficiency (up to 99%)
  - No or few moving parts (low maintenance)

Transformers

- Shift between voltage levels
  - generation 11 kV to 30 kV
  - transmission up to 765 kV
  - distribution around 69 kV
  - residential 240/120 V
- Controlling voltages, power flows
  - regulating transformers
- Isolation (dc current)
- Instrument
  - PTs, CTs

Inductance

- Transformers and other machines have coils of wire wrapped around permeable material
- Transformers are made of one or more inductors on a common core
- We will start with a qualitative description of inductance

Inductance

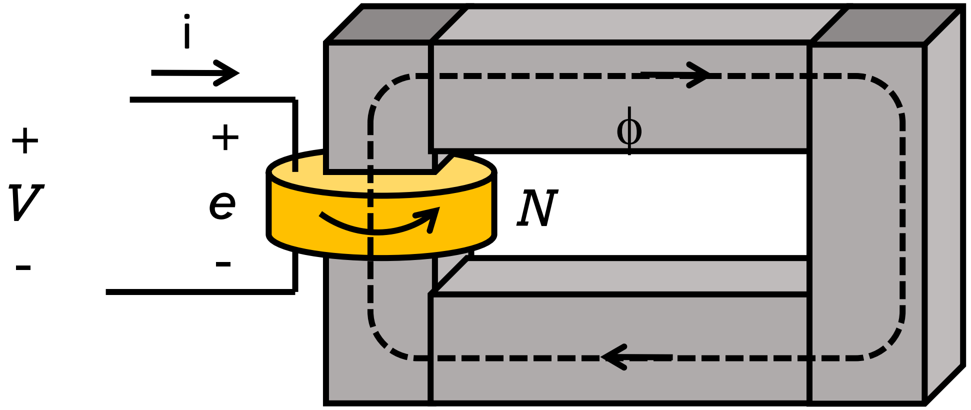
- Inductive reactance X_L exists due to Faraday's Law
  - $jX_L = j\omega L$
- The j operator accounts for the 90 degree phase shift between current and induced voltage
- ω accounts for the dependency on frequency
- L is a description of how strong the current links the flux through the coil
- Next, we examine inductance

Inductance

- Recall that

$$e = N\frac{d\phi}{dt}$$ (note the polarity in the figure)

  - $N\phi$ is also known as the <u>flux linkages</u> (λ)

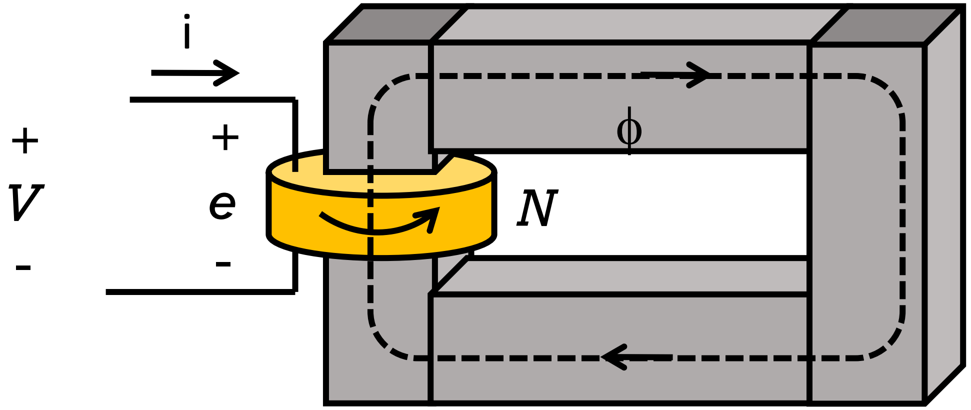


Inductance

- Self-inductance (inductance) is defined as:

$$L \triangleq N \frac{\mathrm{d}\phi}{\mathrm{d}i}$$

- Large inductance: great sensitivity of flux wrt current



Inductance describes how the flux linking a coil changes with the applied current

Self Inductance

- Inductance depends on the physical characteristics of the magnetic circuit
- Recall that

$$\phi = \frac{Ni}{\Re}$$

$$L \triangleq N \frac{d\phi}{di} \qquad \text{therefore}$$

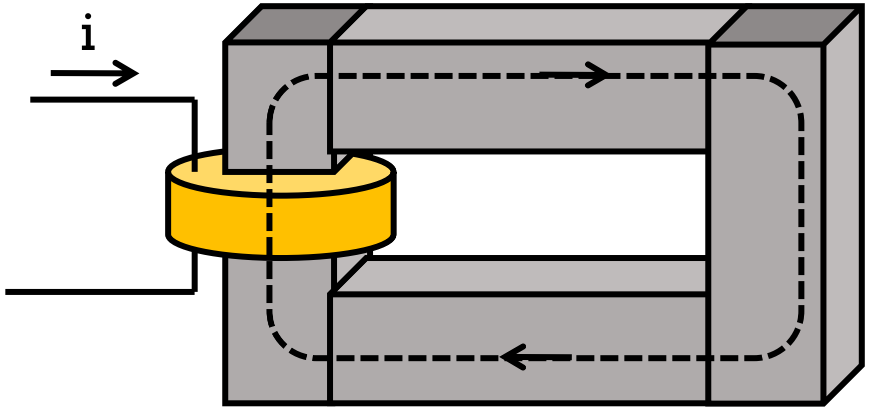
$$L = \frac{N^2}{\Re}$$

- Inductance is constant if the permeability of the magnetic circuit is itself constant (not the case in ferromagnetic materials)
- We will assume that we are operating in the linear region of B-H curve

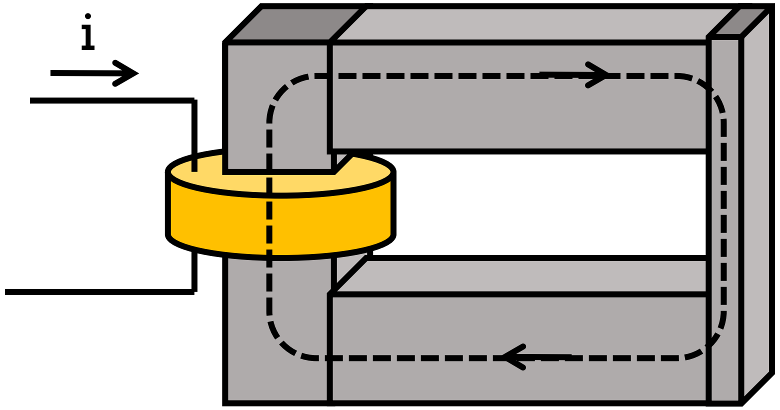
Exercise

Which circuit has greater inductance?

A.



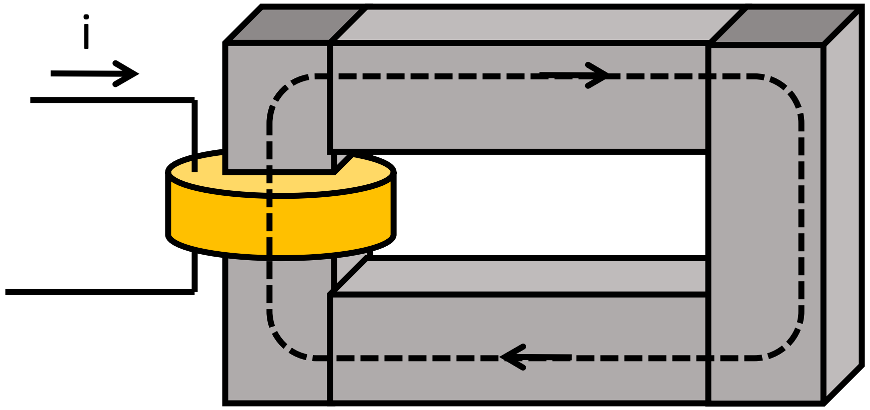
B.

