04-Electric Power

ECEGR 452

Renewable Energy Systems

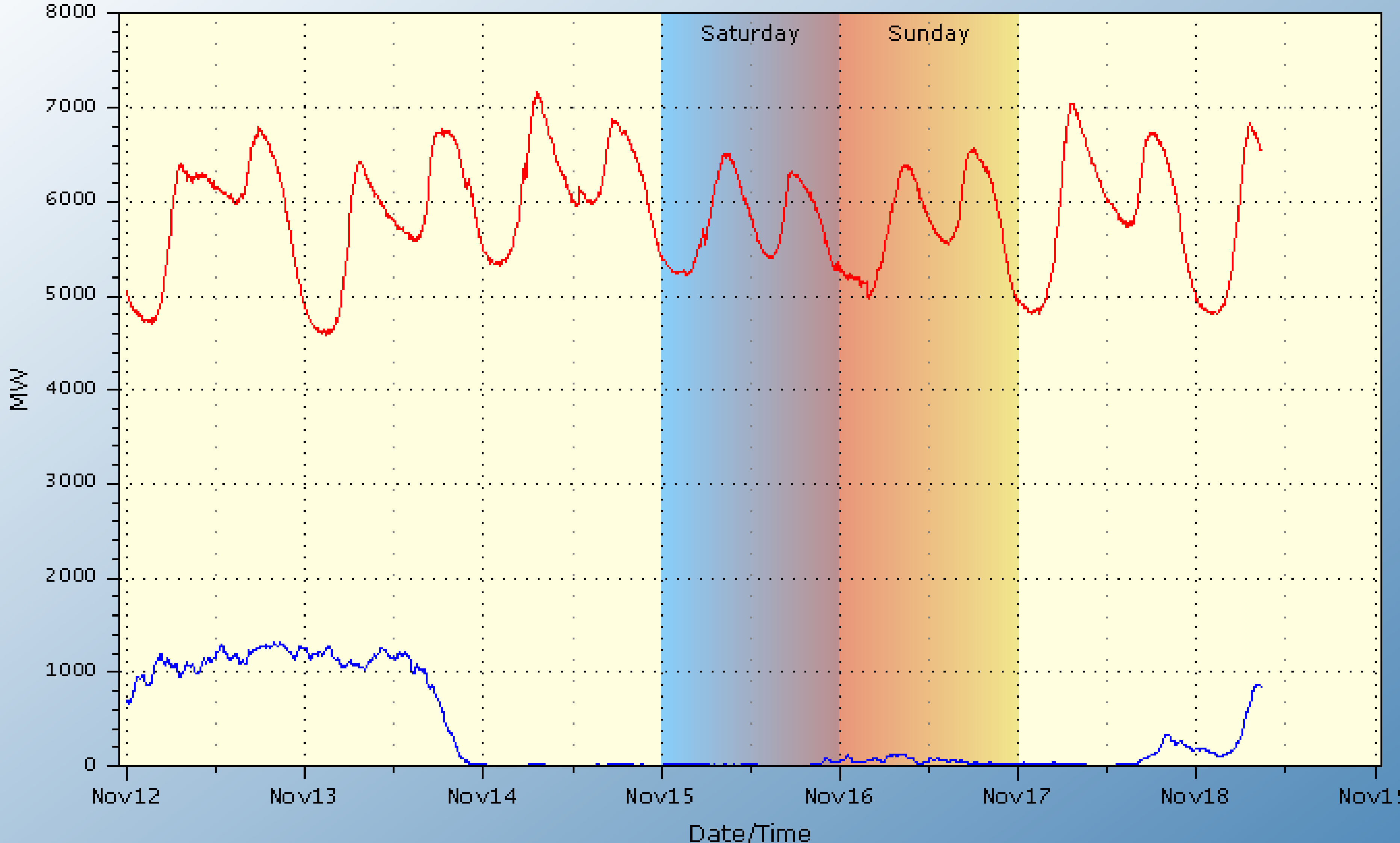
Overview

- Review of Electric Circuits
- Phasor Representation
- Electrical Power
- Power Factor

