# 15-Intro. to Three Phase Transformers 

ECEGR 3500
Text: 11.4
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## Overview

- Transformer Connections
- Transformer Ratings
- Y-Y Transformer Connection
- Delta-Delta Transformer Connection
- Y-Delta Transformer Connection
- Delta-Y Transformer Connection
- Transformer Banks


## Three-Phase Transformer Connections

- Three-phase transformers can be composed of three singlephase transformers or be wound on the same core
- Primary windings
- $\mathrm{a}, \mathrm{a}^{\prime}$
- b, b'

All primary windings have the same number of turns, $N_{1}$

- c, c'
- Secondary windings
- A, A'
- B, B'
- C, C'

All secondary windings have the


## Three-Phase Transformer Connections

- The coils on the primary and secondary can be connected as delta ( $\Delta$ ) or wye (Y)
- Four possible combinations

| PRIMIARY | SECONDARY |
| :---: | :---: |
| Wye | Wye |
| Wye | Delta |
| Delta | Wye |
| Delta | Delta |

## Example Wye-Delta Transformer



## Three-Phase Transformer Ratings

- Electrical ratings of three-phase transformers:
- Voltage (line-line primary and line-line secondary)
- Current (line primary and line secondary)
- Power (three-phase apparent power)
- Ratings indicate the voltage, current, and apparent power the transformer can operate at without damaging it or shortening its lifespan

```
Voltage and current ratings refer to magnitudes.
They are not phasors
```


## Three-Phase Transformer Rating Example

- Ratings also indicate how the primary coils are connected (Delta or Wye) and how the secondary coils are connected (Delta or Wye)
- Consider a transformer rated as " $60 \mathrm{kVA}, 8 \mathrm{kV}(\Delta) / 416 \mathrm{~V}(\underset{\mathrm{~T}}{\mathrm{Y}})$ "

Primary is connected as a Delta


## Three-Phase Transformer Rating Example

- Consider a transformer rated as " $60 \mathrm{kVA}, 8 \mathrm{kV}(\Delta) / 416 \mathrm{~V}(\mathrm{Y})$ "
- Rated three-phase apparent power (total) is 60 kVA
- Not per phase
- Rated apparent power of each phase is $60 / 3=20 \mathrm{kVA}$


## Three-Phase Transformer Rating Example

- Consider a transformer rated as " $60 \mathrm{kVA}, 8 \mathrm{kV}(\Delta) / 416 \mathrm{~V}(\mathrm{Y})$ "
- Line-Line primary voltage is 8 kV
- Primary phase voltage is also 8 kV (because it is delta connected)
- Line-Line secondary voltage is 416 V
- Secondary phase voltage is $\frac{416}{\sqrt{3}}=240 \mathrm{~V}$ (because it is wye-connected)


## Voltage Ratio

- The voltage ratio of a three-phase transformer is the ratio of the rated primary voltage to secondary voltage
- Example: Consider a transformer rated as " $60 \mathrm{kVA}, 8 \mathrm{kV}(\Delta) / 416 \mathrm{~V}(\mathrm{Y})$ "
- The voltage ratio is $8000 / 416=19.23$

```
The voltage ratio for three-phase transformers ALWAYS uses line-line quantities
```


## Turns Ratio

- The turns ratio of a three-phase transformer is the ratio of the rated primary phase voltage to rated secondary phase voltage
- This is the same as the ratio of the number of turns on the primary coil to the number of turns on the secondary coil

$$
a=\frac{V_{\text {ph,primary }}}{V_{\text {ph,secondary }}}=\frac{N_{1}}{N_{2}}
$$

- Example: Consider a transformer rated as " 60 kVA , $8 \mathrm{kV}(\Delta) / 416 \mathrm{~V}(\mathrm{Y})$ "
- The turns ratio is $a=\frac{8000}{416 / \sqrt{3}}=\frac{8000}{240}=33.33$


## Voltage per Turn

- The voltage per turn is the same for the primary and the secondary

$$
V_{\mathrm{T}}=\frac{V_{\mathrm{ph}, \mathrm{primary}}}{N_{1}}=\frac{V_{\mathrm{ph}, \mathrm{secondary}}}{N_{2}}
$$

- Note: the number of turns is often not given on the nameplate of the transformer


## Exercise

- A three-phase transformer is rated at " $90 \mathrm{kVA}, 8 \mathrm{kV}(\mathrm{Y}) / 416 \mathrm{~V}(\mathrm{Y})$ "
- Compute the voltage ratio and the turns ratio


