16-Non-Ideal Transformers

Text 11.5

ECEGR 3500

Electrical Energy Systems

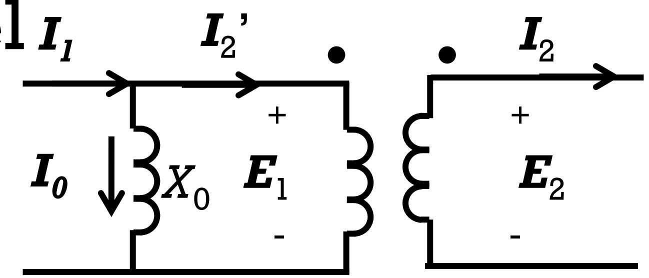
Professor Henry Louie

Magnetizing Reactance

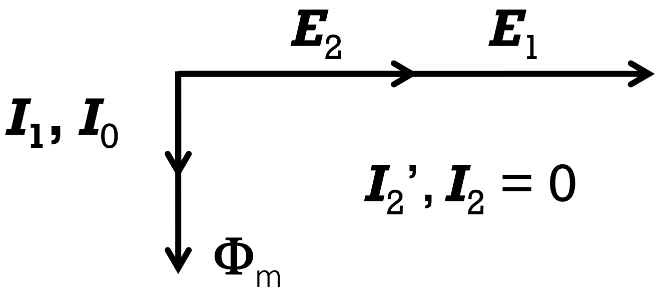
- Non-ideal transformers do not have near infinite permeability

$$\Re = \frac{\ell}{\mu A} \neq 0 \qquad \Im = N_1\mathbf{I}_1 - N_2\mathbf{I}_2 = \Re\mathbf{\Phi}_m \neq 0$$

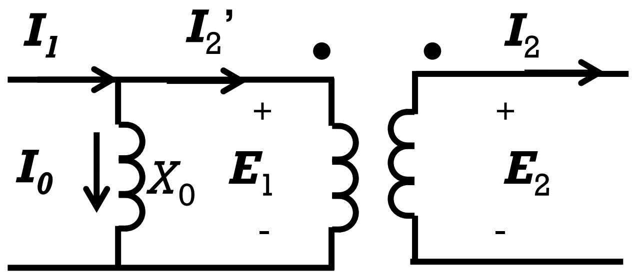
- Add shunt magnetizing reactance (X_0) to ideal transformer model



Magnetizing Reactance



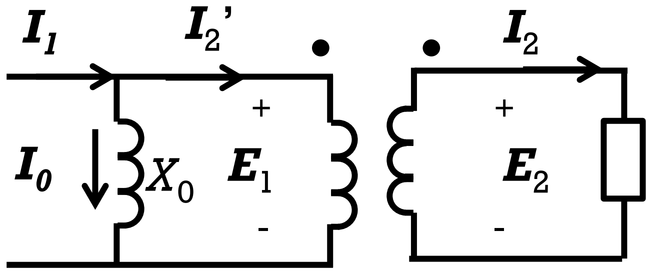
without load

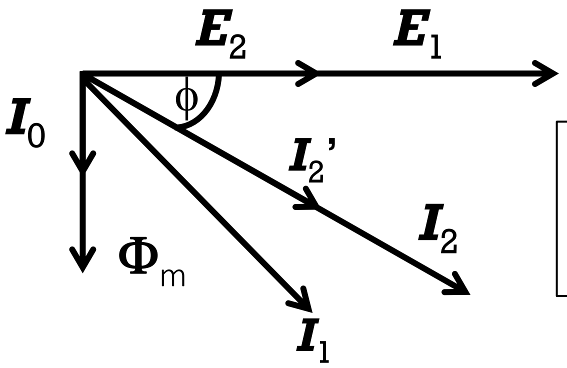


$$N_1 \boldsymbol{I}_2' = \Re \boldsymbol{\Phi}_m + N_2 \boldsymbol{I}_2$$

(vector sum)

with load





Current into and out of xfmr terminals no longer in phase

Exercise

- A transformer has 450 turns on the primary and 50 turns on the secondary. The primary voltage is 6000V. If the magnetizing reactance is $j500\Omega$ compute:
  - The no-load primary current and real power loss of the transformer
  - The primary current if a load impedance of $\boldsymbol{Z} = 10 + j15$ is applied to the secondary.

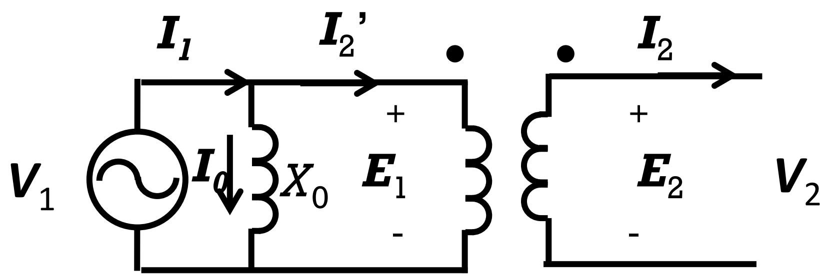
Exercise

$$\boldsymbol{I}_1 = \boldsymbol{I}_0 + \boldsymbol{I}_2^{'} = \frac{\boldsymbol{V}_1}{500\angle 90^\circ} + 0 = 12\angle -90^\circ \text{ A}$$

$$\text{P}_{\text{in}} = \text{Re}\{\boldsymbol{V}_1\boldsymbol{I}_1^*\} = 0\text{W}$$

$$\text{P}_{\text{out}} = \text{Re}\{\boldsymbol{V}_2\boldsymbol{I}_2^*\} = 0\text{W}$$

$$\text{P}_{\text{Loss}} = \text{P}_{\text{in}} - \text{P}_{\text{out}} = 0\text{W}$$



No load: open circuit

Exercise

$$\boldsymbol{I}_0 = \frac{\boldsymbol{V}_1}{500\angle 90^\circ} = 12\angle -90^\circ \text{ A}$$

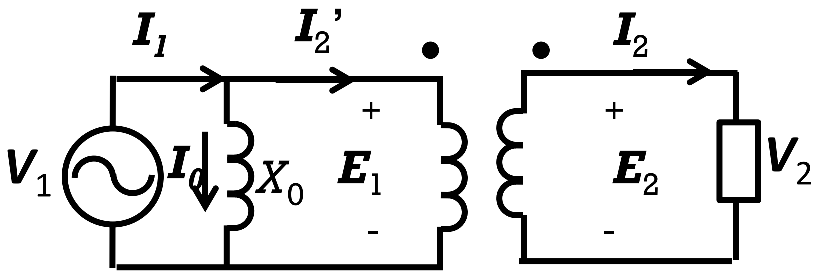
$$\boldsymbol{E}_1 = \boldsymbol{V}_1 = \frac{N_1}{N_2}\boldsymbol{E}_2$$

$$\boldsymbol{E}_2 = 667\angle 0^\circ \text{ V}$$

$$\boldsymbol{I}_2 = \frac{\boldsymbol{E}_2}{10 + j15} = 36.98\angle -56.3^\circ \text{ A}$$