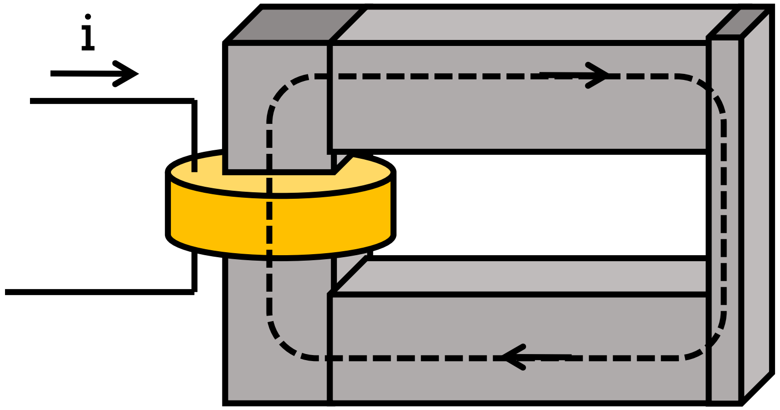


Exercise

Which circuit has greater inductance?

A. B.





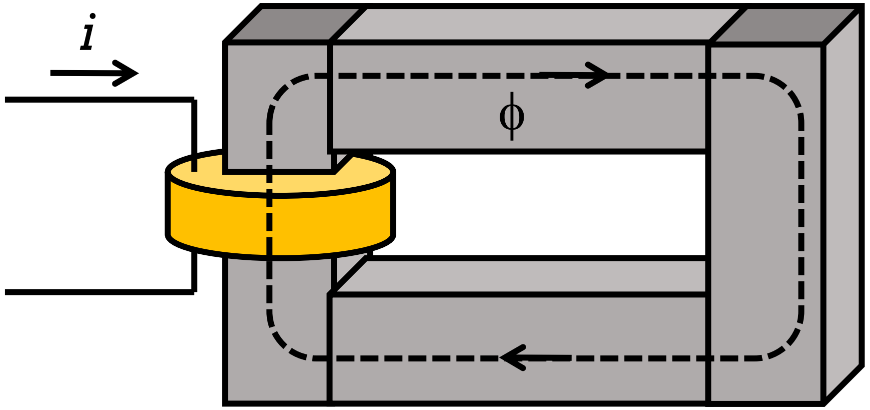
A. Has smaller reluctance. Current gives rise to greater flux so the inductance is larger.

Self Inductance

- Inductance is related to emf by:

$$e = N\frac{d\phi}{dt} = N\frac{d\phi}{di}\frac{di}{dt} = L\frac{di}{dt}$$

- A coil with 1 H of inductance will have 1 volt induced in it if the current changes at a rate of 1 A/s
- <u>If we know the inductance, we do not need to compute the flux</u>

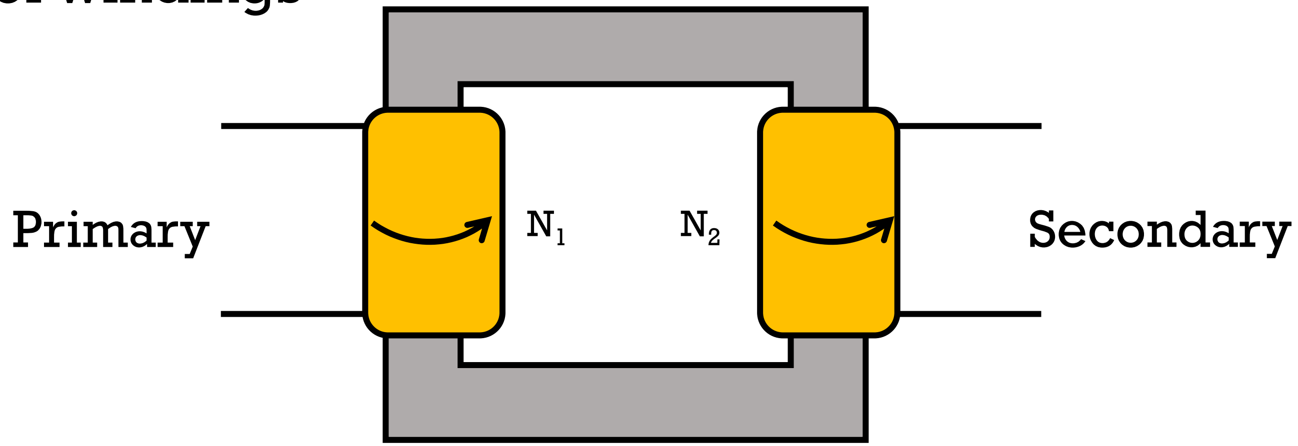


Questions

- Why are transformers used in power systems?
- Is it possible to use a “dc” transformer?
- Are transformers efficient?
- How is the power into a transformer related to the power out of a transformer?

Ideal Single-Phase Transformer

- Two magnetically coupled coils
  - Primary: N_1 turns
  - Secondary: N_2 turns
- Primary and secondary can be arbitrarily assigned
- Note direction of windings