Introduction

- Majority of the electrical energy produced by renewable resources is ultimately transmitted and consumed within the interconnected power system
- Integration of renewable resources into the power system is a challenge
- We need to understand basics of electric power

BPA Balancing Authority Load & Total Wind Generation, Last 7 days
12Nov2008 - 19Nov2008 (last updated 18Nov2008 08:51:30)



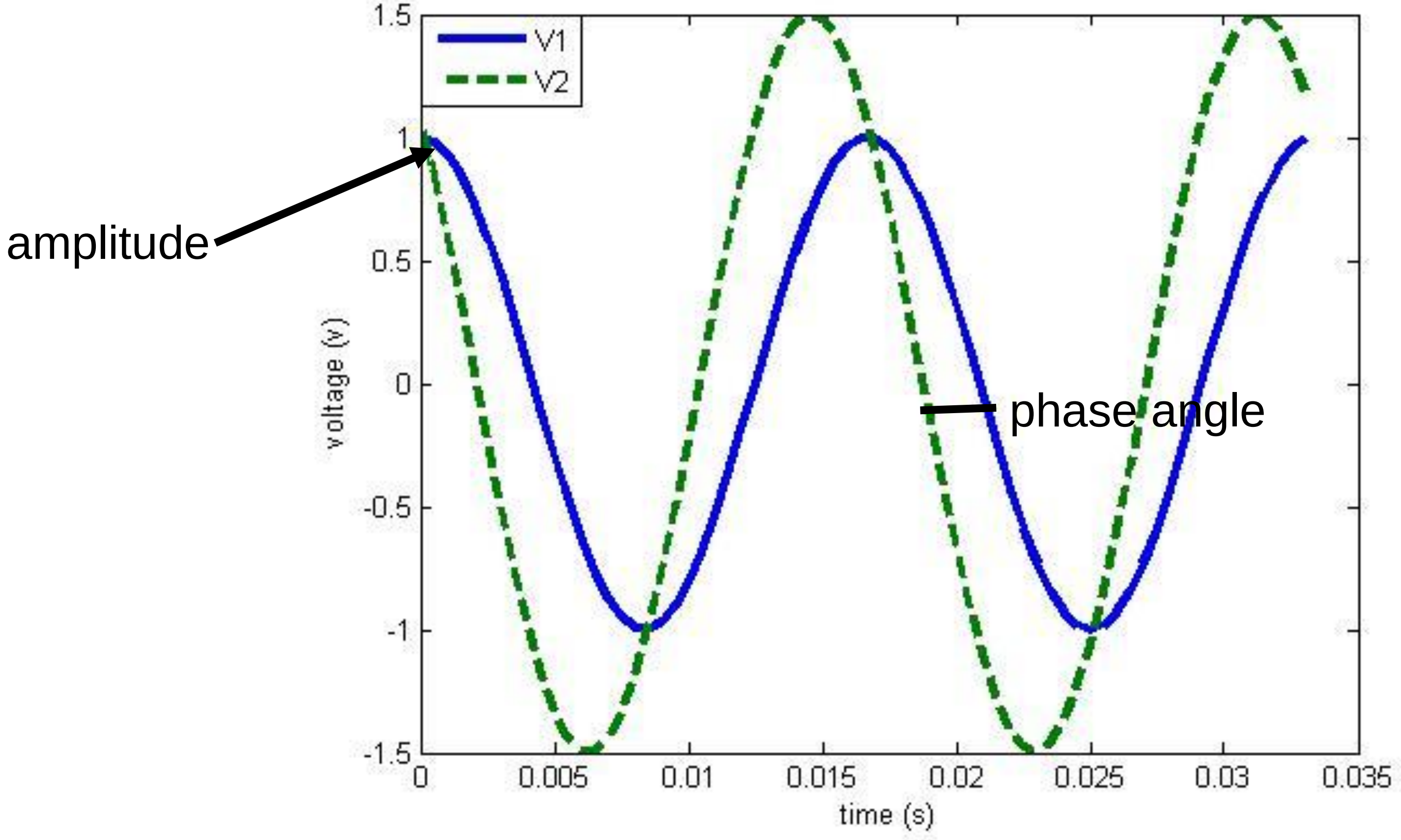
Based on 5-min readings from the BPA SCADA system for points 45583, 79687
Balancing Authority Load in Red, Wind Generation in Blue; Installed Wind Capacity=1489 MW
BPA Technical Operations: Roy Ellis (rcellis@bpa.gov)