## Exercise

- A three-phase transformer is rated at " $90 \mathrm{kVA}, 8 \mathrm{kV}(\mathrm{Y}) / 416 \mathrm{~V}(\mathrm{Y})$ "
- Compute the voltage ratio and the turns ratio
- The voltage ratio is $8000 / 416=19.23$

$$
a=\frac{8000 / \sqrt{3}}{416 / \sqrt{3}}=\frac{4612}{240}=19.23
$$

## Y-Y Transformer

- Each side has a common point, n or N
- Neutral points usually grounded
- Tertiary winding (Delta) sometimes connected to avoid distortion if


Y-connected primary harmonics are present

- Line-neutral voltages appear across the coils on each side
- Insulation is stressed to only $58 \%$ of line voltage


## Y-Y Transformer

- Turns Ratio: a $=N_{1} / N_{2}$
- Voltage Ratio: $V_{\mathrm{ab}} / V_{\mathrm{AB}}$
- We can also show that the voltage ratio is the same as the ratio of phase voltages

$$
\frac{N_{1}}{N_{2}}=\frac{V_{\mathrm{ab}}}{V_{A B}}=\frac{\sqrt{3} V_{\mathrm{an}}}{\sqrt{3} V_{\mathrm{AN}}}=\frac{V_{\mathrm{an}}}{V_{\mathrm{AN}}}=\frac{V_{\text {ph,primary }}}{V_{\text {ph,secondary }}}
$$



Voltage per Turn: $V_{\mathrm{T}}=\frac{V_{\mathrm{ab}}}{N_{1}}=\frac{V_{\mathrm{AB}}}{N_{2}}$

## Y-Y Transformer

- Current Ratio:

$$
\begin{aligned}
& \frac{N_{2}}{N_{1}}=\frac{I_{\mathrm{a}}}{I_{\mathrm{A}}}=\frac{1}{\mathrm{a}} \\
& I_{\mathrm{a}}=I_{\mathrm{A}} \frac{1}{\mathrm{a}}
\end{aligned}
$$



## Y-Y Transformer Analysis

- Mutual flux in each core are out of phase by $120^{\circ}$
- $\Phi_{a}$ links primary and secondary
- Faraday's law: $\boldsymbol{V}_{\text {an }}$ and $\boldsymbol{V}_{\text {AN }}$ must be in phase with each other and lead $\Phi_{\mathrm{a}}$ by $90^{\circ}$
- similar result for b, c phases
- Ampere's Law: $\boldsymbol{I}_{\mathrm{a}}$ and $\boldsymbol{I}_{A}$ are also in phase
- similar result for b, c phases



## Y-Y Phasor Diagram

Phasor diagram for ideal Y-Y connected transformer


## Y-Y Primary \& Secondary Relationships

- Line-Line voltages

$$
\begin{aligned}
& \boldsymbol{V}_{\mathrm{ab}}=\boldsymbol{a} \boldsymbol{V}_{\mathrm{AB}} \\
& \boldsymbol{V}_{\mathrm{bc}}=\boldsymbol{a} \boldsymbol{V}_{\mathrm{BC}} \\
& \boldsymbol{V}_{\mathrm{ca}}=\boldsymbol{a} \boldsymbol{V}_{\mathrm{CA}}
\end{aligned}
$$

- Phase voltages

$$
\boldsymbol{V}_{\mathrm{an}}=\boldsymbol{a} \boldsymbol{V}_{\mathrm{AN}}
$$

$\boldsymbol{V}_{\text {bn }}=\boldsymbol{a} \boldsymbol{V}_{\text {BN }}$
$\boldsymbol{V}_{\mathrm{cn}}=\boldsymbol{a} \boldsymbol{V}_{\mathrm{cN}}$

- Line currents
$\boldsymbol{I}_{\mathrm{a}}=\frac{1}{\mathrm{a}} \boldsymbol{I}_{\mathrm{A}}$
$\boldsymbol{I}_{\mathrm{c}}=\frac{1}{\mathrm{a}} \boldsymbol{I}_{\mathrm{c}}$
$\boldsymbol{I}_{\mathrm{b}}=\frac{1}{\mathrm{a}} \boldsymbol{I}_{\mathrm{B}}$
Recall: phase currents equal line current for $Y$ connections