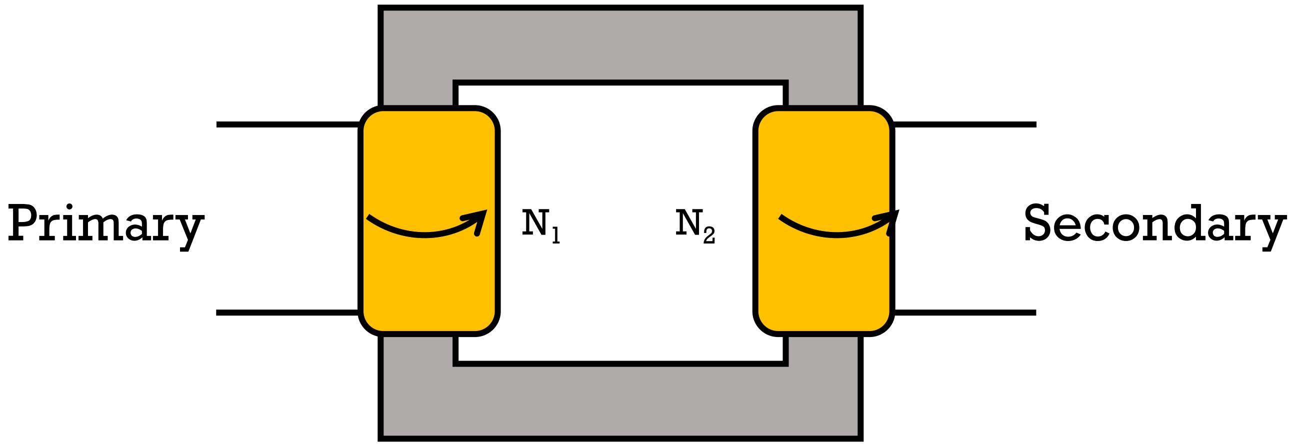
Ideal Single-Phase Transformer

Ideal assumptions

- No flux leakage
- No eddy currents

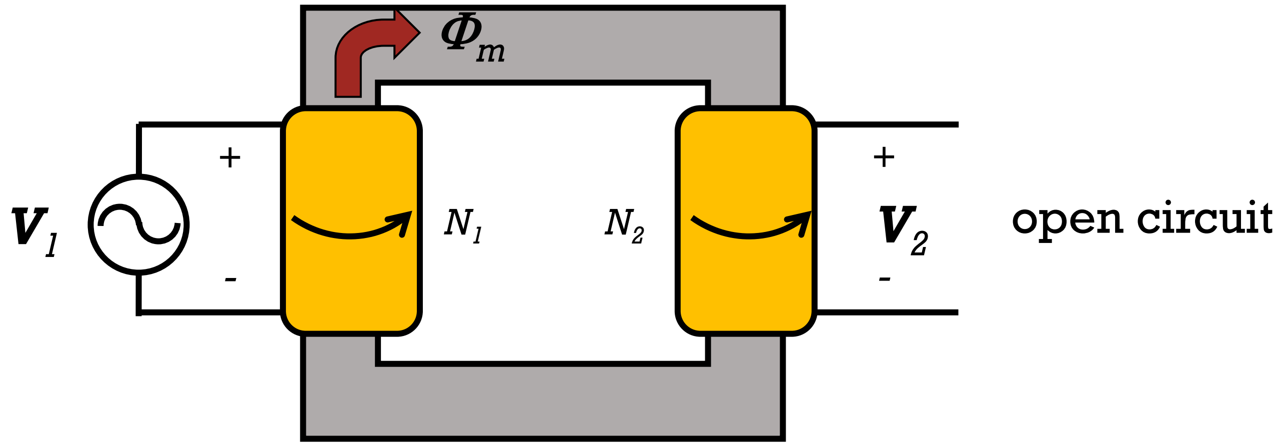
Recall from magnetic circuits lecture

- No winding resistance
- <u>Near infinite core permeability</u>



Single-Phase Transformer

- Primary directly connected to AC voltage source
- Voltage across coil has sinusoidal flux associated with it $e = -\frac{d\phi}{dt}$ (Faraday's Law)

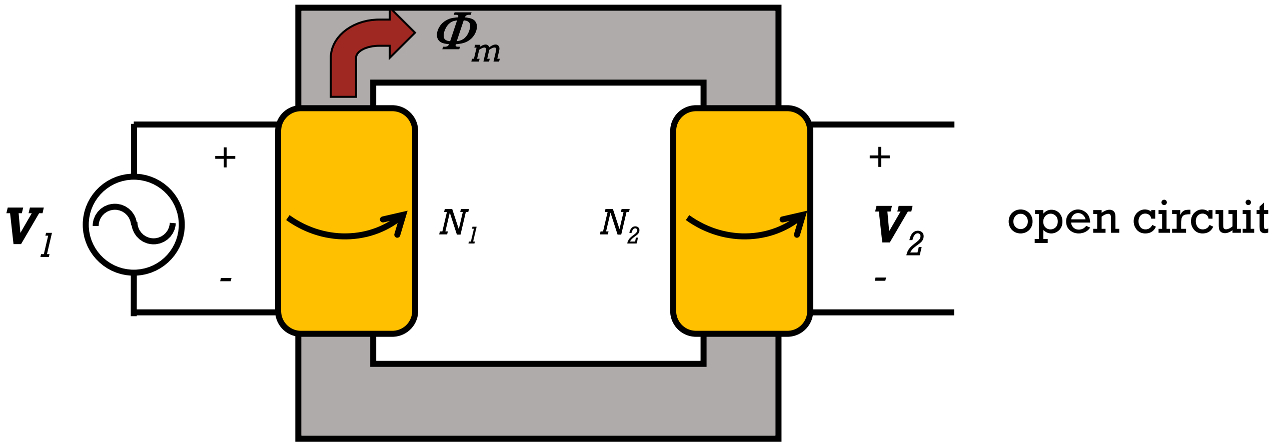


Single-Phase Transformer

- Same flux passes through each coil
  - Φ_m: mutual flux (phasor)
- Therefore:

$$\boldsymbol{V}_1 = \frac{d\lambda_1}{dt} = N_1 \frac{d\Phi_m}{dt}$$

$$\boldsymbol{V}_2 = \frac{d\lambda_2}{dt} = N_2 \frac{d\Phi_m}{dt}$$



- Rewritten: $\frac{V_1}{V_2} = \frac{N_1}{N_2} \triangleq a \triangleq \frac{1}{n}$

"a" is also referred to as the "voltage ratio"

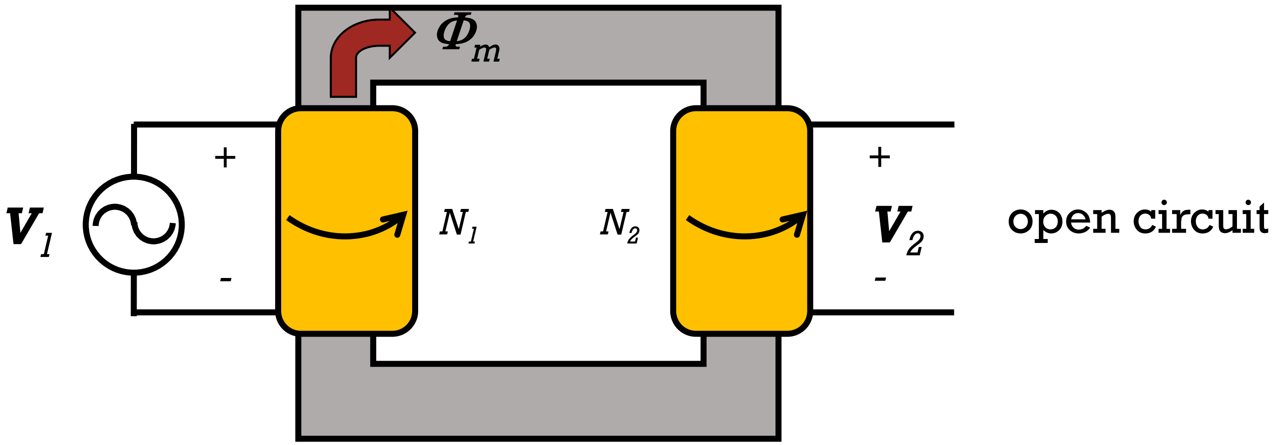
Single-Phase Transformer

- Ratio of voltages is the same as the ratio of turns

$$\frac{V_1}{V_2} = \frac{N_1}{N_2}$$

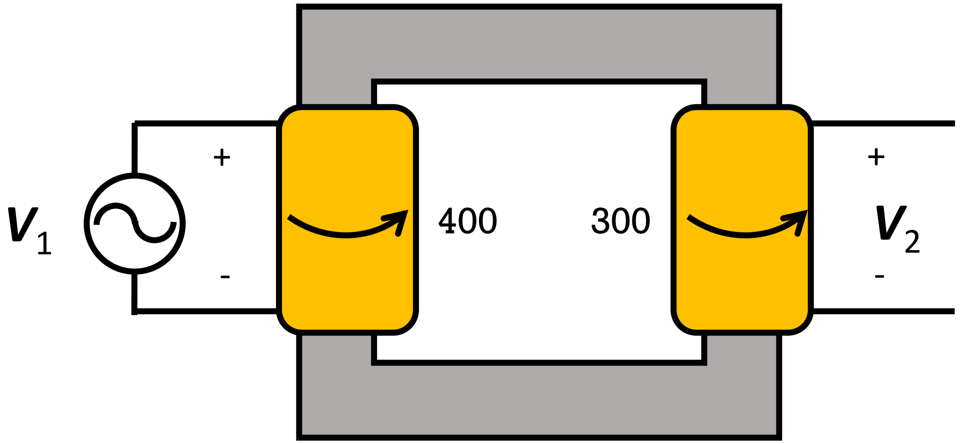
- Possible to transform voltage level from primary to secondary (and vice versa)
- Note: no current flows
  - (near infinite permeability)

