18-Torque and Lorentz Force

ECEGR 3500

Electrical Energy Systems

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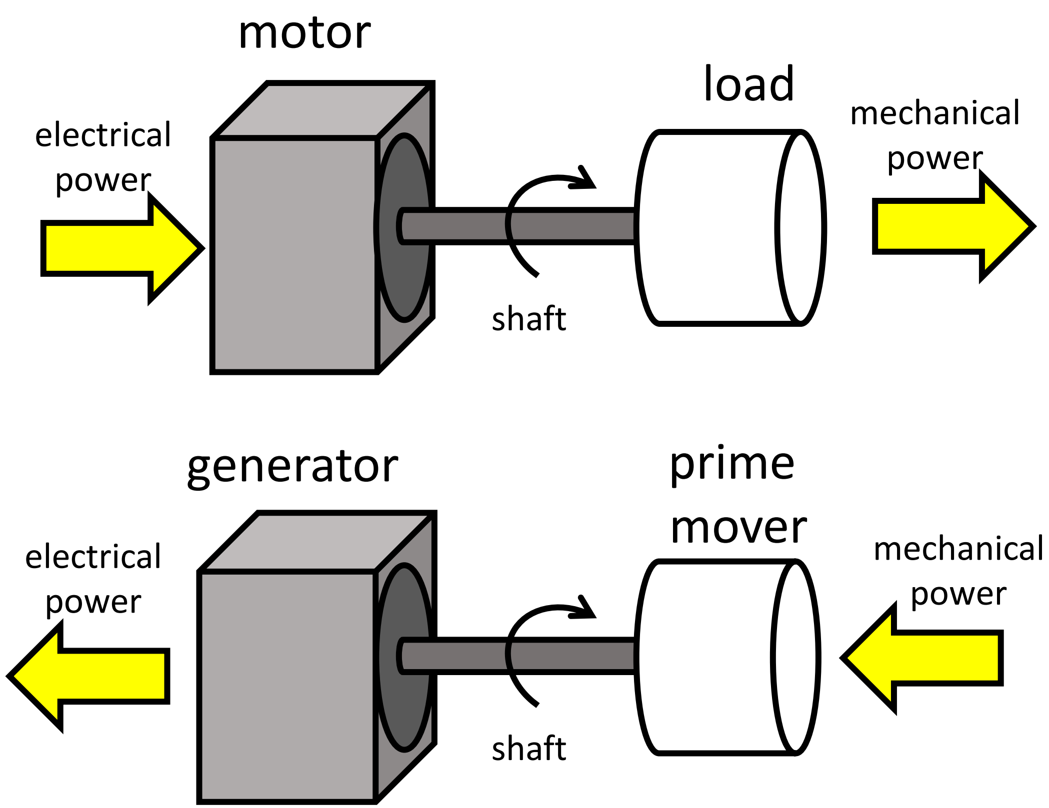
Overview

- Torque
- Power
- Angular Acceleration
- Rotational Dynamics
- Torque vs Speed
- Lorentz Force Equation

Introduction

- We now discuss basic principles of electromechanical energy conversion
- Motor: conversion of electrical energy into mechanical energy
  - Movement of a current carrying conductor due to a magnetic field
- Generator: conversion of mechanical energy into electrical energy
  - Movement of a current carrying conductor by an external force in opposition to a magnetic field

Introduction



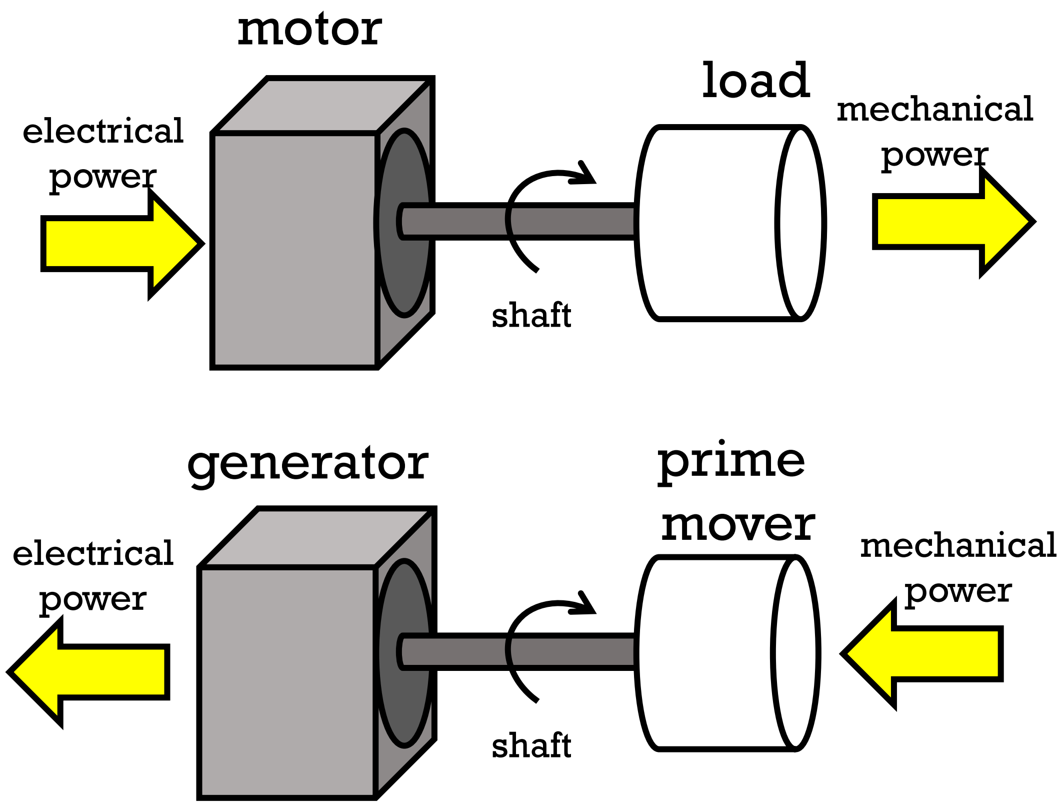
Introduction

- Energy conversion is reversible except for losses
- No such thing as a 100 percent efficient machine
  - Losses are manifested as heat, vibrations, noise
- Focus on machines that use magnetic fields to facilitate the energy conversion process