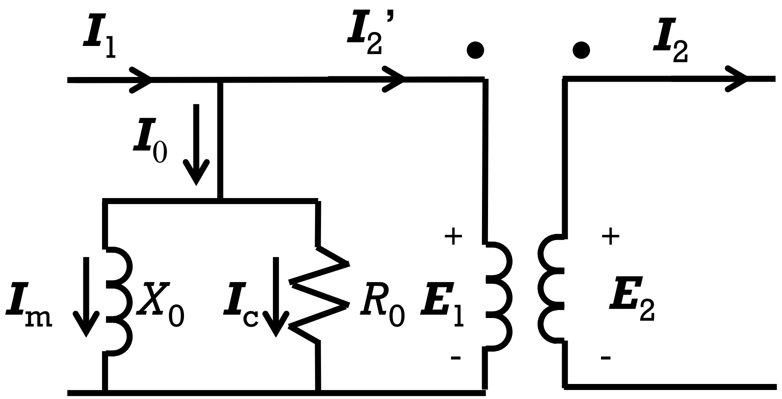
$$\boldsymbol{I}_1 = \boldsymbol{I}_0 + \boldsymbol{I}_2' = \boldsymbol{I}_0 + \frac{50}{450}\boldsymbol{I}_2 = 15.59\angle -81.6^\circ \text{ A}$$

Note: real power in = real power delivered to load



Core Resistance

- Non-ideal transformers have eddy current loss
  - real power loss
  - occurs even with no secondary load
- Model as shunt resistance
- $R_0 >> X_0$



Note: xfmr are designed to have large X_0, R_0 values

Leakage Flux

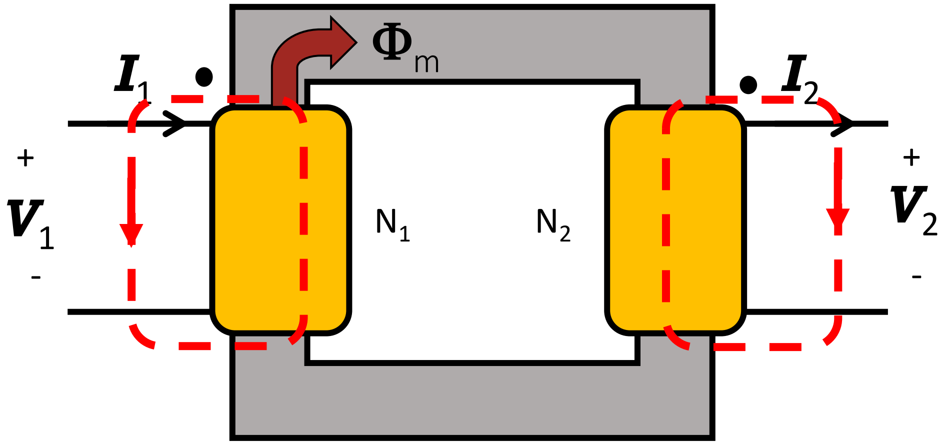
- Non-ideal transformers have leakage flux
- Leakage flux: flux in primary(secondary) coil that is not linked to secondary (primary) coil

$$\boldsymbol{\lambda}_1 = \boldsymbol{\lambda}_{l1} + N_1 \boldsymbol{\Phi}_m$$

$$\boldsymbol{\lambda}_2 = -\boldsymbol{\lambda}_{l2} + N_2 \boldsymbol{\Phi}_m$$

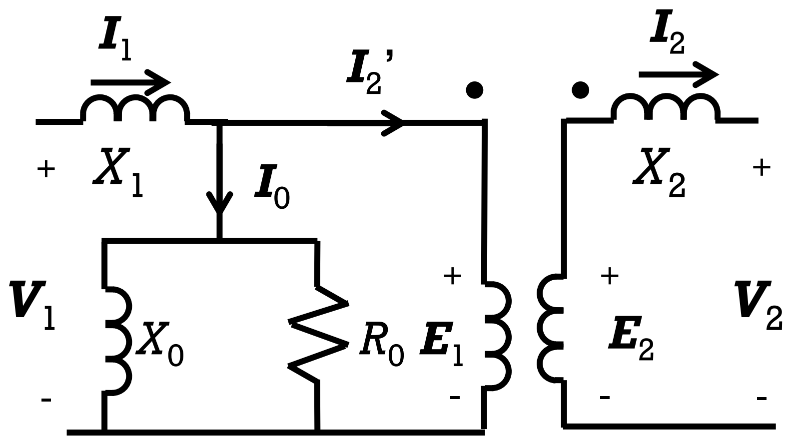
$$\boldsymbol{V}_1 = \frac{d\boldsymbol{\lambda}_1}{dt} = L_{l1}\frac{d\boldsymbol{I}_1}{dt} + N_1\frac{d\boldsymbol{\Phi}_m}{dt}$$

$$\boldsymbol{V}_2 = \frac{d\boldsymbol{\lambda}_2}{dt} = -L_{l2}\frac{d\boldsymbol{I}_2}{dt} + N_2\frac{d\boldsymbol{\Phi}_m}{dt}$$



Leakage Flux

- Model as series reactances on primary and secondary
- Xmfrs are generally designed to have low leakage reactance
  - $X_1 << X_0$