## $\Delta-\Delta$ Transformer

- No neutral point
- $\boldsymbol{V}_{\mathrm{ab}}, \boldsymbol{V}_{\mathrm{AB}}$ in phase
- $\boldsymbol{I}_{\mathrm{ab}}, \boldsymbol{I}_{\mathrm{AB}}$ in phase
- Similar results for b, c phase
- Line-line voltages appear across the coils on primary and secondary
- Best suited for lower voltage applications



## $\Delta-\Delta$ Transformer

- Turns Ratio: a $=N_{1} / N_{2}$
- Voltage Ratio: $V_{\mathrm{ab}} / V_{\text {AB }}$
- We can also show that the voltage ratio is the same as the ratio of phase voltages

$$
\frac{N_{1}}{N_{2}}=\frac{V_{\text {ab }}}{V_{A B}}=\frac{V_{\text {ph } h \text { primary }}}{V_{\text {ph,secondary }}}
$$



Voltage per Turn: $V_{\mathrm{T}}=\frac{V_{\mathrm{ab}}}{N_{1}}=\frac{V_{\mathrm{AB}}}{N_{2}}$

## $\Delta-\Delta$ Transformer

- Current Ratio:

$$
\begin{aligned}
& \frac{N_{2}}{N_{1}}=\frac{I_{\mathrm{ab}}}{I_{\mathrm{AB}}}=\frac{1}{\mathrm{a}} \\
& I_{\mathrm{ab}}=I_{\mathrm{AB}} \frac{1}{a}
\end{aligned}
$$



## $\Delta-\Delta$ Transformer

- Mutual flux in each core are out of phase by $120^{\circ}$
- $\Phi_{\mathrm{a}}$ links primary and secondary
- Faraday's law: $\boldsymbol{V}_{\mathrm{ab}}$ and $\boldsymbol{V}_{\mathrm{AB}}$ must be in phase with each other and lead $\Phi_{\mathrm{a}}$ by $90^{\circ}$
- similar result for b, c phases
- Ampere's Law: $\boldsymbol{I}_{\mathrm{ab}}$ and $\boldsymbol{I}_{\mathrm{AB}}$ are also in phase
- similar result for b, c phases



## $\Delta-\Delta$ Phasor Diagram

Phasor diagram for ideal $\Delta-\Delta$ connected transformer


## $\Delta-\Delta$ Primary \& Secondary Relationships

- Line-Line \& Phase voltages

$$
\begin{aligned}
& \boldsymbol{V}_{\mathrm{ab}}=a \boldsymbol{V}_{\mathrm{AB}} \\
& \boldsymbol{V}_{\mathrm{bc}}=a \boldsymbol{V}_{\mathrm{BC}} \\
& \boldsymbol{V}_{\mathrm{ca}}=\boldsymbol{a} \boldsymbol{V}_{\mathrm{CA}}
\end{aligned}
$$

Recall: phase voltage equals line-line voltage for $\Delta$ connections

- Line currents

$$
\begin{array}{ll}
\boldsymbol{I}_{\mathrm{a}}=\frac{1}{\mathrm{a}} \boldsymbol{I}_{\mathrm{A}} & \boldsymbol{I}_{\mathrm{c}}=\frac{1}{\mathrm{a}} \boldsymbol{I}_{\mathrm{C}} \\
\boldsymbol{I}_{\mathrm{b}}=\frac{1}{\mathrm{a}} \boldsymbol{I}_{\mathrm{B}} &
\end{array}
$$

- Phase currents

$$
\begin{array}{ll}
\boldsymbol{I}_{\mathrm{ab}}=\frac{1}{\mathrm{a}} \boldsymbol{I}_{\mathrm{AB}} & \boldsymbol{I}_{\mathrm{ca}}=\frac{1}{\mathrm{a}} \boldsymbol{I}_{\mathrm{CA}} \\
\boldsymbol{I}_{\mathrm{bc}}=\frac{1}{\mathrm{a}} \boldsymbol{I}_{\mathrm{BC}} &
\end{array}
$$



## Y- $\Delta$ Transformer

- Mutual flux in each core are out of phase by $120^{\circ}$
- $\Phi_{\mathrm{a}}$ links primary and secondary
- Faraday's law: $\boldsymbol{V}_{\text {an }}$ and $\boldsymbol{V}_{\text {AC }}$ must be in phase with each other and lead $\Phi_{\mathrm{a}}$ by $90^{\circ}$
- similar result for b, c phases
- Ampere's Law: $\boldsymbol{I}_{\mathrm{a}}$ and $\boldsymbol{I}_{\mathrm{CA}}$ are also in phase
- similar result for b, c phases