Review of Electric Circuits

- Voltage in AC circuits:
  - $v(t) = V_{max}\cos(\omega t + \theta_V)$
    - V_{max}: voltage amplitude (Volts)
    - ω: frequency (rad/sec)
    - θ_V: phase angle of the voltage (rad)

- Current in AC circuits:
  - $i(t) = I_{max}\cos(\omega t + \theta_i)$
    - I_{max}: amplitude (Amperes)
    - θ_i: phase angle of the voltage (rad)

Review of Electric Circuits



$$v_1(t) = 1.0\cos\left(377t + 0^\circ\right) \qquad v_2(t) = 1.5\cos\left(377t + 45^\circ\right)$$

Review of Electric Circuits

- Conversion of radians (θ_{rad}) to degrees (θ_{deg})

$$\theta_{deg} = \theta_{rad} \frac{180^\circ}{\pi}$$

- Conversion of degrees to radians

$$\theta_{rad} = \theta_{deg} \frac{\pi}{180^\circ}$$

Review of Electric Circuits

- Frequency in North American power systems is 60 Hz
  - f: frequency (Hertz)
  - $\omega = 2\pi f \sim 377$ rad/sec
- Other parts of the world 50 Hz is common
- We assume 60 Hz unless otherwise noted
- Voltage waveform is set as a reference, so $\theta_V = 0^o$

Phasor Transform

- Shorthand for writing sinusoidal functions
- Used for steady-state calculations
- Contains amplitude and phase angle information
    - Assumed that frequency is known
- Relies on Euler's Identity

$$v(t) = v_{max}\cos(\omega t + \theta_v) \longrightarrow \boxed{\text{Phasor Transform}} \longrightarrow \frac{V_{max}}{\sqrt{2}}\angle\theta_v$$

Note: division by square root of 2
is used in power system analysis ("effective phasor")

Example

- Write the phasor representation of
  $v_1(t) = 1.41\cos(377t + 0)$
- Write the phasor representation of
  $v_2(t) = 2.12\cos(377t + 45)$
- Write the phasor representation of
  $v_3(t) = 1.41\cos(t + 0)$

Example

- write the phasor representation of

$$v_1(t) = 1.41\cos(377t + 0)$$

solution

$$\mathbf{V}_1 = 1\angle 0^\circ$$

- write the phasor representation of

$$v_2(t) = 2.12\cos(377t + 45)$$

solution

$$\mathbf{V}_2 = 1.5\angle 45^\circ$$

- write the phasor representation of

$$v_3(t) = 1.41\cos(t + 0)$$

solution

$$\mathbf{V}_3 = 1\angle 0^\circ$$

Notation

- Lecture slides use bold uppercase variables (e.g. **V**, **I**) for phasors and other vectors
- Capital letters (e.g. V, I) or absolute values of phasors (|**V**|, |**I**|) are used to indicate the magnitude of the phasor

  $\mathbf{V} = V\angle\theta$

- Lowercase variables (e.g. v, i) are preferred to represent scalars not associated with phasors and vectors
  - Notable exceptions P, Q for real and imaginary power

Phasor Transform

- We use the effective phasor because

  $P = v_{rms} i_{rms} \cos(\phi)$

  - So we can then write $P = VI\cos(\phi)$
- Unless otherwise specified, <u>assume that voltages and currents are given in RMS and all phasors are "effective phasors"</u>
- Also note:

$$e^{-j90^\circ} = \cos(-90^\circ) + j\sin(-90^\circ) = j\sin(-90^\circ) = -j$$

where j is the imaginary operator $j = \sqrt{-1}$

Phasor Transform

- Different expressions of a voltage phasor

$$\frac{V_{max}}{\sqrt{2}}\angle\theta_v = V_{rms}\angle\theta_v = |\mathbf{V}|\angle\theta_v = V\angle\theta_v = Ve^{j\theta_v}$$