Questions

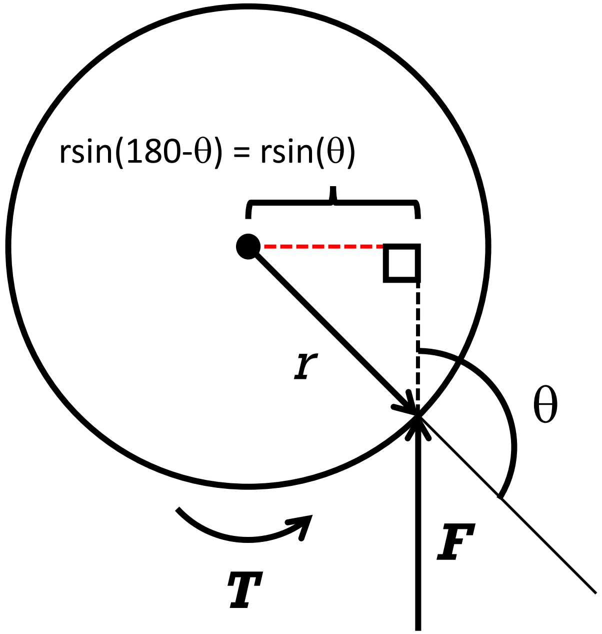
- What determines the speed an object will rotate at?
- Why do bicycles and automobiles have different gears?

Introduction



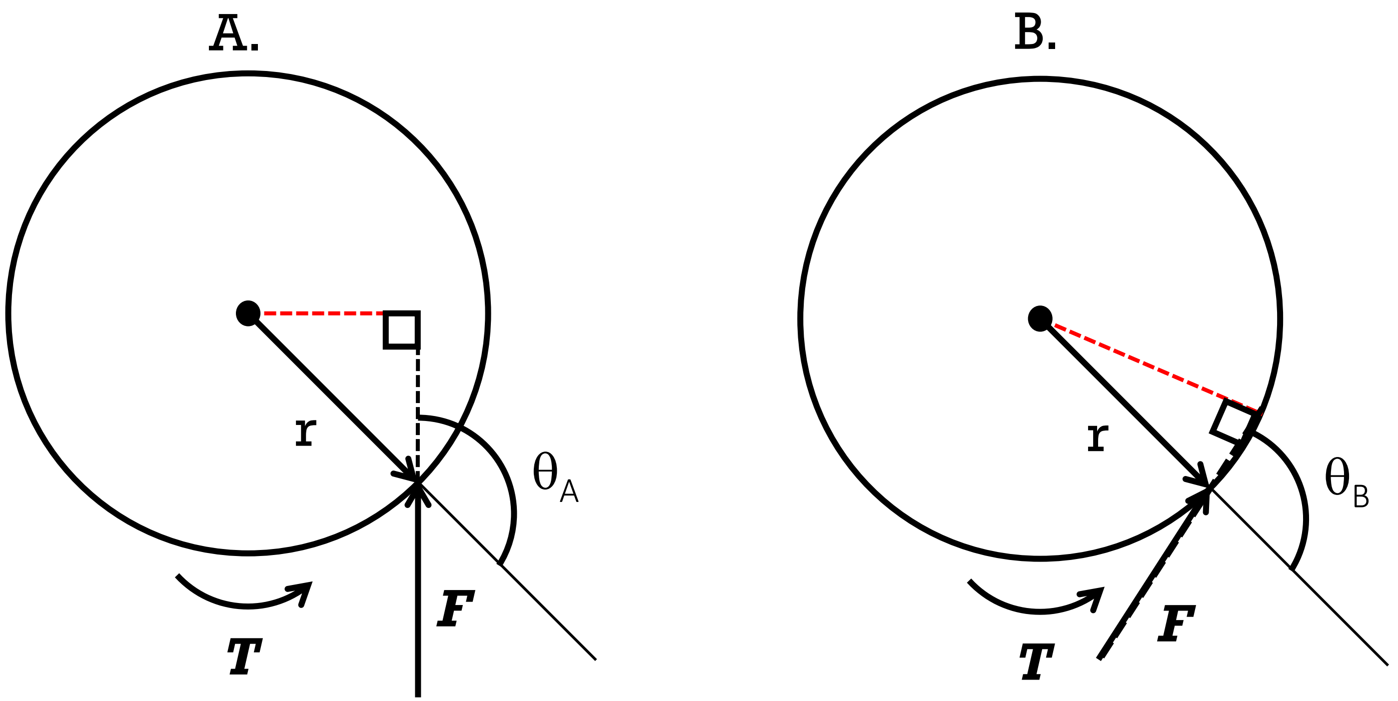
Torque

- Torque: tangential force times radial distance at which it is applied measured from axis of rotation
- $\boldsymbol{T} = \boldsymbol{F}r\sin(\theta)$ (in Nm)