## Y- $\Delta$ Voltage Relationships

- Y-side Phase Voltage

$$
\begin{aligned}
& \boldsymbol{V}_{\mathrm{an}}=\boldsymbol{a} \boldsymbol{V}_{\mathrm{AC}} \\
& \boldsymbol{V}_{\mathrm{bn}}=\boldsymbol{a} \boldsymbol{V}_{\mathrm{BA}} \\
& \boldsymbol{V}_{\mathrm{cn}}=\boldsymbol{a} \boldsymbol{V}_{\mathrm{CB}}
\end{aligned}
$$



## Y- $\Delta$ Voltage Relationships

- Y-side Line-Line Voltage

$$
\begin{aligned}
\boldsymbol{V}_{\mathrm{ab}} & =\boldsymbol{V}_{\mathrm{an}} \sqrt{3} \angle 30^{\circ} \\
& =\boldsymbol{V}_{\mathrm{bn}}\left(1 \angle 120^{\circ}\right)\left(\sqrt{3} \angle 30^{\circ}\right)=\mathrm{a} \boldsymbol{V}_{\mathrm{BA}}\left(1 \angle 120^{\circ}\right)\left(\sqrt{3} \angle 30^{\circ}\right) \\
& =\mathrm{a}\left(\boldsymbol{V}_{\mathrm{AB}} \angle-180^{\circ}+120^{\circ}\right)\left(\sqrt{3} \angle 30^{\circ}\right) \\
& =\mathrm{a} \sqrt{3} \boldsymbol{V}_{\mathrm{AB}} \angle-30^{\circ}
\end{aligned}
$$

The Delta side leads the Y-side by 30 degrees


$$
\boldsymbol{V}_{\mathrm{bc}}=\mathbf{a} \sqrt{3} \boldsymbol{V}_{\mathrm{BC}} \angle-30^{\circ}
$$

$$
\boldsymbol{V}_{\mathrm{ca}}=\mathrm{a} \sqrt{3} \boldsymbol{V}_{\mathrm{CA}} \angle-30^{\circ}
$$

## Phasor Diagram (Y- $\Delta$ )



The Delta side leads the Y-side by 30 degrees (compare $\boldsymbol{V}_{\mathrm{AB}}$ to $\boldsymbol{V}_{\mathrm{ab}}$ )

## Y- $\Delta$ Voltage Relationships

- $\Delta$-side Line-Line Voltage

$$
\begin{aligned}
& \boldsymbol{V}_{\mathrm{AB}}=-\frac{1}{\mathrm{a}} \boldsymbol{V}_{\mathrm{bn}}=-\frac{1}{\mathrm{a} \sqrt{3}} \boldsymbol{V}_{\mathrm{bc}} \angle-30^{\circ} \\
& \boldsymbol{V}_{\mathrm{BC}}=-\frac{1}{\mathrm{a}} \boldsymbol{V}_{\mathrm{cn}}=-\frac{1}{\mathrm{a} \sqrt{3}} \boldsymbol{V}_{\mathrm{ca}} \angle-30^{\circ} \\
& \boldsymbol{V}_{\mathrm{CA}}=-\frac{1}{\mathrm{a}} \boldsymbol{V}_{\mathrm{an}}=-\frac{1}{\mathrm{a} \sqrt{3}} \boldsymbol{V}_{\mathrm{ab}} \angle-30^{\circ}
\end{aligned}
$$



> Recall: phase voltage equals line-line voltage for $\Delta$ connections

## Example

- Consider a Y-Delta transformer with $\mathrm{a}=1$. If $\boldsymbol{V}_{\mathrm{ab}}$ is 480 V , compute $\left|\boldsymbol{V}_{\mathrm{AB}}\right|$



## Example

- Consider a Y-Delta transformer with $\mathrm{a}=1$. If $\boldsymbol{V}_{\mathrm{ab}}$ is 480 V , compute $\left|\boldsymbol{V}_{\mathrm{AB}}\right|$

$$
\left|\boldsymbol{V}_{\mathrm{AB}}\right|=-\frac{1}{\mathrm{a} \sqrt{3}} \boldsymbol{V}_{\mathrm{bc}} \angle-30^{\circ}=\left|-\frac{1}{1 \times \sqrt{3}} 480 \angle-90^{\circ}-30^{\circ}\right|=277 \mathrm{~V}
$$