- Current:

$$\frac{I_{max}}{\sqrt{2}}\angle\theta_I = I_{rms}\angle\theta_I = |\mathbf{I}|\angle\theta_I = I\angle\theta_I = Ie^{j\theta_I}$$

- Impedance:

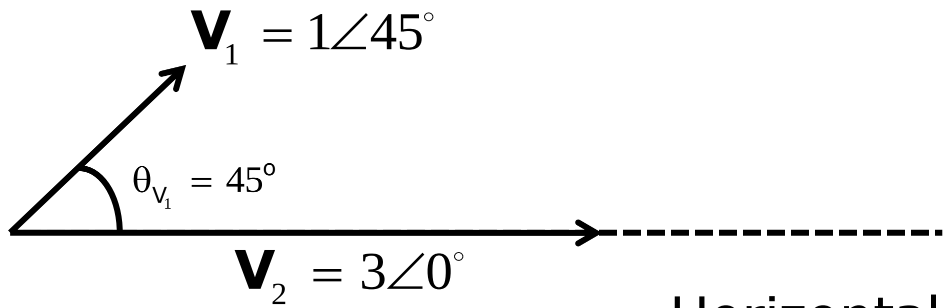
$$|\mathbf{Z}|\angle\theta_Z = Z\angle\theta_Z = Ze^{j\theta_Z}$$

- Define: $\phi \triangleq \theta_v - \theta_i$ (remember this!)