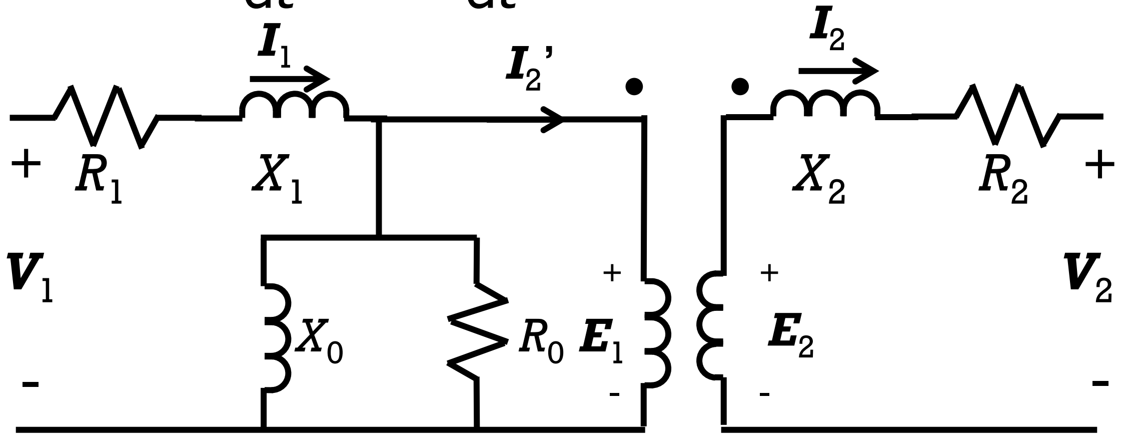


Winding Resistance

- Include winding resistance
- $\mathrm{R}_1 < X_1, R_2 < X_2$

$$\boldsymbol{V}_1 = \mathrm{R}_1\boldsymbol{I}_1 + \frac{d\boldsymbol{\lambda}_1}{dt} = \mathrm{R}_1\boldsymbol{I}_1 + L_{l1}\frac{d\boldsymbol{I}_1}{dt} + \mathrm{N}_1\frac{d\boldsymbol{\Phi}_m}{dt}$$

$$\boldsymbol{V}_2 = -\mathrm{R}_2\boldsymbol{I}_2 + \frac{d\boldsymbol{\lambda}_2}{dt} = -\mathrm{R}_2\boldsymbol{I}_2 - L_{l2}\frac{d\boldsymbol{I}_2}{dt} + \mathrm{N}_2\frac{d\boldsymbol{\Phi}_m}{dt}$$



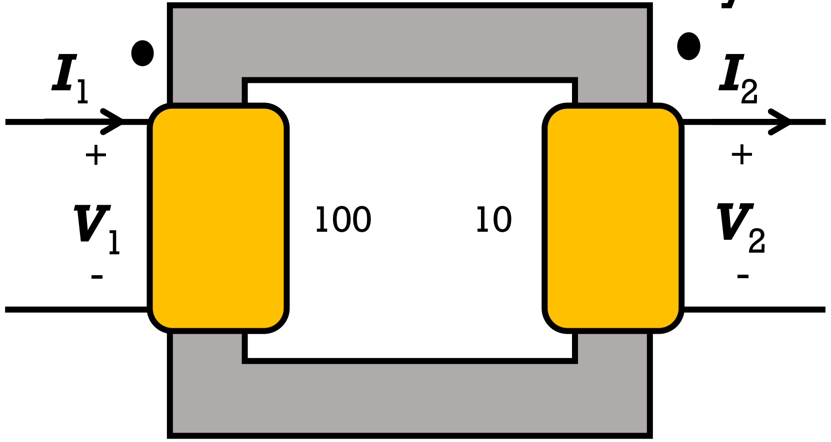
Winding Resistance

If you were designing a transformer with the shown number of turns, would you rather:

A. use the same gauge wire on the primary and secondary

B. use larger diameter on the primary

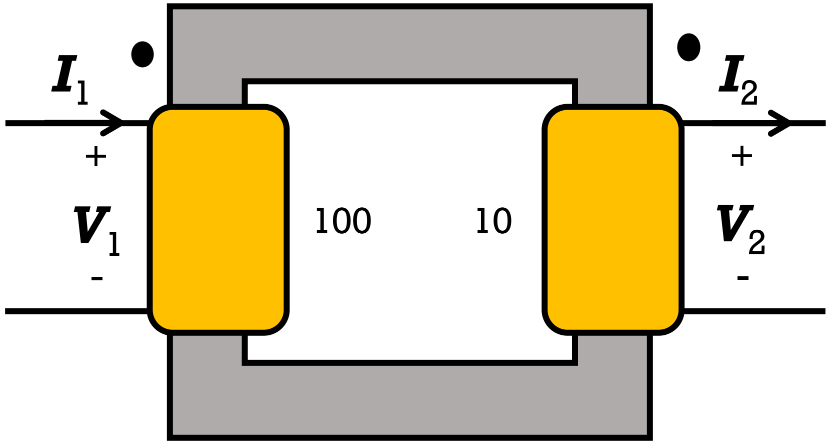
C. use larger diameter wire on the secondary



Winding Resistance

More current is flowing through the secondary, so it requires lower resistance to dissipate the same heat. You should use larger diameter wire.

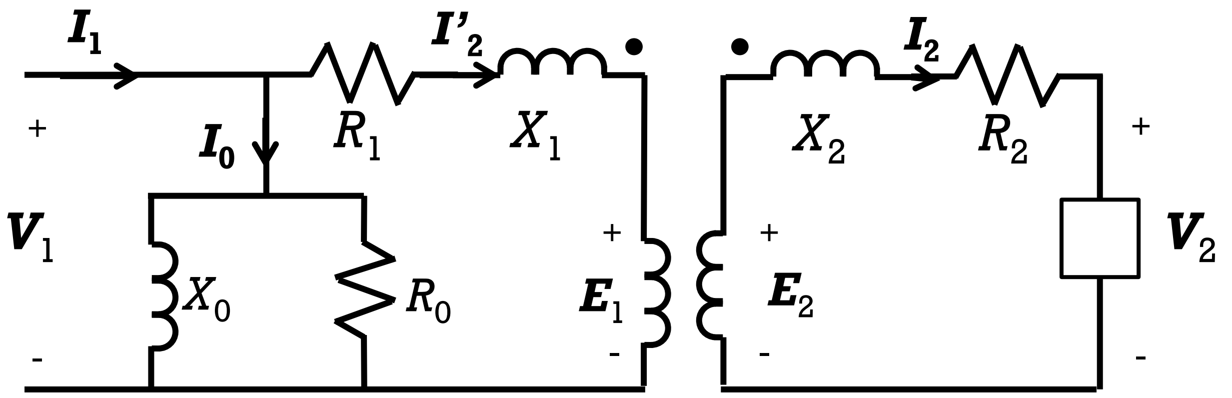
Side with fewer turns (lower voltage, higher current) has lower resistance wire



Example

- Let:
  - $X_o = 20{,}000\ \Omega$
  - $R_o = 40{,}000\ \Omega$
  - $R_1 = 2.56\ \Omega$
  - $R_2 = 0.010\ \Omega$
  - $X_1 = 3.84\ \Omega$
  - $X_2 = 0.015\ \Omega$
  - $N_1 = 3200$
  - $N_2 = 200$
  - $Z_{load} = 10\ \Omega$
  - $|\mathbf{V}_1| = 8000$ V