Letting $\boldsymbol{V}_{\text {an }}$ be the reference so that

$$
\boldsymbol{V}_{\mathrm{bc}}=480 \angle-90^{\circ}
$$

## Example—alternative solution

- Consider a Y-Delta transformer with $\mathrm{a}=1$. If $\boldsymbol{V}_{\mathrm{ab}}$ is 480 V , compute $\left|\boldsymbol{V}_{\mathrm{AB}}\right|$
- The voltage across each primary winding is 277 V . Since the turns
 ratio is 1 , then 277 V also appears across each secondary coil. The secondary is connected as a delta, so the secondary line-line voltage is 277 V


## Y- $\Delta$ Current Relationships

- Y-side Phase Current

$$
\boldsymbol{I}_{\mathrm{A}}=\boldsymbol{I}_{\mathrm{CA}}-\boldsymbol{I}_{\mathrm{AB}}=\sqrt{3} \boldsymbol{I}_{\mathrm{CA}} \angle 30^{\circ}(\text { by KCL })
$$



$$
\boldsymbol{I}_{\mathrm{an}}=\boldsymbol{I}_{\mathrm{a}}=\frac{1}{\mathrm{a}} \boldsymbol{I}_{\mathrm{CA}}
$$



$$
\begin{aligned}
= & \frac{1}{\mathrm{a} \sqrt{3}} \boldsymbol{I}_{\mathrm{A}} \angle-30^{\circ} \\
\boldsymbol{I}_{\mathrm{bn}} & =\boldsymbol{I}_{\mathrm{b}}=\frac{1}{\mathrm{a}} \boldsymbol{I}_{\mathrm{AB}}=\frac{1}{\mathrm{a} \sqrt{3}} \boldsymbol{I}_{\mathrm{B}} \angle-30^{\circ} \\
\boldsymbol{I}_{\mathrm{cn}} & =\boldsymbol{I}_{\mathrm{c}}=\frac{1}{\mathrm{a}} \boldsymbol{I}_{\mathrm{BC}}=\frac{1}{\mathrm{a} \sqrt{3}} \boldsymbol{I}_{\mathrm{c}} \angle-30^{\circ}
\end{aligned}
$$

The Delta side leads the Y-side by 30 degrees

## Phasor Diagram (Y- $\Delta$ )



## Y- $\Delta$ Current Relationships

- $\Delta$-side Phase current

$$
\begin{aligned}
& \boldsymbol{I}_{\mathrm{AB}}=\mathrm{a} \boldsymbol{I}_{\mathrm{bn}} \\
& \boldsymbol{I}_{\mathrm{BC}}=\mathrm{a} \boldsymbol{I}_{\mathrm{cn}} \\
& \boldsymbol{I}_{\mathrm{CA}}=\mathrm{a} \boldsymbol{I}_{\mathrm{an}}
\end{aligned}
$$

- $\Delta$-side Line current


$$
\begin{array}{rlrl}
\boldsymbol{I}_{\mathrm{A}} & =\boldsymbol{I}_{\mathrm{CA}}-\boldsymbol{I}_{\mathrm{AB}}=\sqrt{3} \boldsymbol{I}_{\mathrm{CA}} \angle 30^{\circ} \\
& =\sqrt{3} \mathrm{a} \boldsymbol{I}_{\mathrm{an}} \angle 30^{\circ} & \\
\boldsymbol{I}_{\mathrm{B}} & =\sqrt{3} \mathrm{a} \boldsymbol{I}_{\mathrm{bn}} \angle 30^{\circ} & & \text { The Delta side leads the } \\
\boldsymbol{I}_{\mathrm{C}} & =\sqrt{3} \mathrm{a} \boldsymbol{I}_{\mathrm{cn}} \angle 30^{\circ} & & \text { Y-side by 30 degrees }
\end{array}
$$

## Shifting by $+/-30^{\circ}$

Changing the winding connections changes the shift from +30 to -30 degrees


The Delta side lags the
Y-side by 30 degrees

## Exercise

- Show that $\boldsymbol{V}_{\mathrm{AB}}$ lags $\boldsymbol{V}_{\mathrm{ab}}$ by $30^{\circ}$