Phasors

Phasors have a direct geometric interpretation

- Polar form



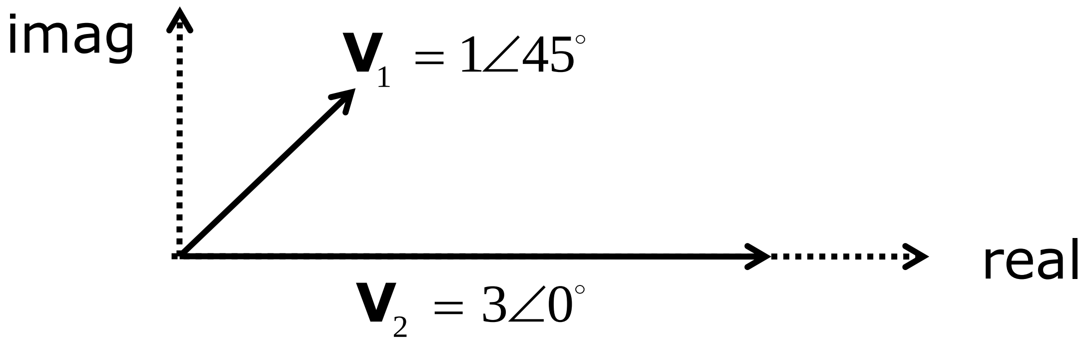
Horizontal is reference

Phasors

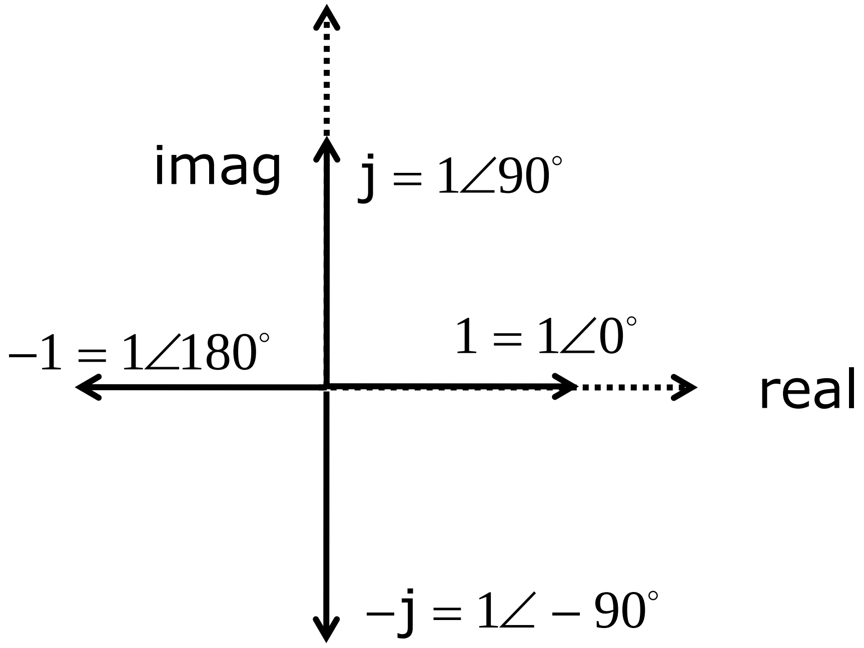
- Another way of specifying phasors is in rectangular form
  - Let the Y-axis be the imaginary (j) axis
  - Let the X-axis be the real axis
- Resolving into real and imaginary components

$$\mathbf{V}_1 = 1\angle 45^\circ = 0.707 + j0.707$$

$$\mathbf{V}_2 = 3\angle 0^\circ = 3 + j0$$



Phasors

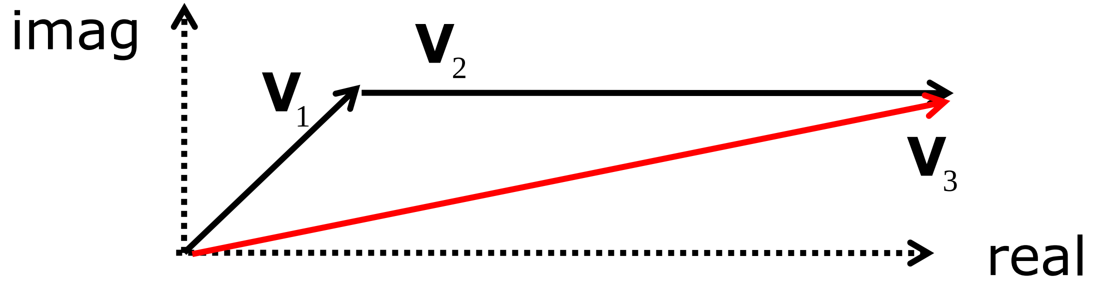

imag
$j = 1\angle 90^\circ$
$-1 = 1\angle 180^\circ$
$1 = 1\angle 0^\circ$
real
$-j = 1\angle -90^\circ$

Addition of Phasors

- Addition and subtraction of phasors are simple using rectangular form
  - Simply add/subtract the real values and add/subtract the imaginary values

$$\mathbf{V}_3 = \mathbf{V}_1 + \mathbf{V}_2 = 3.707 + j0.707$$

Addition is "tip to tail"
Subtraction is "tail to tip"



Multiplication of Phasors

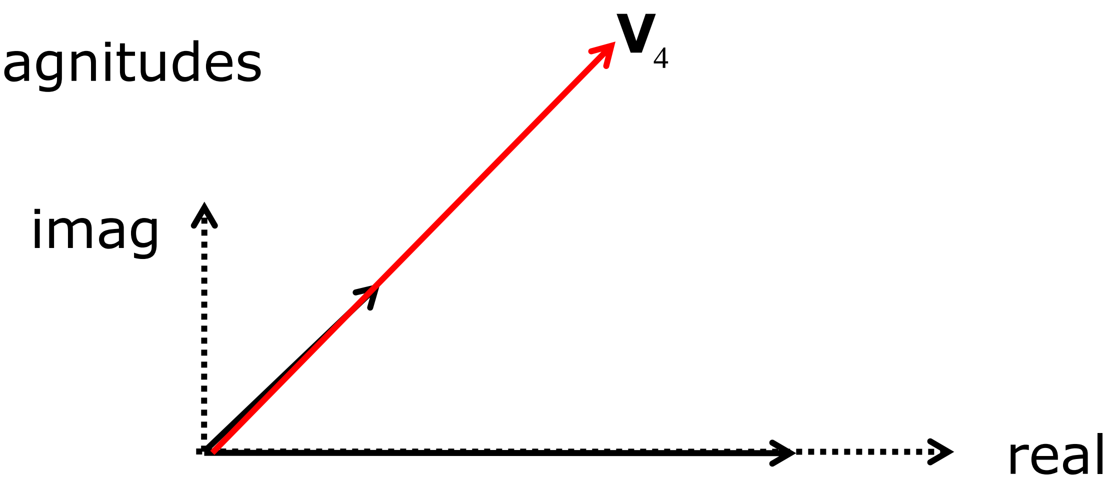
- Multiplication and division are easier in polar form
  - For multiplication: multiply magnitudes, add angles
  - For division: divide magnitudes, subtract angles