Exercise

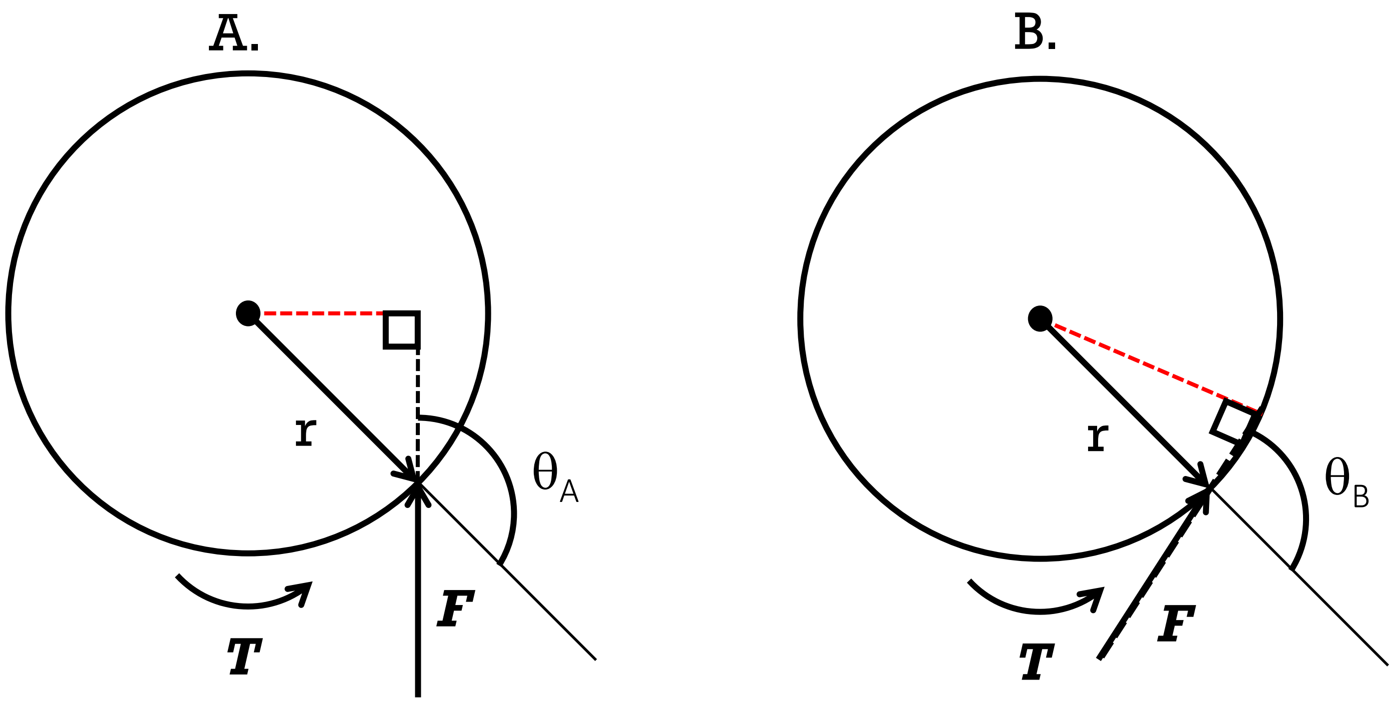
Which experiences greater torque?



Exercise

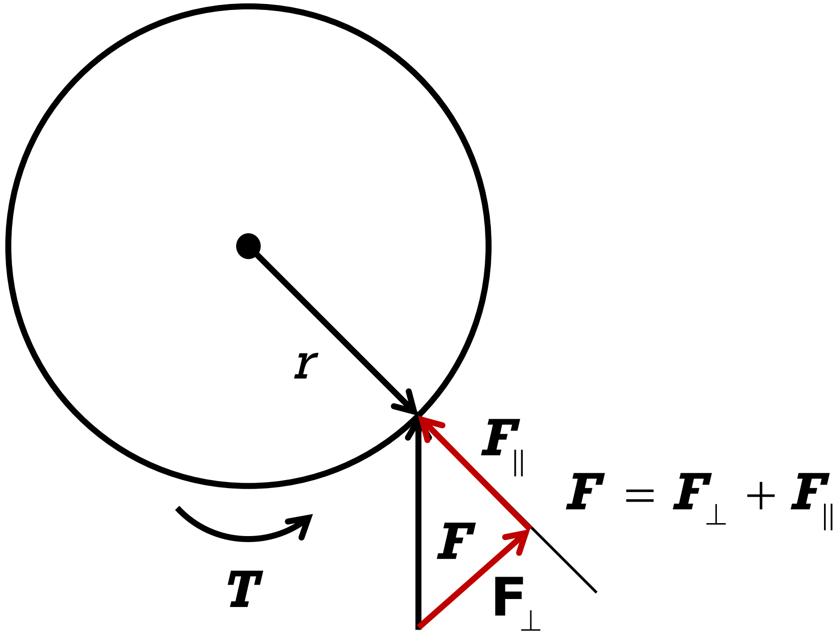
B. experiences greater torque

$\boldsymbol{Fr}\sin(\theta_B) > \boldsymbol{Fr}\sin(\theta_A)$



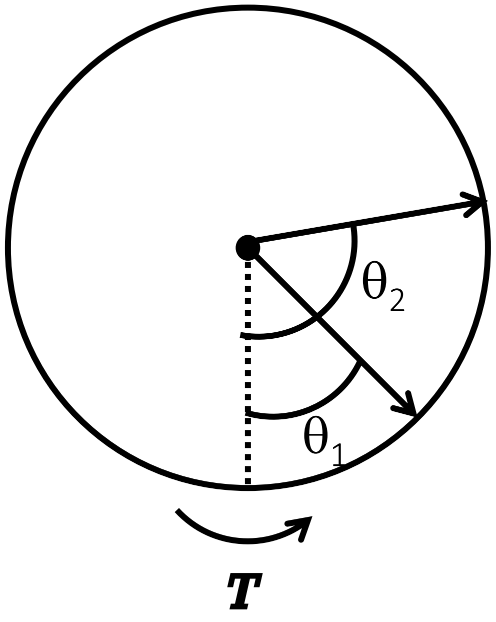
Alternative Forms

- **T** = **r** x **F** (cross product, use right hand rule for direction)
  - Thumb out of the paper CCW
  - Thumb into paper: CW
- Torque: tangential force multiplied by r: $\boldsymbol{T} = r\boldsymbol{F}_{\perp}$