## Exercise



The Delta side lags the Y-side by 30 degrees (compare $\boldsymbol{V}_{\mathrm{AB}}$ to $\boldsymbol{V}_{\mathrm{ab}}$ )

## Exercise

- Show that $\boldsymbol{I}_{\mathrm{A}}$ lags $\boldsymbol{I}_{\mathrm{a}}$ by $30^{\circ}$



## Exercise

 $I_{c r} I_{c}$

## $\Delta$ - Y Transformer Analysis

- $\Delta-\mathrm{Y}$ transformers are similar to $\mathrm{Y}-\Delta$ transformers, but with the primary and secondary sides switched
- Phase shifts of +30 degrees or -30 degrees can be obtained


## Phase Shifts

- The North American standard is to shift +30 degrees from the low voltage side to the high voltage side
- Remember, the gain is unaffected by whether the shift is positive or negative
- If we are only concerned about the magnitude of voltage and current, then the shift can be ignored


## $\Delta$-Y Transformer Analysis



## $\Delta$-Y Transformer



## $\Delta$-Y Transformer Analysis



## $\Delta$-Y Transformer Analysis



## $\Delta$-Y Voltage Relationships

- $\Delta$-side Line-Line Voltage

$$
\begin{aligned}
& \boldsymbol{V}_{\mathrm{ab}}=a \boldsymbol{V}_{\mathrm{AN}}=\frac{a}{\sqrt{3}} \boldsymbol{V}_{\mathrm{AB}} \angle-30^{\circ} \\
& \boldsymbol{V}_{\mathrm{bc}}=a \boldsymbol{V}_{\mathrm{BN}}=\frac{a}{\sqrt{3}} \boldsymbol{V}_{\mathrm{BC}} \angle-30^{\circ}
\end{aligned}
$$

$$
\boldsymbol{V}_{\mathrm{ca}}=a \boldsymbol{V}_{\mathrm{CN}}=\frac{a}{\sqrt{3}} \boldsymbol{V}_{\mathrm{CA}} \angle-30^{\circ}
$$

- Y-side Line-Line Voltage

$$
\begin{aligned}
& \boldsymbol{V}_{\mathrm{AB}}=\frac{\sqrt{3}}{\mathrm{a}} \boldsymbol{V}_{\mathrm{ab}} \angle 30^{\circ} \\
& \boldsymbol{V}_{\mathrm{BC}}=\frac{\sqrt{3}}{\mathrm{a}} \boldsymbol{V}_{\mathrm{bc}} \angle 30^{\circ} \\
& \boldsymbol{V}_{\mathrm{CA}}=\frac{\sqrt{3}}{\mathrm{a}} \boldsymbol{V}_{\mathrm{ca}} \angle 30^{\circ}
\end{aligned}
$$

## $\Delta$-Y Transformer Analysis

- $\Delta$-side Phase Current
$\boldsymbol{I}_{\mathrm{ab}}=\frac{1}{\mathrm{a}} \boldsymbol{I}_{\mathrm{A}}$
$\boldsymbol{I}_{\mathrm{bc}}=\frac{1}{\mathrm{a}} \boldsymbol{I}_{\mathrm{B}}$
$\boldsymbol{I}_{\mathrm{ca}}=\frac{1}{\mathrm{a}} \boldsymbol{I}_{\mathrm{C}}$
- $\Delta$-side Line current

$$
\begin{aligned}
& \boldsymbol{I}_{\mathrm{a}}=\frac{\sqrt{3}}{\mathrm{a}} \boldsymbol{I}_{\mathrm{A}} \angle-30^{\circ} \\
& \boldsymbol{I}_{\mathrm{b}}=\frac{\sqrt{3}}{\mathrm{a}} \boldsymbol{I}_{\mathrm{B}} \angle-30^{\circ} \\
& \boldsymbol{I}_{\mathrm{c}}=\frac{\sqrt{3}}{\mathrm{a}} \boldsymbol{I}_{\mathrm{c}} \angle-30^{\circ}
\end{aligned}
$$