$$\mathbf{V}_4 = \mathbf{V}_1\mathbf{V}_2 = (1\angle 45^\circ)(3\angle 0^\circ)$$

$$\mathbf{V}_4 = 3\angle 45^\circ$$

add angles

multiply magnitudes



Example

If $\mathbf{V} = 1\angle 10^{\circ}$, what is j**V**?

A. $1\angle 10^{\circ}$

B. $1\angle 100^{\circ}$

C. $1\angle 190^{\circ}$

D. 0

Example

If $\mathbf{V} = 1\angle 10^\circ$, what is j**V**?

A. $1\angle 10^\circ$

B. $1\angle 100^\circ$

C. $1\angle 190^\circ$

D. 0

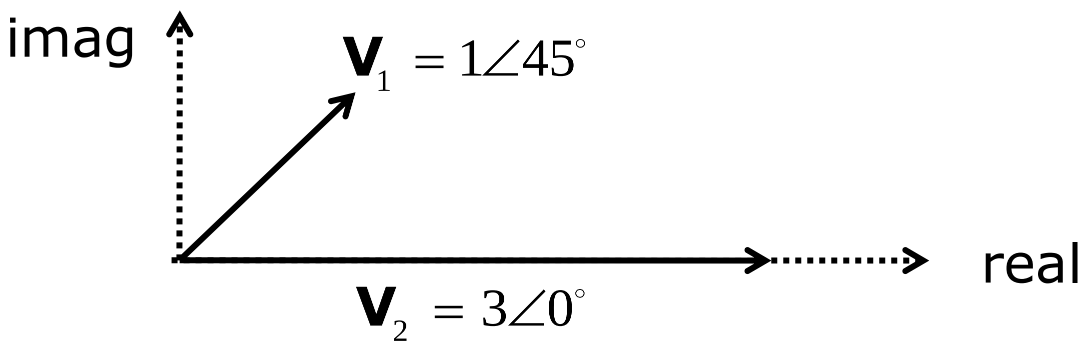
$$j\mathbf{V} = (1\angle 90^\circ)(1\angle 10^\circ) = 1\angle 100^\circ$$

Phasors

- Another way of specifying phasors is in rectangular form
  - Let the Y-axis be the imaginary (j) axis
  - Let the X-axis be the real axis
- Resolving into real and imaginary components

$$\mathbf{V}_1 = 1\angle 45^\circ = 0.707 + j0.707$$

$$\mathbf{V}_2 = 3\angle 0^\circ = 3 + j0$$



Phasor Analysis of Resistors

- For resistors

$$i(t) = i_{max}\cos(\omega t + \theta_i)$$

$$v(t) = i(t)R$$

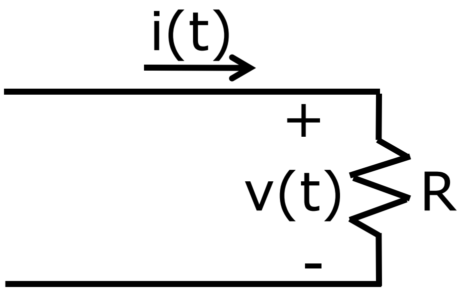
$$v(t) = Ri_{max}\cos(\omega t + \theta_i)$$