Work

- Torque through an angle is work
- For constant torque: $W = \boldsymbol{T}(\theta_2 - \theta_1)$ (Joules)
  - θ_1: starting angle (radians)
  - θ_2: ending angle (radians)



Power

- Power is rate of work

$$P = \frac{\mathrm{d}W}{\mathrm{d}t} = T\frac{\mathrm{d}\theta}{\mathrm{d}t} = T\omega_m \text{ (watts)}$$

$$T = \frac{P}{\omega_m}$$

- where:
  - ω_m: angular velocity (rad/sec)

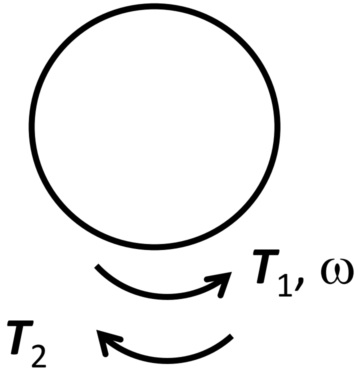
Angular Acceleration

- A shaft experiencing torque will accelerate according to:

$$\alpha = (T_1 - T_2)J$$

- Where
  - α: angular acceleration (rad/s^2)
  - J: mass moment of inertia (kgm^2)

If $T_1 = T_2$: speed unchanged If $T_1 < T_2$: slows down If $T_1 > T_2$: speeds up



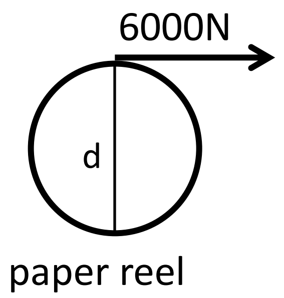
Example

A large reel of paper installed at the end of a paper machine has a diameter of 1.8m and a moment of inertia of 4500 kgm^2. It is coupled to a variable speed dc motor revolving at 120 rpm. The paper is kept under constant tension of 6000 N.

Compute:

the torque exerted on the reel

the power output by the motor



Example

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Compute:

the torque exerted on the reel

$$T = Fr = 6000 \text{ x } 0.9 = 5400 \text{ Nm}$$

the power output by the motor

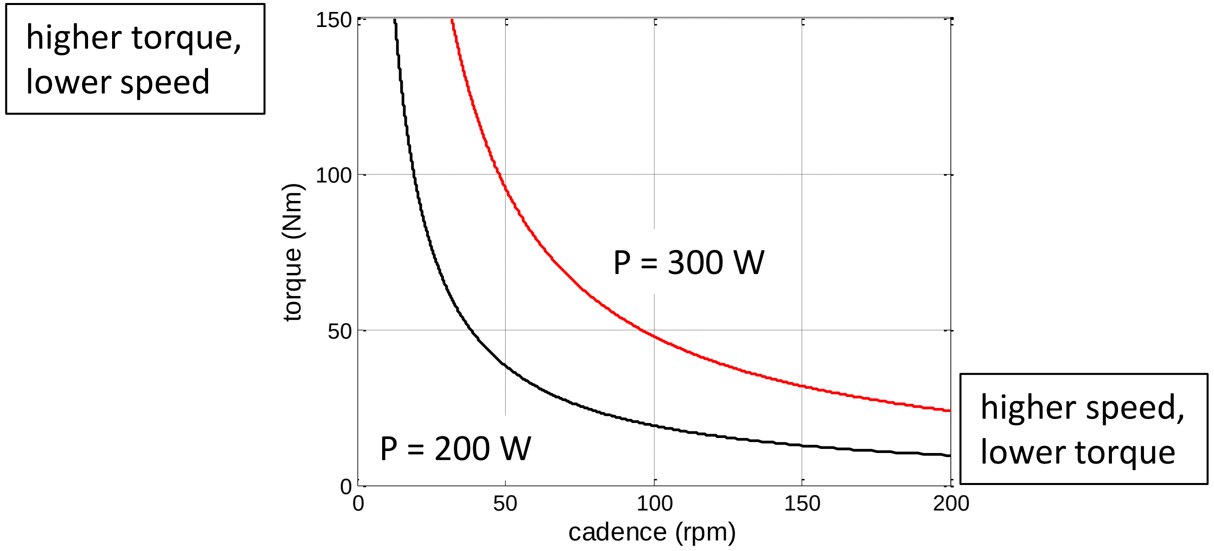
$$P = 5400 \text{ x } 120 \text{ x } 2\pi/60 = 67.85 \text{ kW}$$

Torque vs Speed

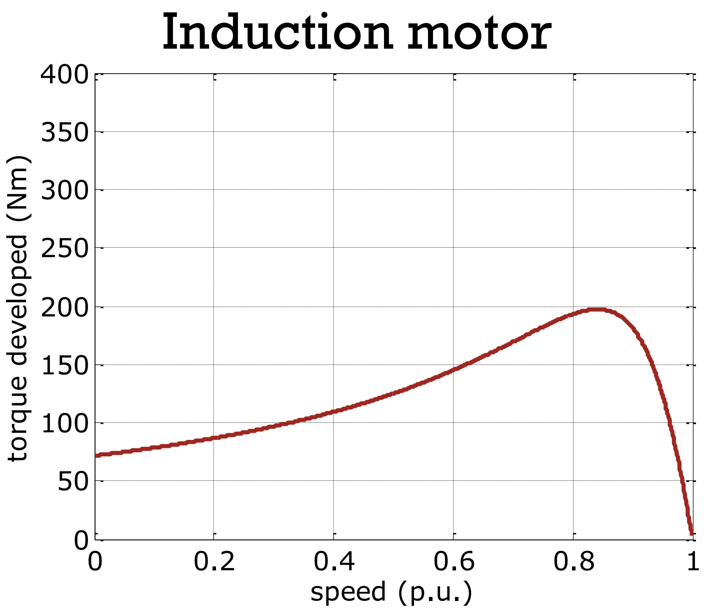
- The power, torque and speed relationship is of interest to electric motors and generators
- Governs rotational dynamics of system
- Different machines have different torque vs speed relationships for a given power level