Find $\boldsymbol{I}_1$, and the input power



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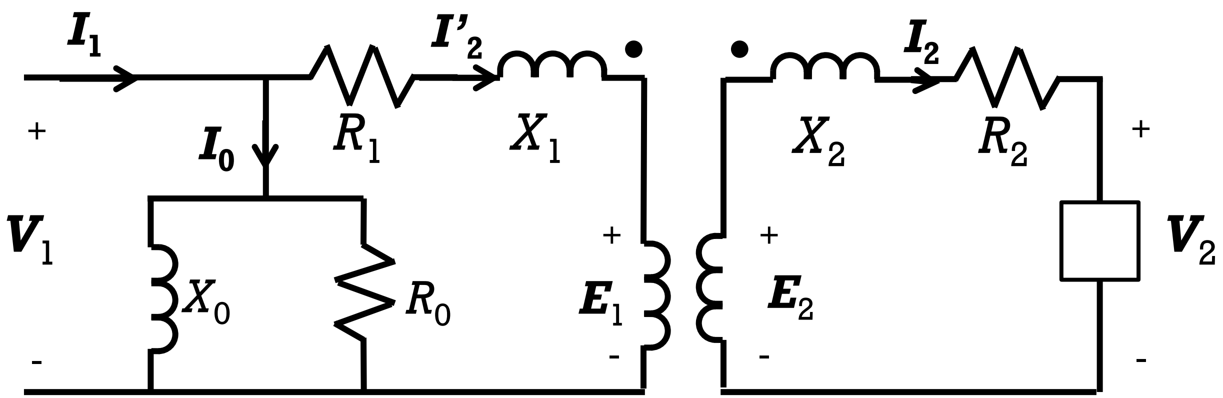
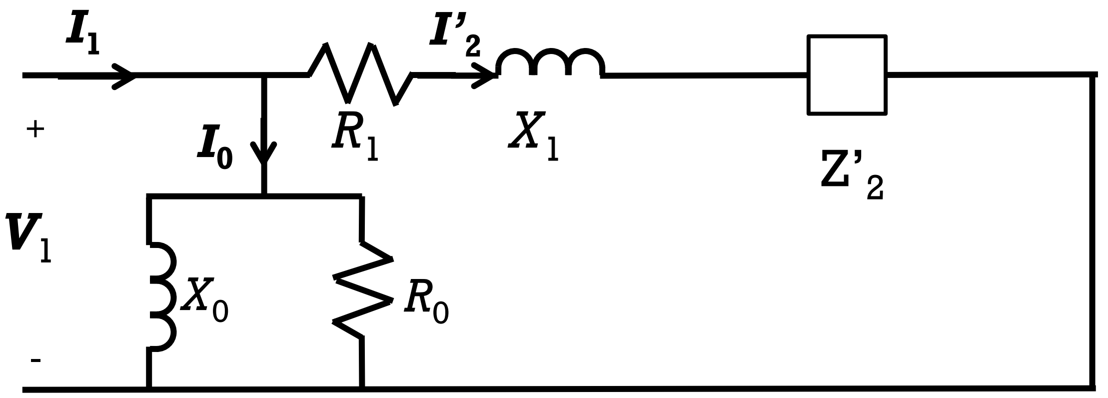
  - $Z_{load} = 10\ \Omega$
  - $|V_1| = 8000\ \text{V}$

$$a = \frac{N_1}{N_2} = \frac{3200}{200} = 16$$

$$Z_2 = R_2 + jX_2 + Z_{load} = 10.01 + j0.015\Omega$$

$$Z_2^{'} = a^2 Z_2 = 2563 + j3.84\Omega$$





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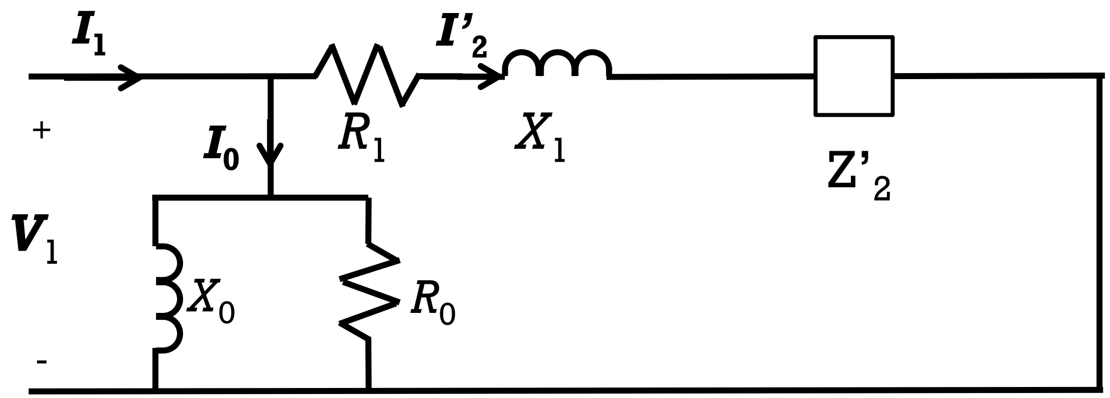
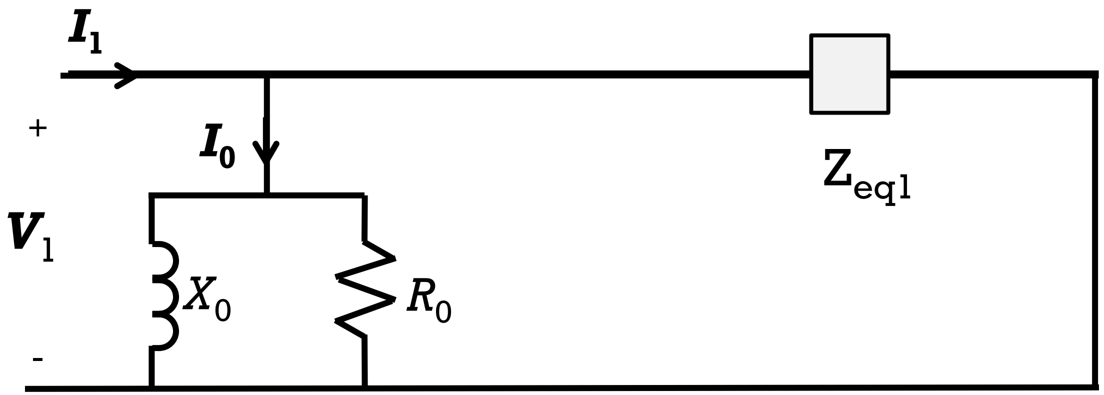
  - $|V_1| = 8000$ V