$$\phi = BA = \frac{\mu NiA}{\ell}$$



Exercise

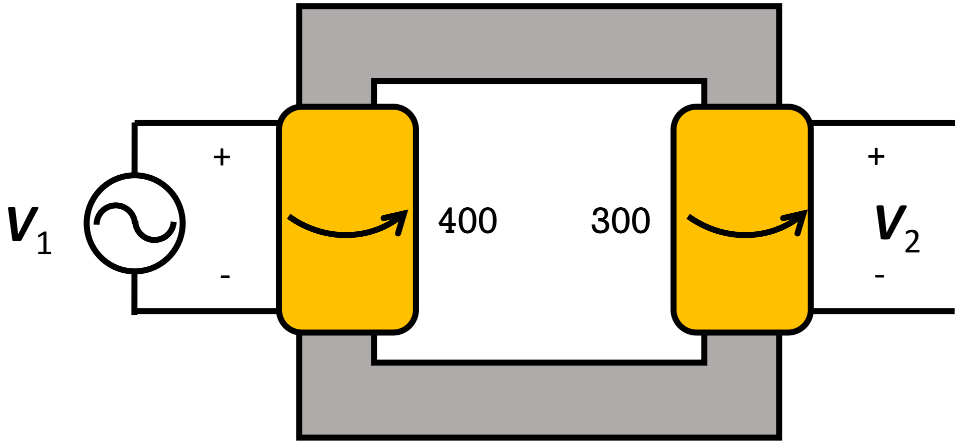
If $|\boldsymbol{V}_1| = 120\text{V}$, is $|\boldsymbol{V}_2|$ greater than 120V?



Exercise

If $|\boldsymbol{V}_1| = 120$V, is $|\boldsymbol{V}_2|$ greater than 120V?

Less than 120V.



The winding with more turns has greater voltage

Voltage per Turn

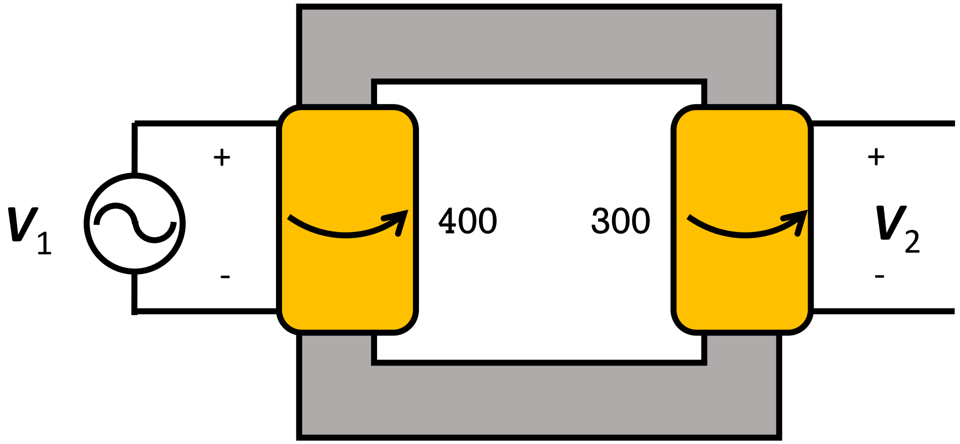
- We define the voltage per turn as:

$$V_{\mathrm{T}} = \frac{V_1}{N_1} = \frac{V_2}{N_2}$$

- The voltage per turn is the same for the primary and secondary of the transformer

Exercise

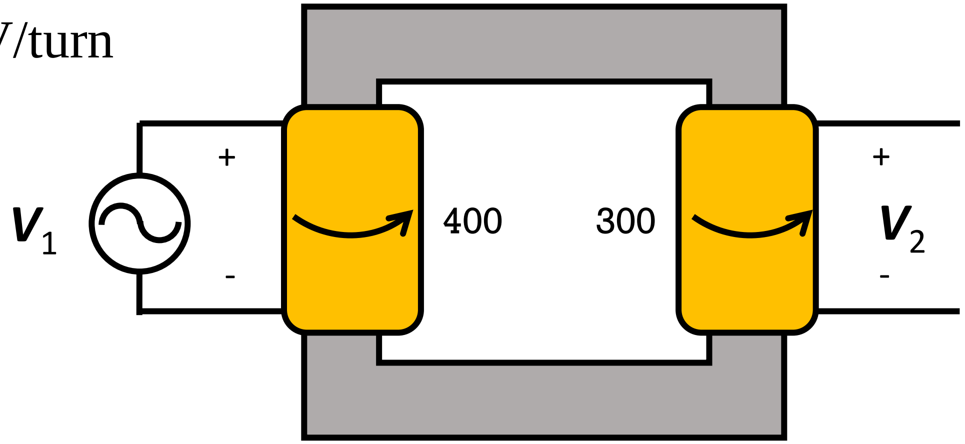
If $|\boldsymbol{V}_1| = 120\text{V}$, what is the voltage per turn of the transformer?



Exercise

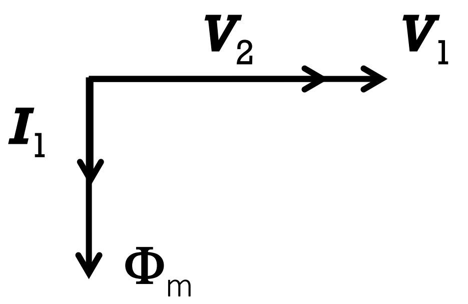
If $|\boldsymbol{V}_1| = 120$V, what is the voltage per turn of the transformer?

$$V_T = \frac{V_1}{N_1} = \frac{120}{400} = 0.30 \text{ V/turn}$$



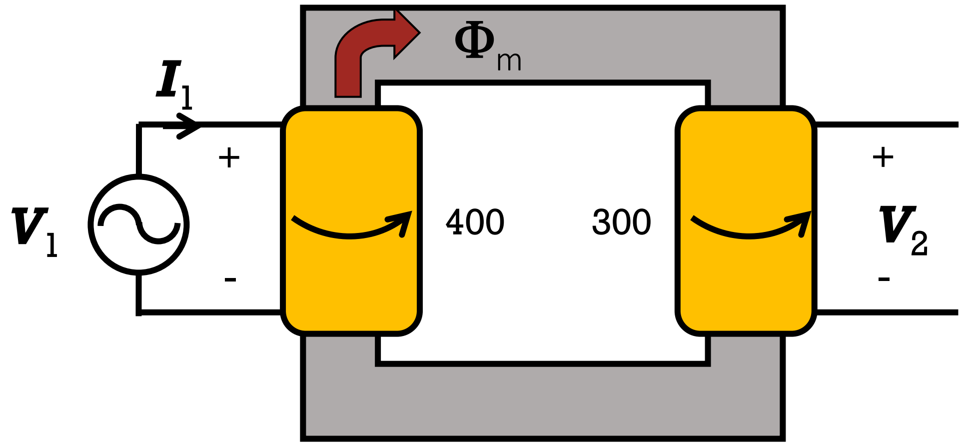
Single-Phase Transformer

Phasor Diagram
(finite permeability, no load)