Y-side Line currents

$$
\begin{aligned}
& \boldsymbol{I}_{\mathrm{A}}=\frac{a}{\sqrt{3}} \boldsymbol{I}_{\mathrm{a}} \angle 30^{\circ}=\mathrm{a} \boldsymbol{I}_{\mathrm{ab}} \\
& \boldsymbol{I}_{\mathrm{B}}=\frac{a}{\sqrt{3}} \boldsymbol{I}_{\mathrm{b}} \angle 30^{\circ}=\mathrm{a} \boldsymbol{I}_{\mathrm{bc}} \\
& \boldsymbol{I}_{\mathrm{C}}=\frac{a}{\sqrt{3}} \boldsymbol{I}_{\mathrm{c}} \angle 30^{\circ}=\mathrm{a} \boldsymbol{I}_{\mathrm{ca}}
\end{aligned}
$$

equals phase current for Y connections

## Phase Shifts

| Primary | Secondary | Phase <br> Displacement |
| :---: | :---: | :---: |
| Wye | Wye | None |
| Delta | Delta | None |
| Wye | Delta | $+/-30$ degrees |
| Delta | Wye | $+/-30$ degrees |

## Exercise

- Consider an ideal $\Delta$-Y transformer, with 20 turns on each primary coil, and 80 turns on each secondary coil. If the primary side values are:

$$
\begin{aligned}
& \boldsymbol{V}_{\mathrm{ab}}=208 \angle 0^{\circ} \mathrm{V} \\
& \boldsymbol{I}_{\mathrm{a}}=10 \angle 5^{\circ} \mathrm{A}
\end{aligned}
$$

- Compute: $\boldsymbol{V}_{\mathrm{AB}}, \boldsymbol{V}_{\mathrm{AN}}$ and $\boldsymbol{I}_{\mathrm{A}}$


## Exercise

- Consider an ideal $\Delta$ - Y transformer, with 20 turns on each primary coil, and 80 turns on each secondary coil. If the primary side values are:

$$
\begin{aligned}
& \boldsymbol{V}_{\mathrm{ab}}=208 \angle 0^{\circ} \mathrm{V} \\
& \boldsymbol{I}_{\mathrm{a}}=10 \angle 5^{\circ} \mathrm{A}
\end{aligned}
$$

- Compute: $\boldsymbol{V}_{\mathrm{AB}}, \boldsymbol{V}_{\mathrm{AN}}$ and $\boldsymbol{I}_{\mathrm{A}}$

Per convention, we have shifted the low voltage side (delta) by +30 degrees when going to the secondary side (wye).

$$
\begin{aligned}
& a=\frac{N_{1}}{N_{2}}=\frac{20}{80}=0.25 \\
& \boldsymbol{V}_{\mathrm{AB}}=\boldsymbol{V}_{\mathrm{ab}} \sqrt{3} \frac{1}{a} \angle 30^{\circ}=1441 \angle 30^{\circ} \mathrm{V} \\
& \boldsymbol{V}_{\mathrm{AN}}=\boldsymbol{V}_{\mathrm{AB}} \frac{1}{\sqrt{3}} \angle-30^{\circ}=832 \angle 0^{\circ} \mathrm{V}
\end{aligned}
$$

## Exercise

- Consider an ideal $\Delta$ - Y transformer, with 20 turns on each primary coil, and 80 turns on each secondary coil. If the primary side values are:

$$
\begin{aligned}
& \boldsymbol{V}_{\mathrm{ab}}=208 \angle 0^{\circ} \mathrm{V} \\
& \boldsymbol{I}_{\mathrm{a}}=10 \angle 5^{\circ} \mathrm{A}
\end{aligned}
$$

- Compute: $\boldsymbol{V}_{\mathrm{AB}}, \boldsymbol{V}_{\mathrm{AN}}$ and $\boldsymbol{I}_{\mathrm{A}}$


## Practical Considerations

- Medium-voltage industrial facilities often use $\Delta$-Y incoming transformers
- Y-side is grounded through a resistor
- Reduces ground current during fault
- Reduces voltage dip during fault



## Summary

- Y-Y, Delta-Delta transformers result in magnitude changes of a from primary to secondary
- No phase shifting occurs
- Y-Y transformers grant access to neutral point, which is usually grounded to prevent distortion
- Delta-Delta transformers have no neutral point, but are less prone to distortion
- Per phase analysis can be used on Y-Y, Delta-Delta transformers
- Ensure impedances and voltages are properly converted
- There is a complex gain (magnitude and phase angle shift) associated with Y-Delta and Delta-Y transformers (either $+/-30$ degrees)


## Reading [on your own]

- Also read:
- 11.2 (Multi-Winding Transformer)
- 11.3 (Autotransformer)