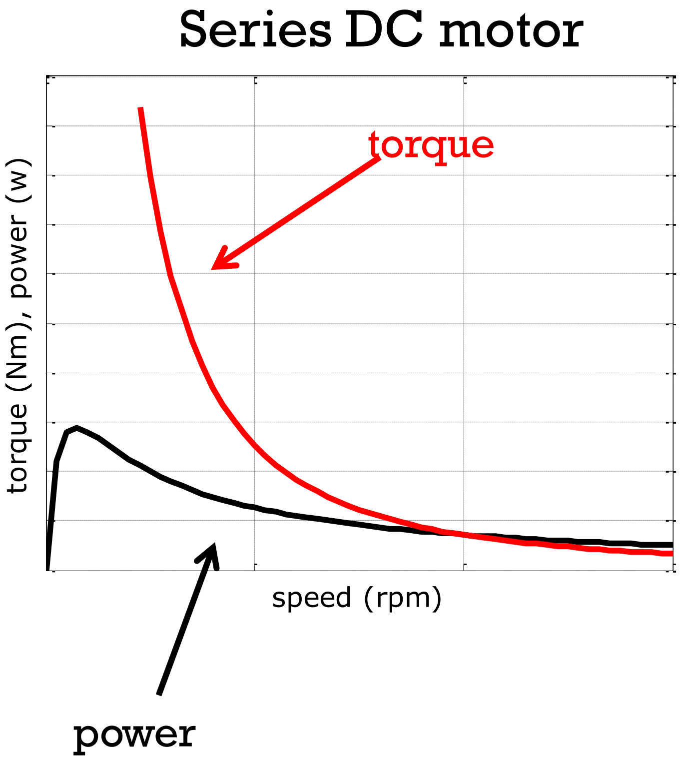
Torque vs Speed

- Recall: $P = T\omega$
  - $T = P/\omega$
- For a hypothetical constant power machine:



Torque vs Speed Examples





Lorentz Force Equation

- Lorentz Force Equation

$$\boldsymbol{F} = q(\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B})$$

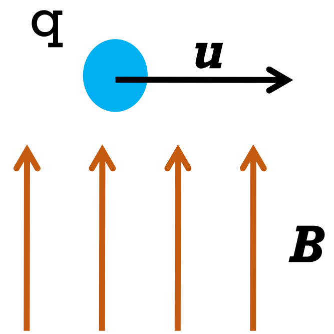
- where
  - $\boldsymbol{F}$: force (Newton)
  - $\boldsymbol{E}$: electric field (V/m)
  - $\boldsymbol{v}$: velocity (m/s)
  - $\boldsymbol{B}$: flux density (Wb/m^2)

Ampere's Force Law

- Assume a charge q, is moving with velocity **u** through a magnetic field
- By the Lorentz Force equation

$$\boldsymbol{F} = \boldsymbol{q}(\boldsymbol{E} + \boldsymbol{u} \times \boldsymbol{B})$$

$$\boldsymbol{F} = \boldsymbol{qu} \times \boldsymbol{B}$$



- A force is exerted on the charge

  in the direction out of the slide

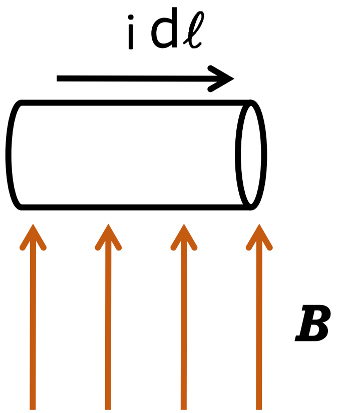
Ampere's Force Law

- A moving charge is current, therefore

$$\boldsymbol{F} = q\boldsymbol{v} \times \boldsymbol{B}$$

$$\boldsymbol{F} = \int_{c} i d\boldsymbol{\ell} \times \boldsymbol{B}$$

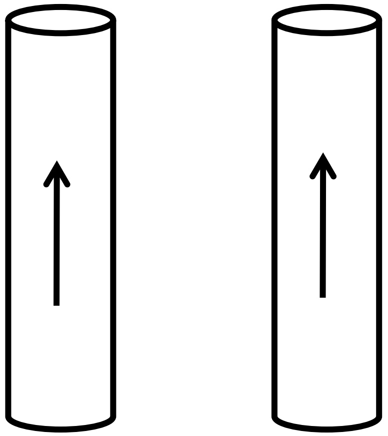
- This is Ampere's force law
- Used to compute torque in machines



Note: $d\ell$ (vector) is the direction of the current

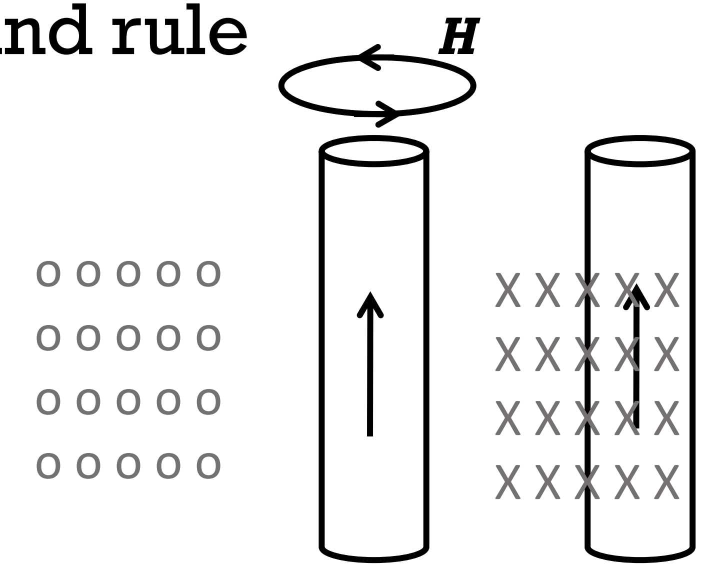
Exercise

- Consider two conductors, each with current I flowing in the same direction
- Are the conductors attracted to each other or repelled from each other?