V_1, V_2 in phase.
Φ_m lags voltage by 90°
Current in phase with Φ_m

Note: $|\Phi|$ arbitrarily drawn



Single-Phase Transformer

- Now a resistive load is connected to the secondary
- $\boldsymbol{V}_2$ causes $\boldsymbol{I}_2$ to flow
- Examining mmf

$$\Im = N_1\mathbf{I}_1 - N_2\mathbf{I}_2 = \Re\boldsymbol{\Phi}_m$$

$$\Re = \frac{\ell}{\mu A}$$

- Infinite permeability

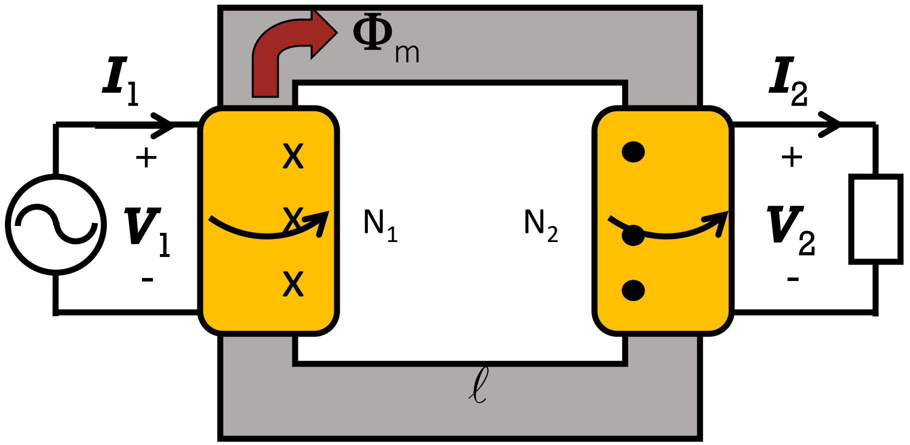
$$\Im = N_1\boldsymbol{I}_1 - N_2\boldsymbol{I}_2 = 0$$

$$N_1\boldsymbol{I}_1 = N_2\boldsymbol{I}_2$$

- Current gain

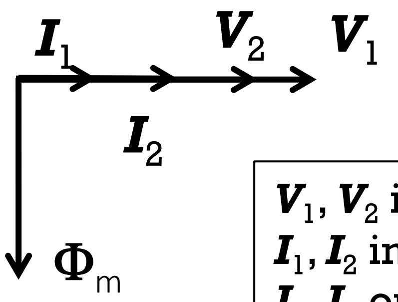
$$\frac{\boldsymbol{I}_2}{\boldsymbol{I}_1} = \frac{N_1}{N_2} = a$$

Compare to: $\frac{V_1}{V_2} = \frac{N_1}{N_2}$



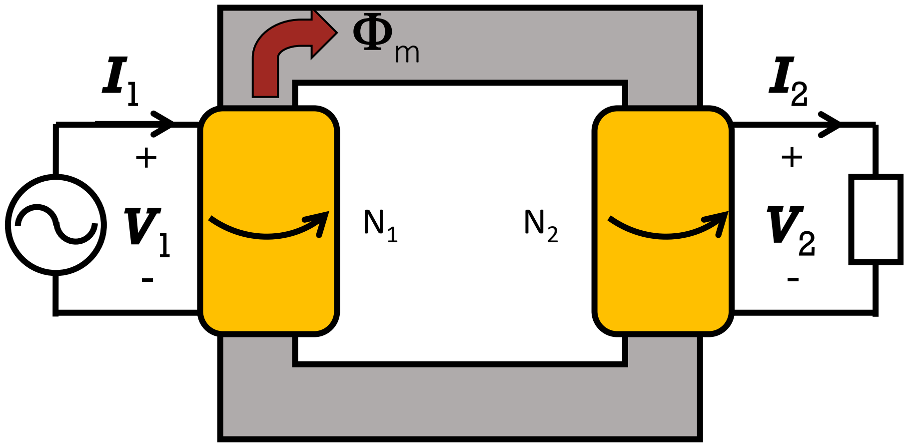
Single-Phase Transformer

Phasor Diagram
(ideal xfmr, resistive load)



V_1, V_2 in phase
I_1, I_2 in phase
I_1, I_2 out of phase with Φ_m
Φ_m lags voltage by 90°

Note: $|\Phi|$ arbitrarily drawn



Exercise

In an ideal transformer serving a load, if $|\boldsymbol{V}_1| > |\boldsymbol{V}_2|$, then is $|\boldsymbol{I}_1| > |\boldsymbol{I}_2|$?

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No. The transformer would be creating energy.

Exercise

How are the transformer input and output power related? Find α in $P_1 = \alpha P_2$

Exercise

- Power into the transformer

$$P_1 = \mathrm{Re}\{\boldsymbol{V}_1\boldsymbol{I}_1{*}\}$$

- Power out of the transformer

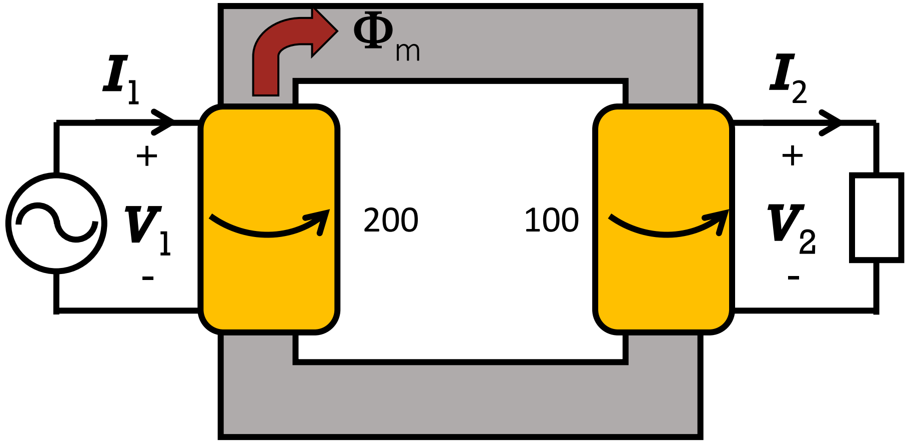
$$P_2 = \mathrm{Re}\{\boldsymbol{V}_2\boldsymbol{I}_2^*\} = \mathrm{Re}\{\frac{1}{a}\boldsymbol{V}_1\boldsymbol{I}_2^*\} = \mathrm{Re}\{\frac{1}{a}\boldsymbol{V}_1 a\boldsymbol{I}_1^*\} = P_1$$

- Power is conserved

Exercise

- Now a load with PF = 0.707 lagging is connected to the secondary
- Draw the phasor diagram of

$\boldsymbol{V}_1, \boldsymbol{V}_2, \boldsymbol{I}_1, \boldsymbol{I}_2, \Phi_\mathrm{m}$