$$Z_{eq1} = R_1 + jX_1 + Z_2^{'} = 2565 + j3.855\Omega$$





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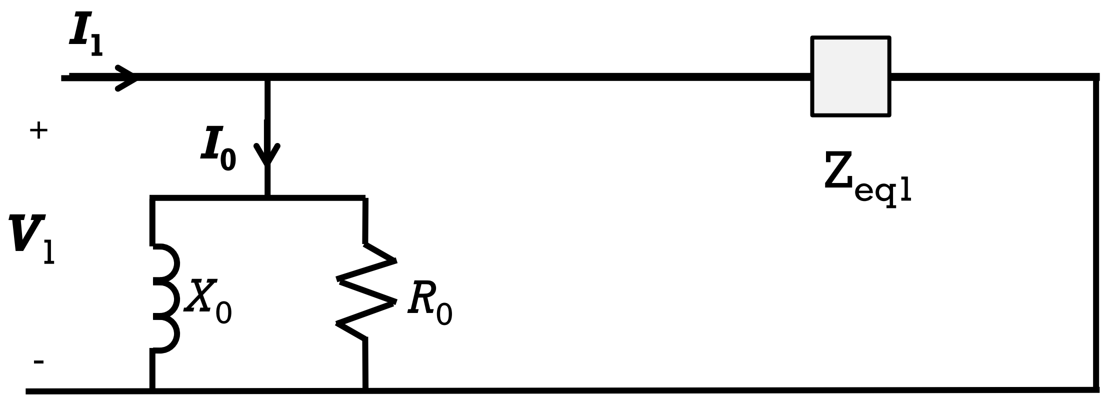
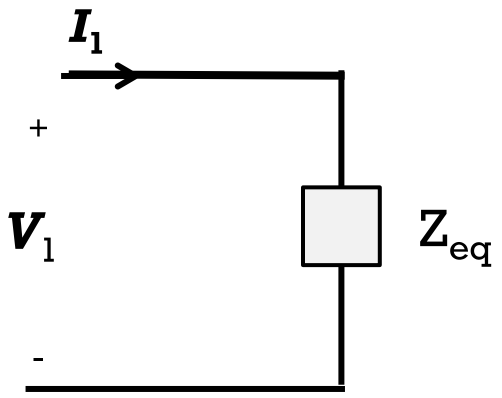
  - $N_2 = 200$
  - $Z_{load} = 10\ \Omega$
  - $|V_1| = 8000\ \text{V}$

$$Z_{eq} = R_0 \parallel jX_0 \parallel Z_{eq1}$$

$$\frac{1}{Z_{eq}} = \frac{1}{R_0} + \frac{1}{jX_0} + \frac{1}{Z_{eq1}}$$

$$Z_{eq} = 2375 + j289.6\Omega$$





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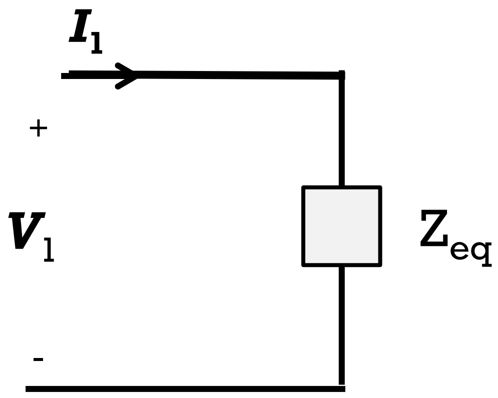
  - $Z_{load} = 10\ \Omega$
  - $|V_1| = 8000\ \text{V}$

$$\boldsymbol{I}_1 = \frac{\boldsymbol{V}_1}{Z_{eq}} = 3.32 - j0.405 = 3.34\angle -6.95\text{A}$$

$$\text{P}_{\text{in}} = \text{Re}\{\boldsymbol{V}_1 \boldsymbol{I}_1^*\} = 26{,}550\ \text{W}$$



Example

- Now find the output power
- Let:
  - $X_o = 20{,}000\ \Omega$
  - $R_o = 40{,}000\ \Omega$
  - $R_1 = 2.56\ \Omega$
  - $R_2 = 0.010\ \Omega$
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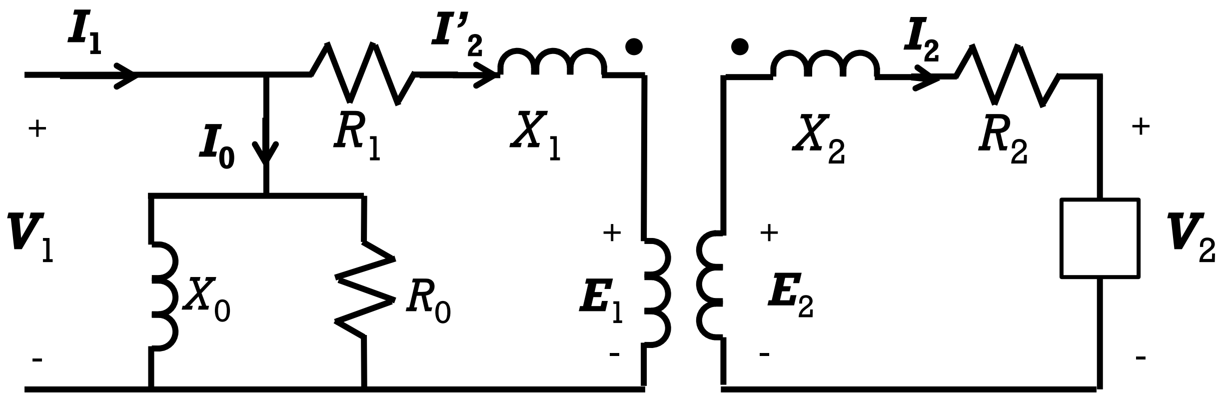
  - $N_1 = 3200$
  - $N_2 = 200$
  - $Z_{load} = 10\ \Omega$
  - $|\mathbf{V}_1| = 8000$ V