Exercise

- Consider the magnetic field associated with conductor 1
- From the right hand rule

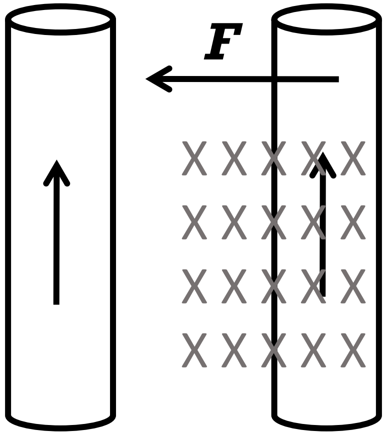


B field into the paper

Exercise

- Using $\boldsymbol{F} = q\boldsymbol{v} \times \boldsymbol{B}$

  $\boldsymbol{F} = \int_c i d\ell \times \boldsymbol{B}$
- Force on conductor 2 is toward conductor 1
- We can also see that the force on conductor 1 is toward conductor 2

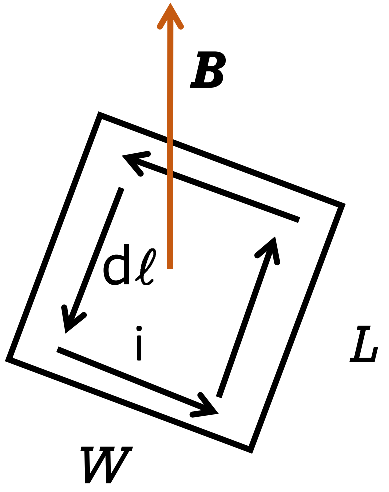


B field into the paper

Torque on a Current Loop

- Consider a loop with width W and length L and current i flowing through it
- Assume a uniform magnetic field $\boldsymbol{B}$ is present and perpendicular to the loop

$$\boldsymbol{F} = \int_{c} i d\ell \times \boldsymbol{B}$$

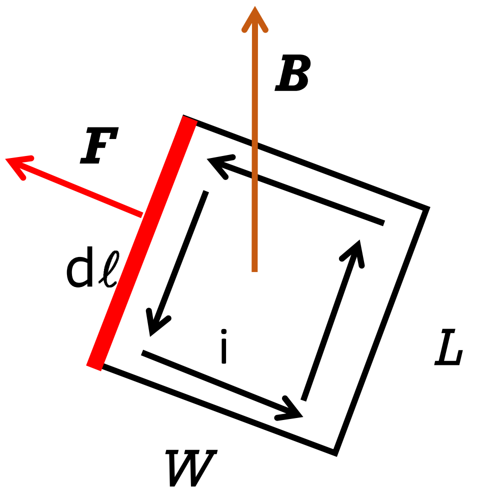


Torque on a Current Loop

- Consider the side in red
- The direction of the force on this side is computed from

$$\boldsymbol{F} = \int_c \mathrm{i}d\ell \times \boldsymbol{B}$$

and therefore is in the direction shown



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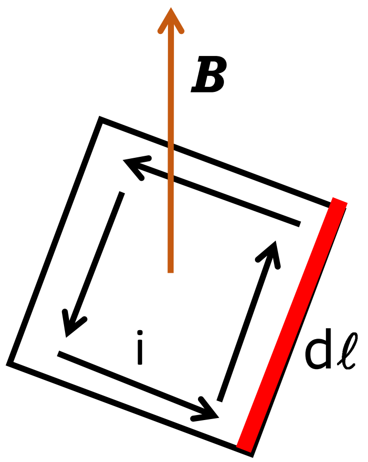
Exercise

Find the direction force on the side colored in red

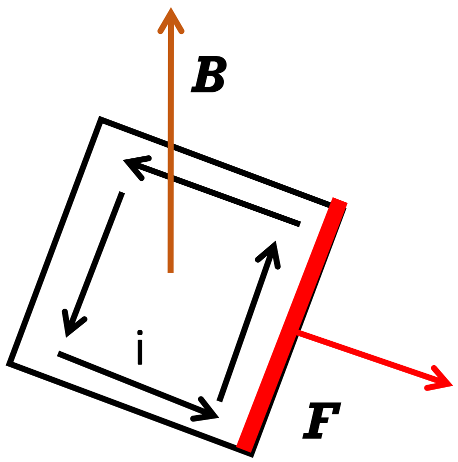
$$\boldsymbol{F} = \int_{c} i\,d\ell \times \boldsymbol{B}$$



Exercise

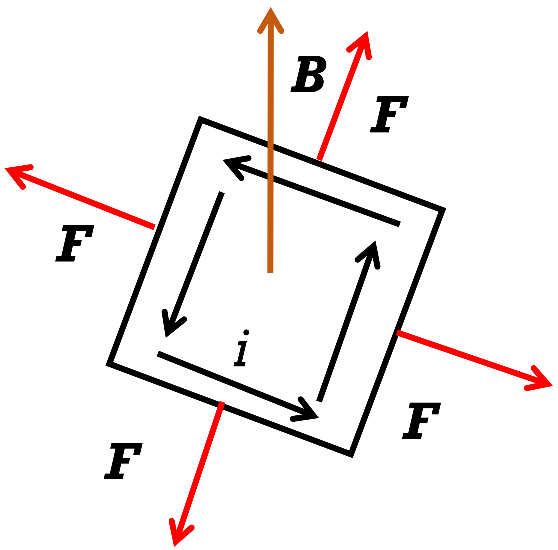
It will be equal in magnitude and opposite in direction as the other side

$$\boldsymbol{F} = \int_{c} i d\ell \times \boldsymbol{B}$$



Torque on a Current Loop

- We can show that the forces on each side of the conductor net to zero
- No torque developed
- No movement of the conductor