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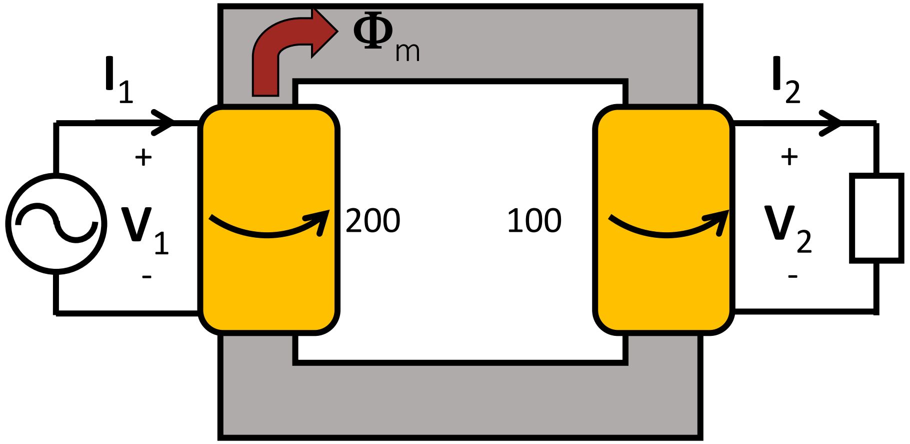
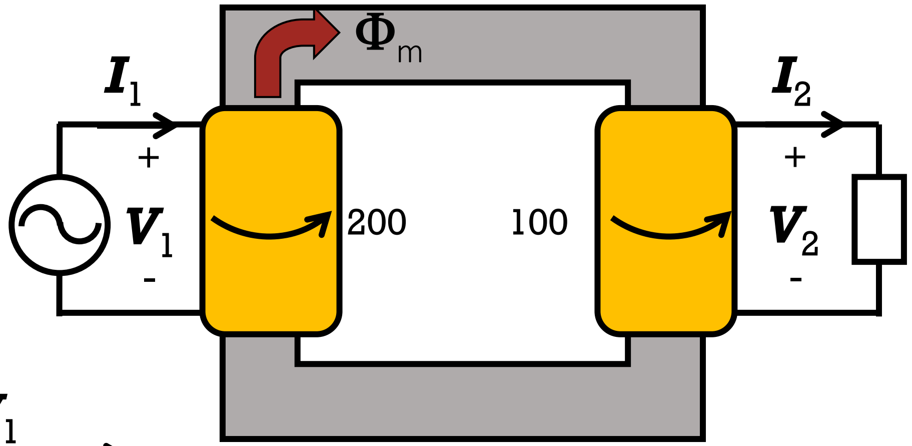
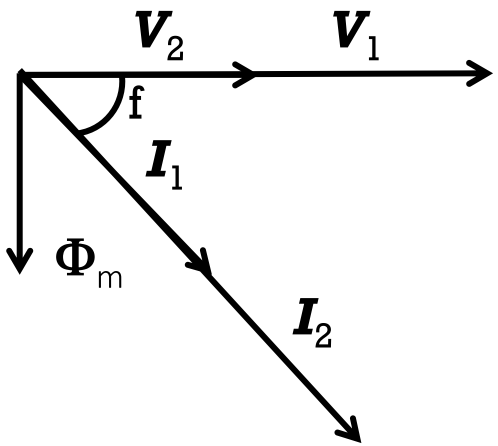


$|\Phi_m|$, $|I|$ can be arbitrarily drawn with respect to each other

Exercise

- $\boldsymbol{V}_1, \boldsymbol{V}_2$ in phase: $\mathbf{V}_1 = \mathbf{V}_2 \dfrac{N_1}{N_2}$
- Φ_m lags voltage by 90°
- $\boldsymbol{I}_2$ lags $\mathbf{V}_2$ by 45°: $\phi = \cos^{-1}(0.707) = 45°$
- $\boldsymbol{I}_1, \boldsymbol{I}_2$ in phase: $\boldsymbol{I}_1 = \boldsymbol{I}_2 \dfrac{N_2}{N_1}$

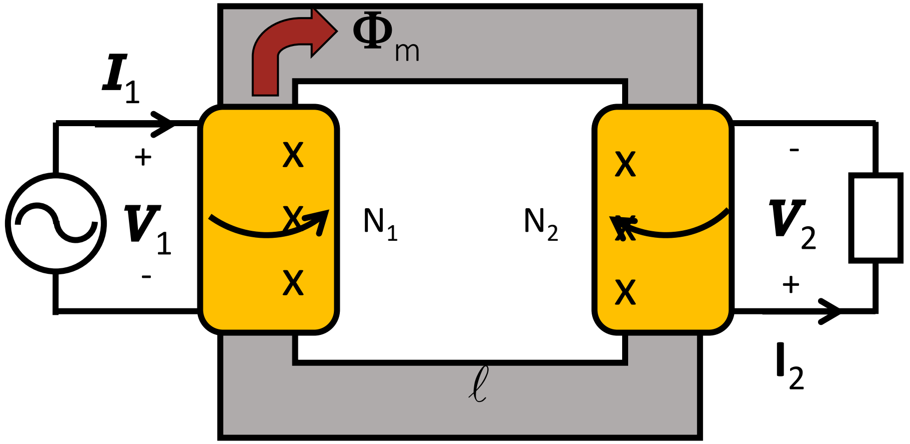




$\mathbf{I}_1, \mathbf{I}_2$ not in phase with Φ_m.

Transformer Polarity

What if the secondary coil was wound the opposite direction?



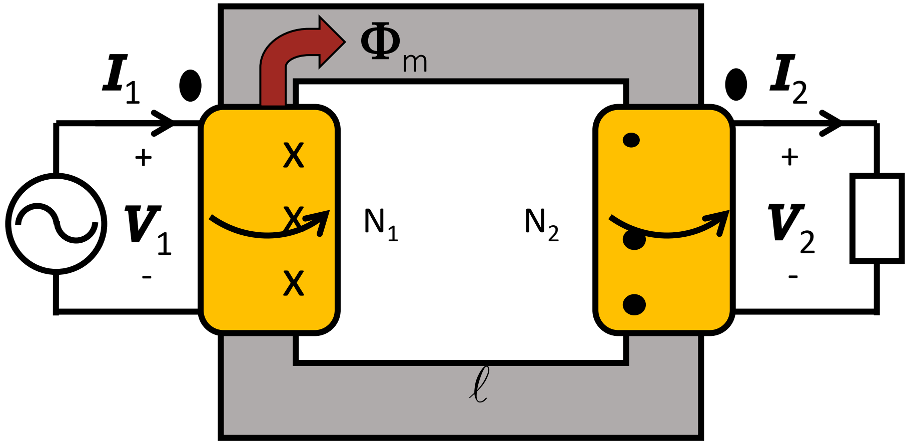
Examining the mmf:

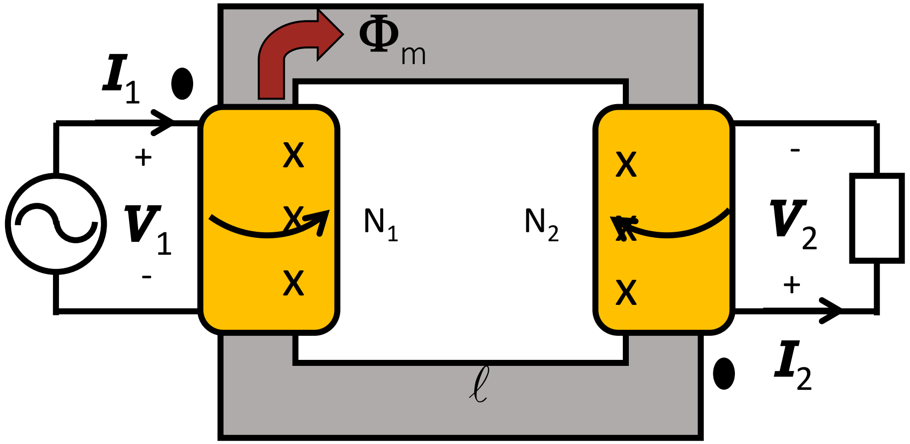
$$\mathfrak{I} = N_1\boldsymbol{I}_1 + N_2\boldsymbol{I}_2 = 0$$
$$N_1\boldsymbol{I}_1 = -N_2\boldsymbol{I}_2$$

Current and voltage polarity reverses

Transformer Polarity

- Dot polarity:
  - Current entering polarity-marked terminals create flux in the same direction
  - When current enters one polarity-marked terminal, it leaves the other
  - Voltage of polarity-marked terminals are in phase (e.g. they are positive at the same time)

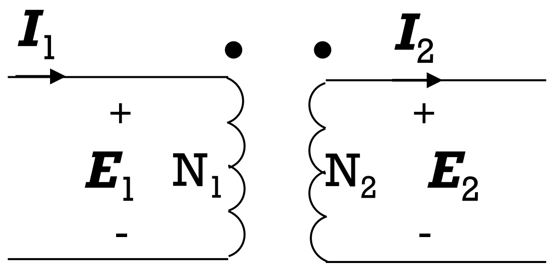




Transformer polarity is dictated by the direction of windings

Circuit Model

- New circuit element: Ideal Transformer
- Voltage relationship $\boldsymbol{E}_1 = a\boldsymbol{E}_2$
- Current relationship $\boldsymbol{I}_1 = \frac{1}{a}\boldsymbol{I}_2$



Ideal Transformer

Recall: $a = \frac{N_1}{N_2}$

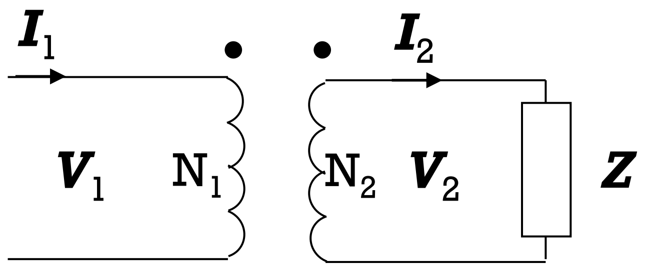
Circuit Model

- Now a load is connected to the secondary
- Solving for $\boldsymbol{I}_1$

$$\boldsymbol{I}_1 = \frac{1}{a}\boldsymbol{I}_2$$

$$\boldsymbol{I}_1 = \frac{1}{a}\frac{\boldsymbol{V}_2}{\boldsymbol{Z}} \quad \text{using } \boldsymbol{I}_2 = \frac{\boldsymbol{V}_2}{\boldsymbol{Z}}$$

$$\boldsymbol{I}_1 = \frac{\boldsymbol{V}_1}{a^2\boldsymbol{Z}} \quad \text{using } \boldsymbol{V}_1 = a\boldsymbol{V}_2$$



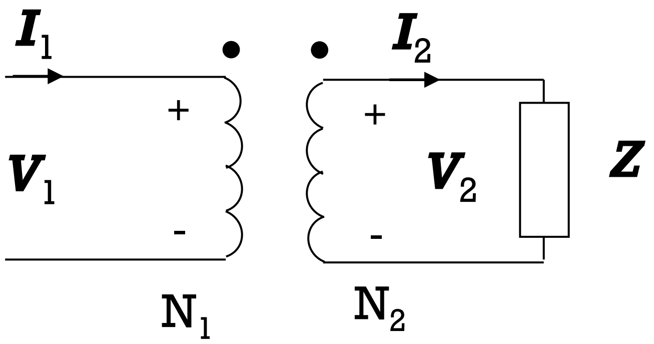
Ideal Transformer

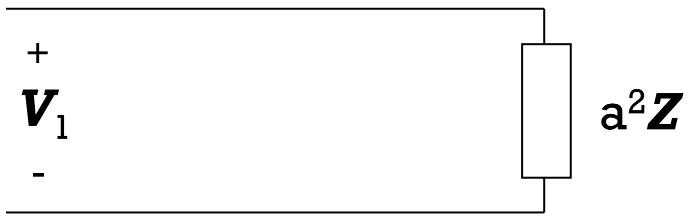
Circuit Model

From this result, <u>it is possible to analyze the circuit only using primary-side voltage and current</u> ($\boldsymbol{V}_1, \boldsymbol{I}_1$)

$$\boldsymbol{I}_1 = \frac{\boldsymbol{V}_1}{a^2 \boldsymbol{Z}}$$

Secondary impedance referred to the primary side





equivalent circuit

Example

Consider an ideal transformer with $N_1 = 100$ and $N_2 = 500$. The primary is connected to a 100 V source. A load of 100 Ohms is connected to the secondary.

Find the power delivered to the load.

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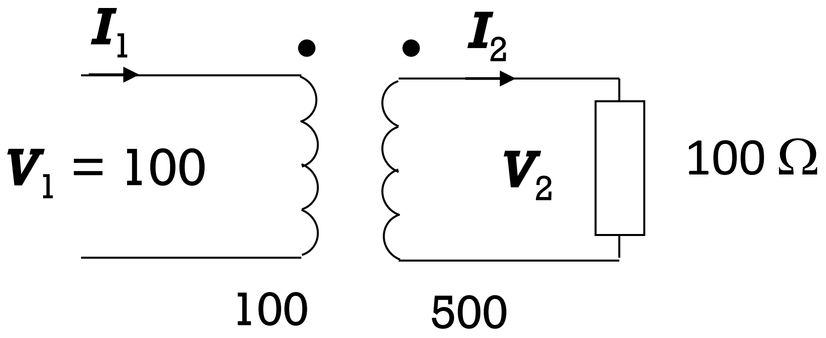
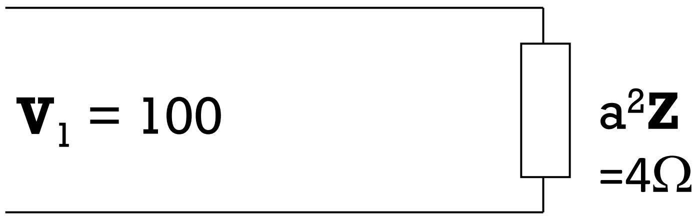
$$a = \frac{N_1}{N_2} = \frac{100}{500} = 0.2$$





equivalent circuit

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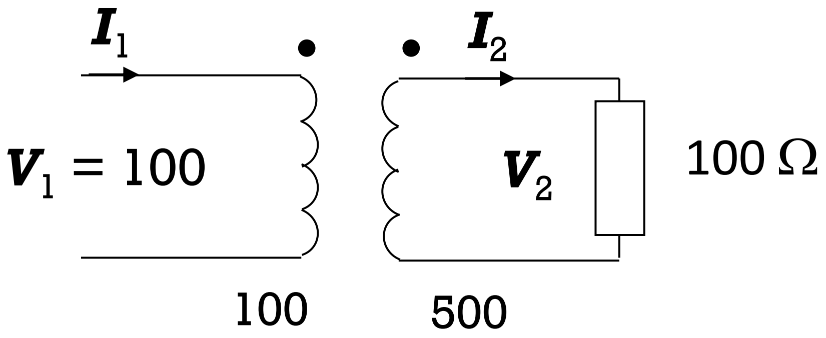
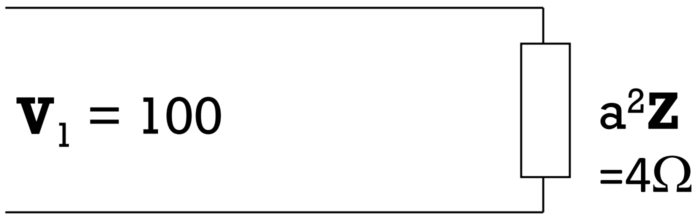
Find the power delivered to the load.