Example

- Now find the output power

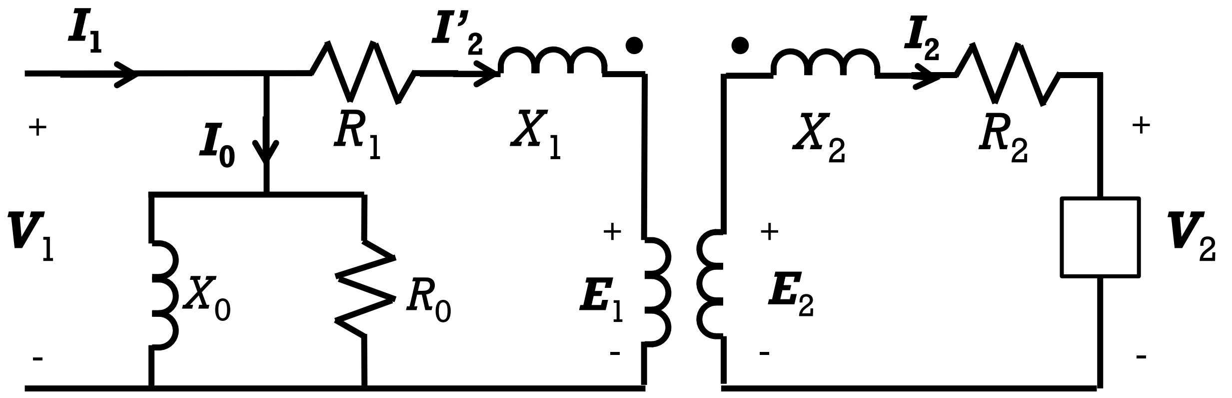
$$\boldsymbol{I}_1 = \frac{\boldsymbol{V}_1}{Z_{eq}} = 3.32 - j0.405 = 3.34\angle -6.95\text{A}$$

$$\boldsymbol{I}_0 = \frac{\boldsymbol{V}_1}{R_0} + \frac{\boldsymbol{V}_1}{jX_0} = 0.2 - j0.4\Omega$$

$$\boldsymbol{I}_2^{'} = \boldsymbol{I}_1 - \boldsymbol{I}_0 = 3.12 - j0.005 = 3.12\angle -0.09^o\,\text{A}$$

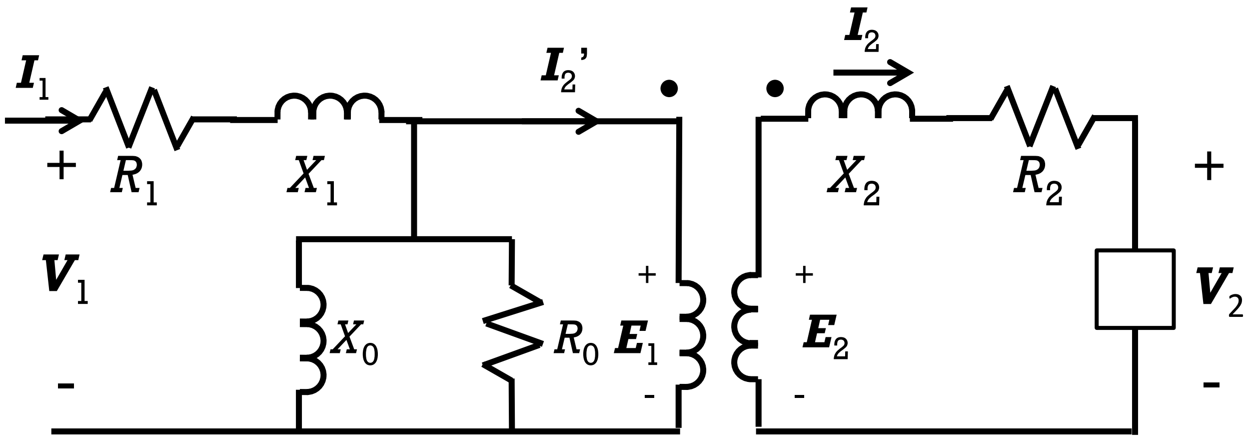
$$\boldsymbol{I}_2 = \boldsymbol{I}_2^{'}a = 49.9 - j0.075 = 49.9\angle -0.09^o\,\text{A}$$

$$\text{P}_{out} = |\,\boldsymbol{I}_2\,|^2\ \text{Re}\left\{Z_{load}\right\} = 24{,}900\text{W}$$



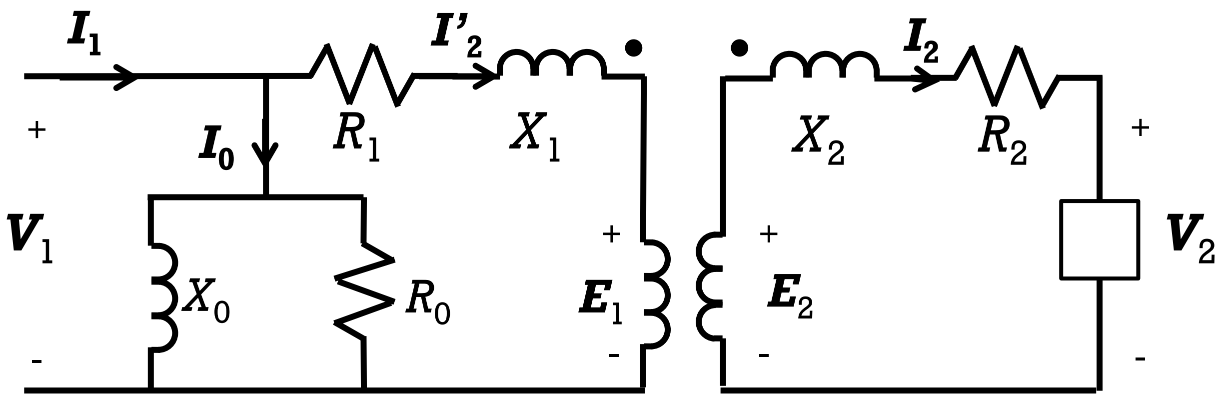
Approximate Circuit

- Often desirable to simplify the transformer model
- More accurate than ideal, less accurate than exact
- Voltage drop across $\boldsymbol{Z}_1 = R_1 + jX_1$ is designed to be small