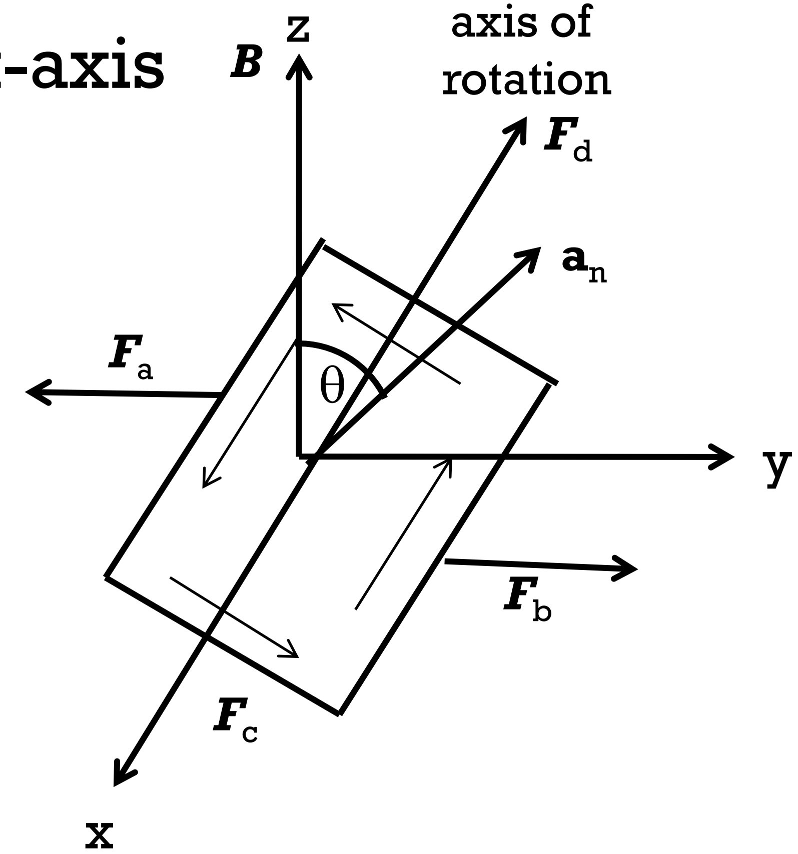


Torque on a Current Loop

- Consider a loop rotated on the x-axis
- From $\boldsymbol{F} = \int_c i d\ell \times \boldsymbol{B}$

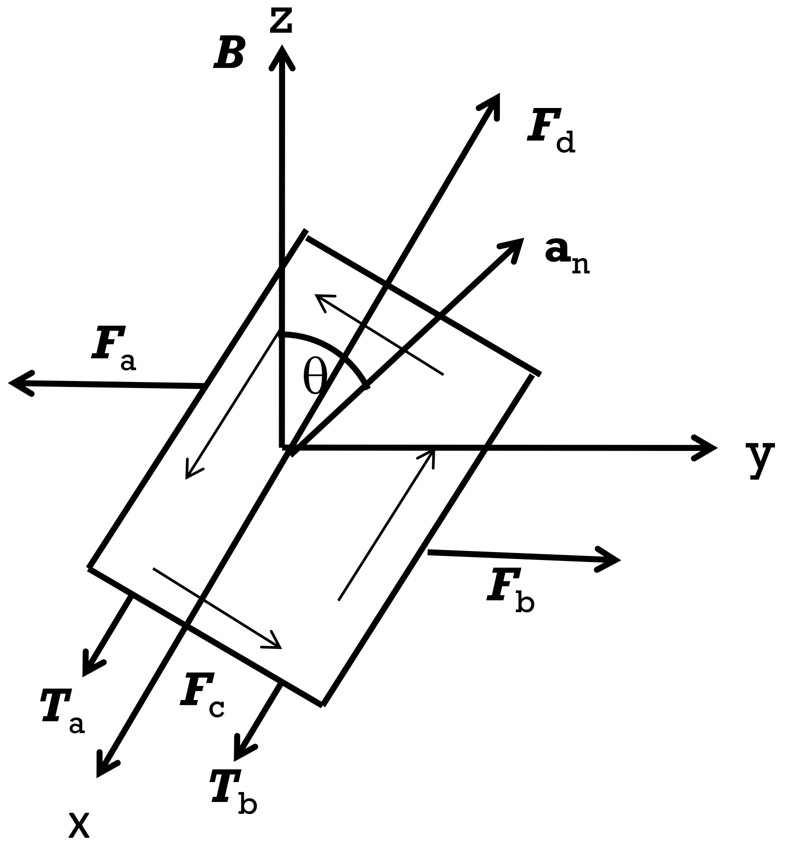
$$\boldsymbol{F}_a = -\boldsymbol{B}iL\boldsymbol{a}_y$$

$$\boldsymbol{F}_b = \boldsymbol{B}iL\boldsymbol{a}_y$$



Torque on a Current Loop

- A torque develops that tends to rotate the loop



Torque on a Current Loop

- The torque $\boldsymbol{T}$ is: $\boldsymbol{T} = \boldsymbol{r} \times \boldsymbol{F}$
- Therefore the torque on the a and b sides is:

$$\boldsymbol{T}_{\mathrm{a}} = BiL\left(W/2\right)\sin\theta\,\boldsymbol{a}_{\mathrm{x}}$$

$$\boldsymbol{T}_{\mathrm{b}} = BiL\left(W/2\right)\sin\theta\,\boldsymbol{a}_{\mathrm{x}}$$