$$P = \frac{|\boldsymbol{V}_1|^2}{R} = \frac{10{,}000}{4} = 2{,}500\ \mathrm{W}$$





equivalent circuit

Transformer Ratings

- Electrical ratings of transformers:
  - Voltage (primary and secondary)
  - Current (primary and secondary)
  - Power (apparent power)
- Ratings indicate the voltage, current, and apparent power the transformer can operate at without damaging it or shortening its lifespan

Voltage and current ratings refer to <u>magnitudes</u>.
They are not phasors

Transformer Rating Example

- Consider a transformer rated as "10 kVA, 8kV/240V"

rated power

- There is a single rated power for the transformer (not a primary rating and secondary rating)
- Transformer power rating = primary voltage rating x primary current rating = secondary voltage rating x secondary current rating

Transformer Rating Example

- Consider a transformer rated as "10 kVA, 8kV/240V"

- The rated primary current is

$$I_1 = \frac{S}{V_1} = \frac{10,000}{8,000} = 1.25 \text{ A}$$

- The rated secondary current is

$$I_2 = \frac{S}{V_2} = \frac{10,000}{240} = 41.67 \text{ A}$$

Transformer Rating Example

- Consider a transformer rated as "10 kVA, 8kV/240V"

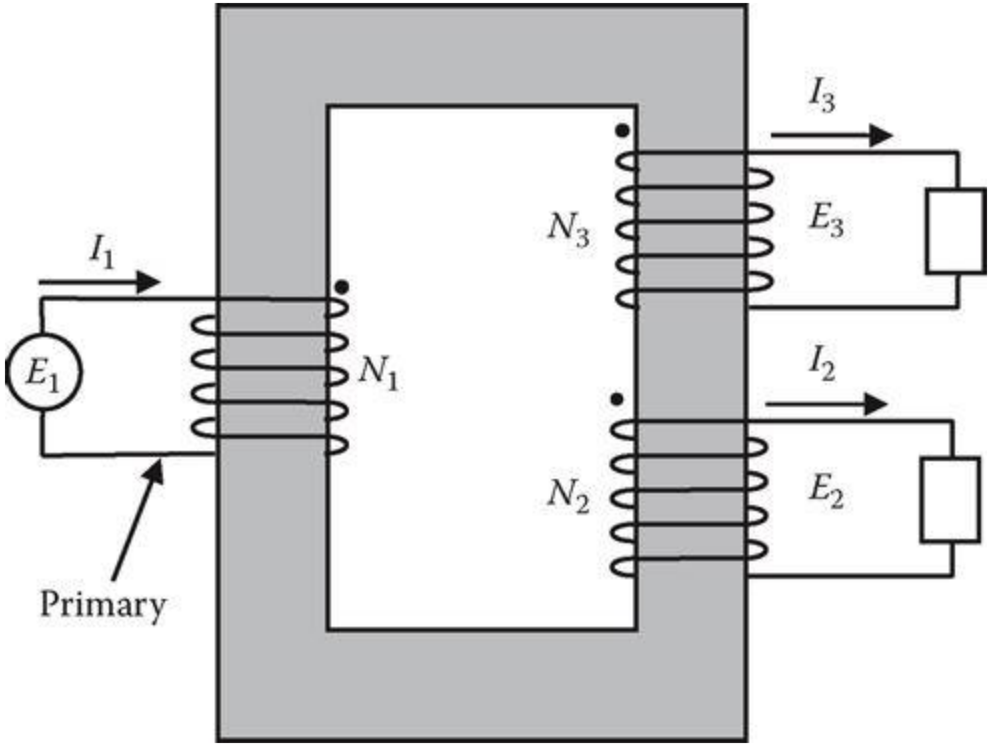
primary rated voltage

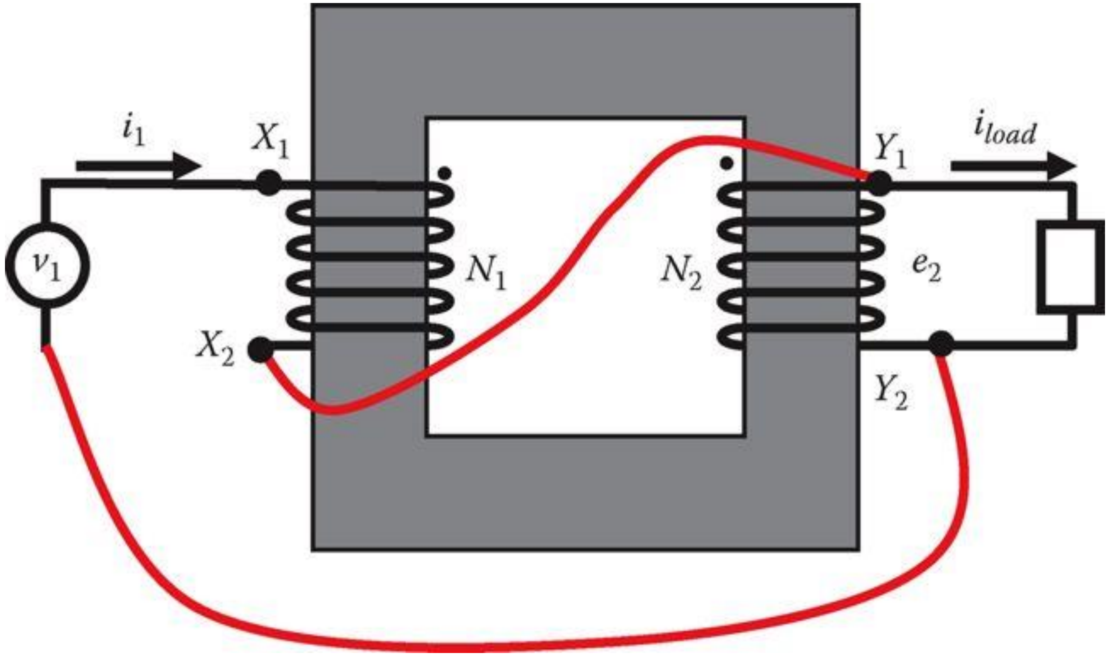
secondary rated voltage

- The voltage ratio is therefore:

$$a = \frac{V_1}{V_2} = \frac{8000}{240} = 33.33$$

Multi-Winding and Autotransformers





Read 11.2, 11.3 of Text

Summary

- Transformers are magnetically coupled coils
- Ratio of turns from primary to secondary is the "turns ratio". Side with greater number of turns has higher voltage, but lower current
- Ideal transformers: Power in = Power out
- Equivalent circuit is used to analyze transformers. Impedances can be transferred from secondary to primary by scaling by a^2