Approximate Circuit

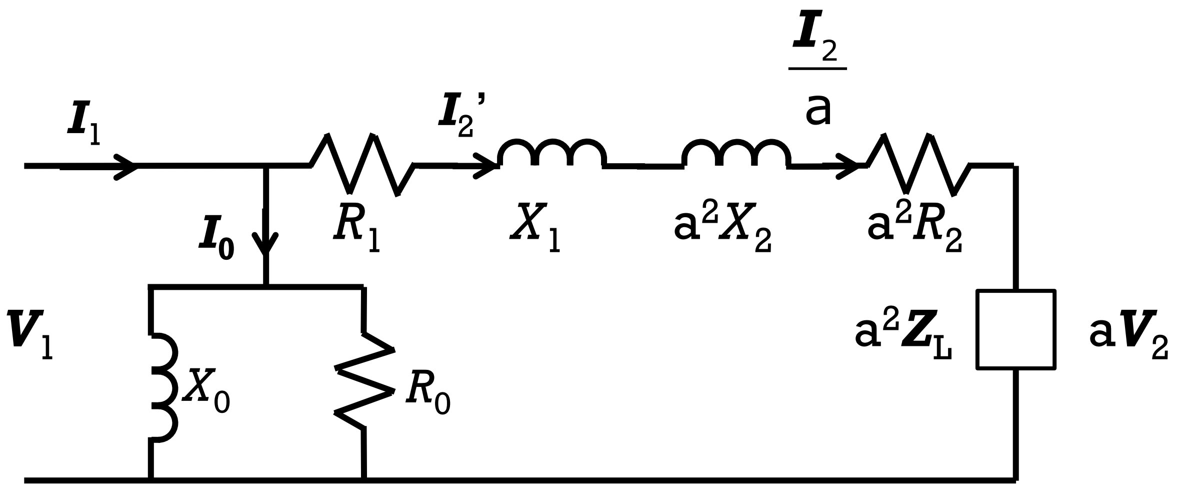
- Move $\boldsymbol{Z}_1$ to other side of shunt elements
- Next, eliminate the ideal transformer by referring the secondary elements to the primary



Approximate Circuit

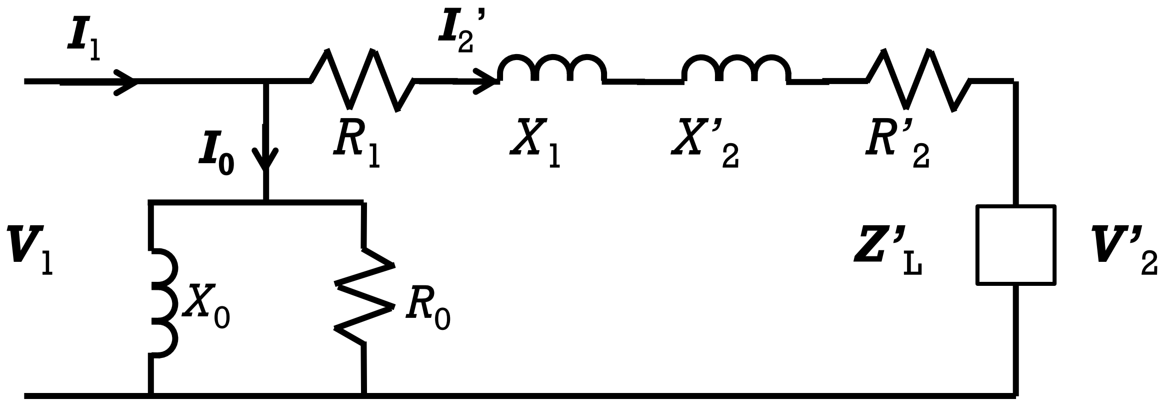
- Move $\boldsymbol{Z}_1$ to other side of shunt elements
- Next, eliminate the ideal transformer by referring the secondary elements to the primary

Recall: $\frac{V_1}{V_2} = \frac{N_1}{N_2} \triangleq a$



Approximate Circuit

- Letting:
  - $R'_2 = a^2R_2$
  - $X'_2 = a^2X_2$
  - $Z'_L = a^2Z_L$
  - $V'_2 = aV_2$

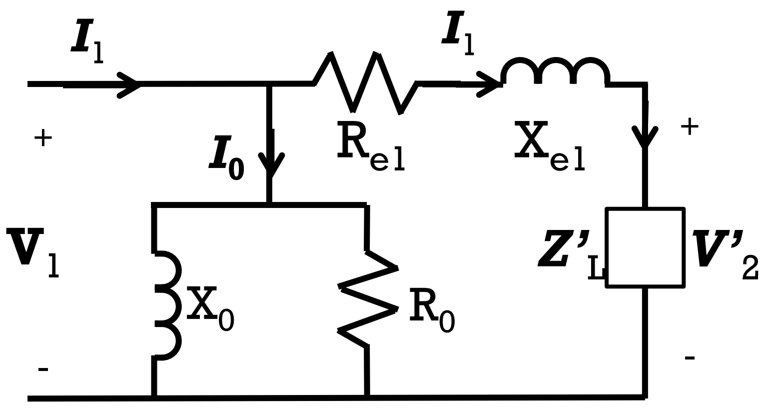


Approximate Circuit

- Combine series elements

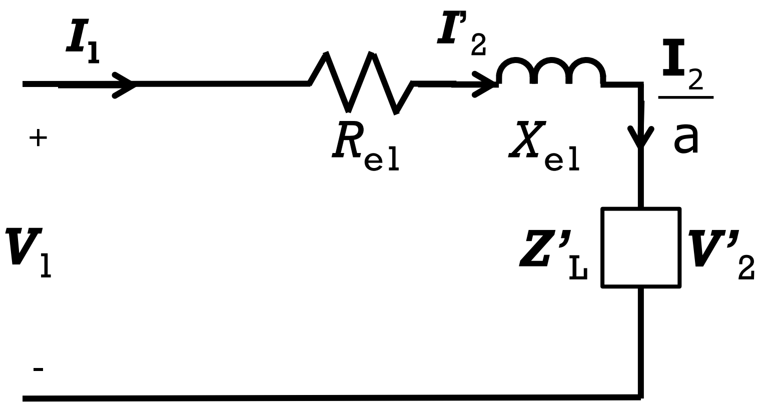
$$R_{e1} = R_1 + R_2^{'}$$
$$X_{e1} = X_1 + X_2^{'}$$

It is also possible to refer to the impedances from the secondary side.



Approximate Circuit

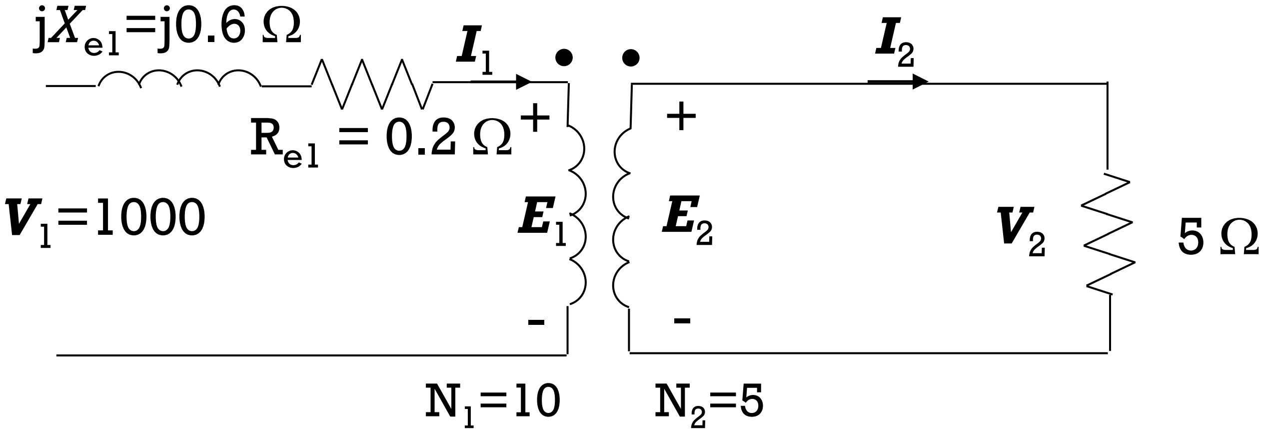
- Further approximations are possible
  - Ignore shunt branch
  - Ignore resistances
- Problem statement will indicate which model to use