- The total torque on the loop is:

$$\boldsymbol{T} = BiA\sin\theta\,\boldsymbol{a}_{\mathrm{x}} \qquad \text{using } A = LW$$

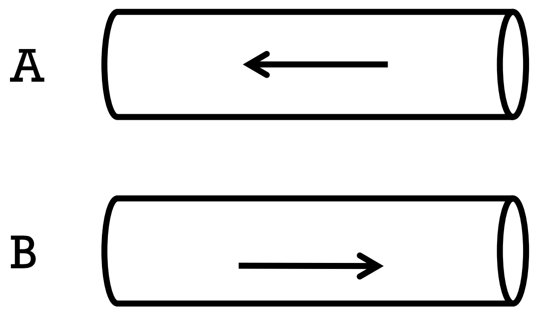
- If there are N coils:

$$\boldsymbol{T} = BiAn\sin\theta\,\boldsymbol{a}_{\mathrm{x}}$$

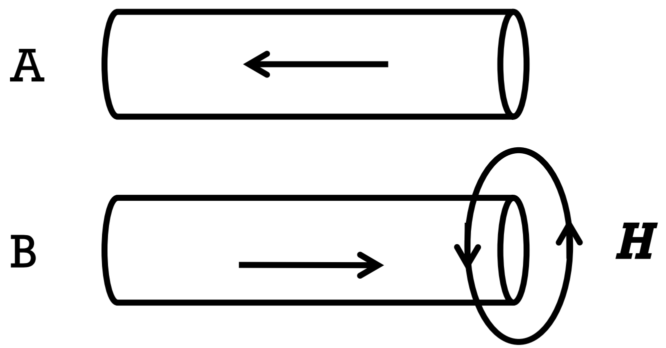
Summary

- Torque = radius x force
- Power = torque x speed
- Machines exhibit different torque vs speed characteristics and are important in determining the application of the machine
- Lorentz Force equation tells us how strong (and what direction) a force a current-carrying conductor experiences when exposed to a magnetic field

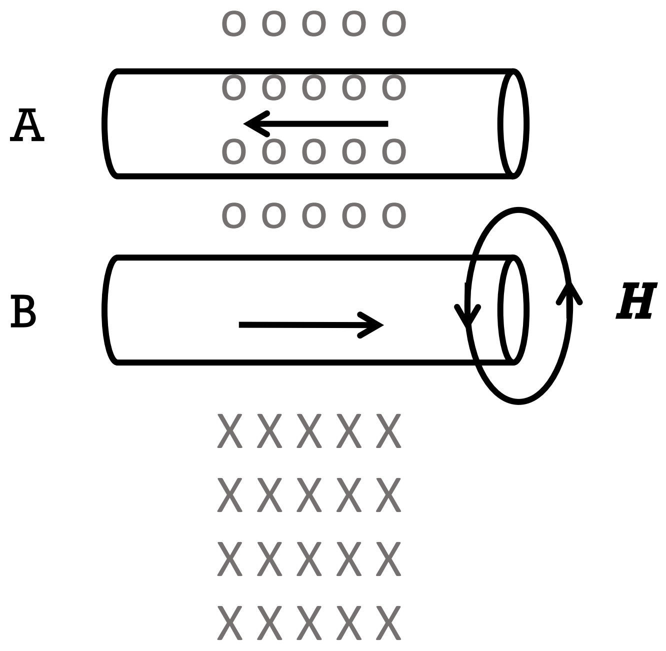
- Consider a segment of our circuit
- Conductor A and B are shown
- At this instant, let the current in each conductor have the direction shown (opposite directions)
- Will the conductors experience a force?



- Let’s first consider the effect that conductor B has on conductor A
- According to Ampere’s Law, the current in conductor B has a circulating magnet field (H) around it
- The direction is found by applying the right hand rule as shown



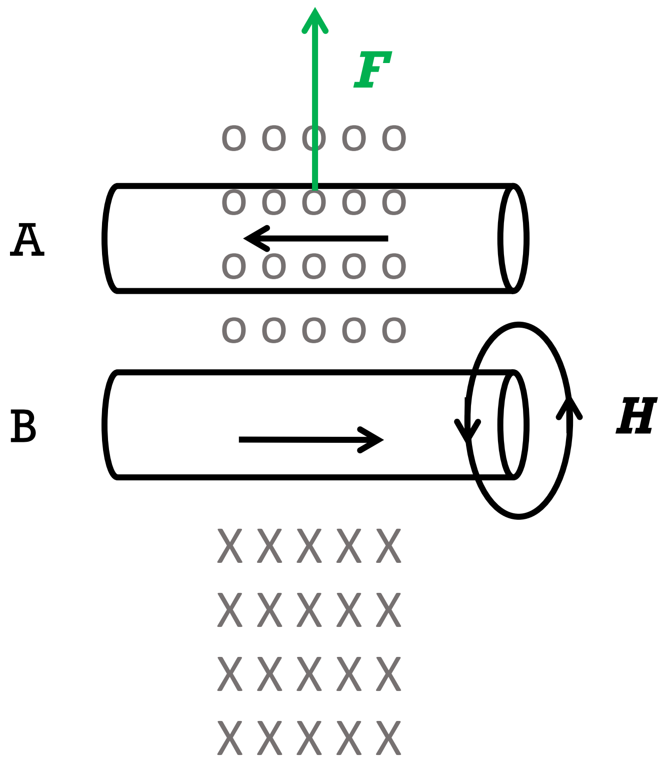
- The direction of the H field is such that it comes out of the screen where conductor A is and into the screen below where conductor B is



- Applying the Lorentz Force equation for a straight wire with length vector ***L*** (in the direction of the current flow)

F = i***L*** x ***B*** (we expect the B field to be in the same direction as the H field)

and then applying its corresponding right hand rule, we see that conductor A is pushed away from conductor B



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- By the same reasoning, we can show that conductor B is pushed away from conductor A