Example

- Consider a single-phase xfmr with the following specifications:
  - primary turns: 10
  - secondary turns: 5
  - winding resistance: 0.2 Ohms
  - leakage reactance: 0.6 Ohms
  - infinite permeability
- If the primary is connected to a 1000 V source and the secondary to a 5 Ohm load, find the power supplied to the load
- Assume the xfmr impedances are referred from the primary and include the secondary impedances

Example



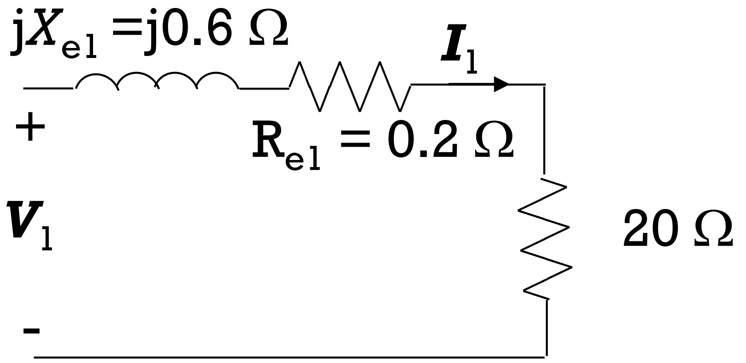
First calculate the ratio:

$$a = \frac{N_1}{N_2} = 2$$

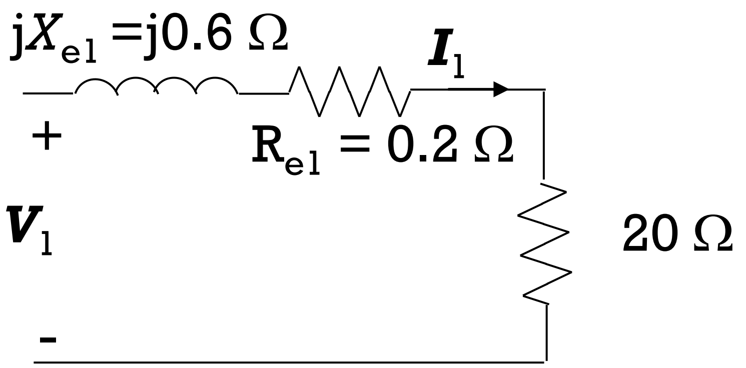
transform the impedance

$$Z_1 = a^2 Z_2 = 20\ \Omega$$

redraw the circuit



Example

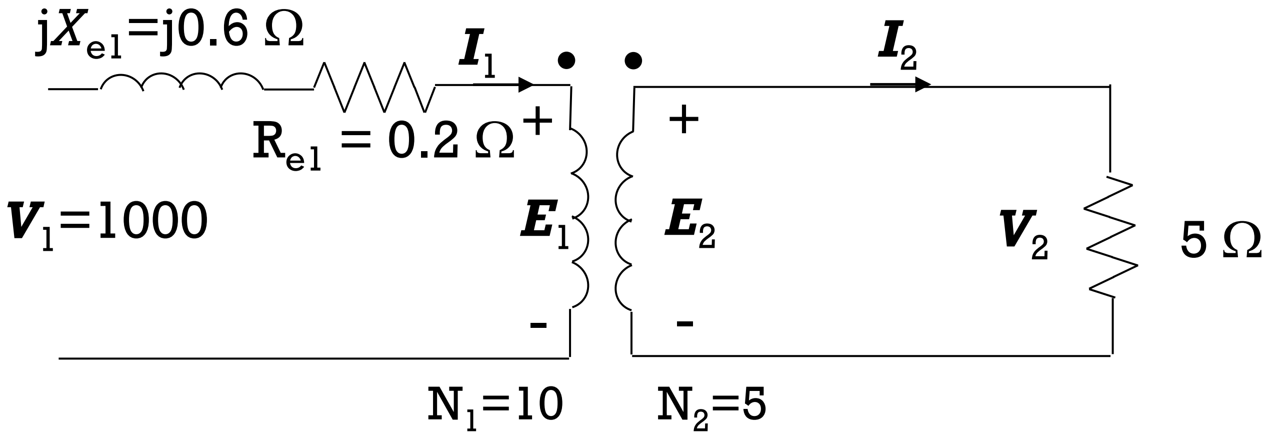


$$\boldsymbol{I}_1 = \frac{1000\angle 0}{20.2 + j0.6} = 49.48\angle -1.7^\circ \ \text{A}$$

$$\text{P} = |\boldsymbol{I}_1^2| Z_L' = 48.97 \ \text{kW}$$

Example

Another approach keeping the ideal transformer element:



$$\boldsymbol{I}_1 = \frac{1000\angle 0}{20.2 + j0.6} = 49.48\angle -1.7^\circ \;\text{ A}$$

$$\boldsymbol{E}_1 = \boldsymbol{V}_1 - \boldsymbol{I}_1(0.2 + j0.6) = 989.66\angle -1.7^\circ \;\text{ V}$$

$$\boldsymbol{V}_2 = \boldsymbol{E}_2 = \left(989.66\angle -1.7^\circ\right)\left(\frac{1}{a}\right) = 494.83\angle -1.7 \;\text{ V}$$

$$\boldsymbol{I}_2 = \left(49.48\angle -1.7^\circ\right)(2) = 98.96\angle -1.7^\circ \;\text{ A}$$

$$\mathrm{P}_2 = |\mathbf{V}_2||\mathbf{I}_2|\cos(0) = 48.97 \;\text{ kW}$$

Reading [on your own]

- 11.5.2 Transformer Efficiency
- 11.5.3 Voltage Regulation

Summary

- Non-ideal xfmrs include: magnetization reactance, leakage reactance, winding resistance and core loss
- Approximations can be made to simplify circuit analysis (series impedances are small, shunt impedances are large)