- Transforming into phasor form:

$$\mathbf{V} = RIe^{j\theta_i}$$

$$\mathbf{V} = R\mathbf{I}$$

Voltage and current are in phase



Phasor Analysis of Inductors

- For inductors

$$i(t) = i_{max}\cos(\omega t + \theta_i)$$

$$v(t) = L\frac{di}{dt}$$

$$v(t) = -L\omega i_{max}\sin(\omega t + \theta_i) = -L\omega i_{max}\cos(\omega t + \theta_i - 90^o)$$

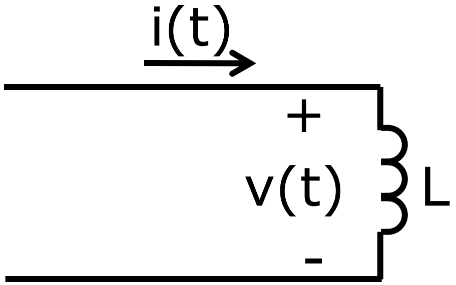
- Transforming into phasor form:

$$v(t) = -L\omega i_{max}\cos(\omega t + \theta_i - 90^o)$$

$$\Rightarrow \mathbf{V} = -L\omega I e^{j\theta_i} e^{-j90^o}$$

$$\mathbf{V} = jL\omega I e^{j\theta_i} = jL\omega \mathbf{I}$$

using $e^{j-90^\circ} = -j$



Phasor Analysis

$\mathbf{V} = j\omega L\mathbf{I}$

- Define $X_L = \omega L$ (inductive reactance)
- Therefore
  - $\mathbf{V} = jX_L\mathbf{I}$ **I** lags **V** by 90 deg.
- A similar derivation for capacitors yields
  - $X_C = 1/(\omega C)$ (capacitive reactance)
  - $\mathbf{V} = -jX_C\mathbf{I}$ **I** leads **V** by 90 deg.

Phasor Analysis

- We can rewrite Ohm's Law to include complex impedances
- **V** = **IZ**
  - **Z**: complex impedance (Ohms)
    - **Z** = R + jX_L - jX_C (if in series)
    - 1/**Z** = 1/R + j/X_L - j/X_C (if in parallel)
- **Z** will have a magnitude and phase associated with it $\mathbf{Z} = Z\angle\theta_z$

Complex Power

P is also known as Real Power, Active Power, Average Power

$$P = |\mathbf{V}_s||\mathbf{I}|\cos(\phi)$$

$$P = \mathrm{Re}\left\{\mathbf{VI}^*\right\}$$

* is the complex conjugate operator,
it denotes a change in sign of the imaginary part

Conjugation is needed so that the difference in phase between voltage and current is considered, rather than their sum

Complex Power

- Let **S** be the complex power defined as

$$\mathbf{S} = \mathbf{V}\mathbf{I}^*$$

then

$$P = \mathrm{Re}\left\{\mathbf{V}\mathbf{I}^*\right\} = \mathrm{Re}\left\{\mathbf{S}\right\}$$

- Let Q be the reactive power defined as

$$Q = \mathrm{Im}\left\{\mathbf{V}\mathbf{I}^*\right\}$$

- Then

$$Q = \mathrm{Im}\left\{\mathbf{S}\right\}$$

Q is also known as imaginary power

- And therefore:

$$\mathbf{S} = P + jQ$$

S is also known as apparent power

Complex Power

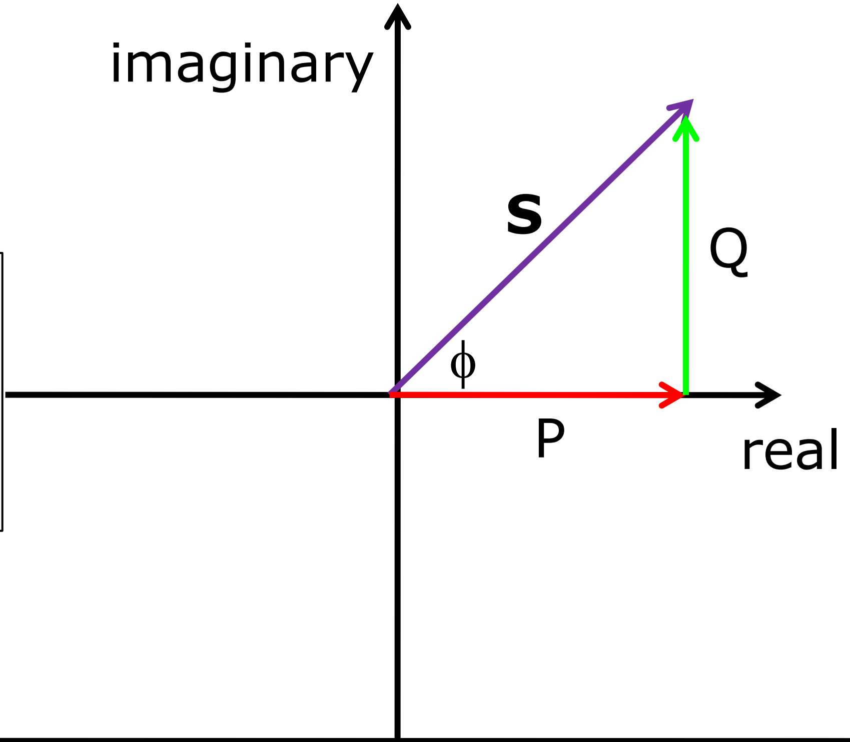
- Technically, units of **S**, Q and P are watts*
- To avoid confusion, alternate units are used in practice
  - S: Volt-Amps (VA)
  - Q: Volt-Amps Reactive (VAR)
- Inductors, capacitors consume/supply reactive power, Q
- S and Q are defined values
  - a meaningful physical interpretation is elusive

*See C. Gross "On VA's, VAR's, and Other Traditions in Electric Power Engineering"

Power Triangle

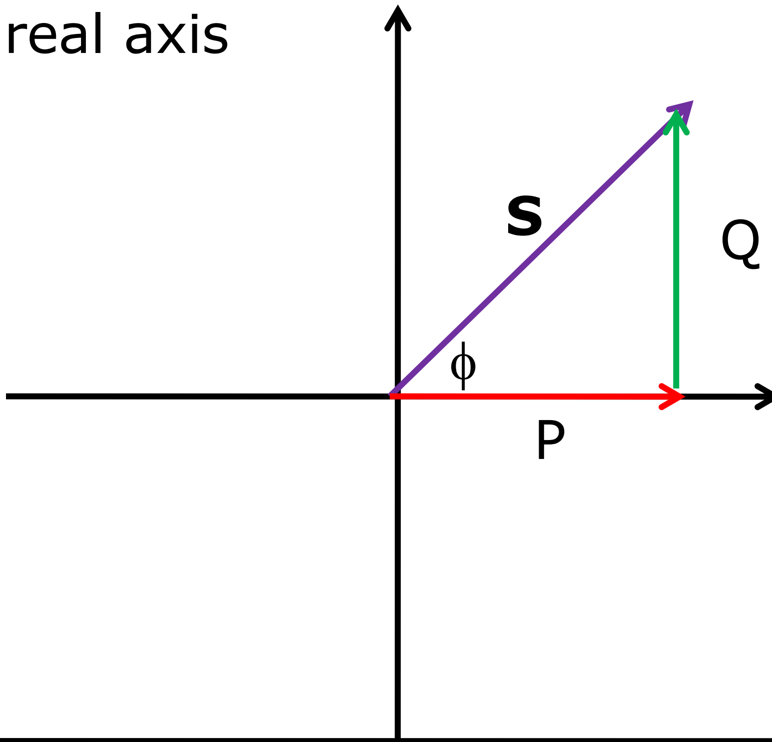
- Relationships between **S**, P and Q can be shown graphically
- **S** = P + jQ

Note:
P, Q are scalars representing the real and imaginary parts of **S**



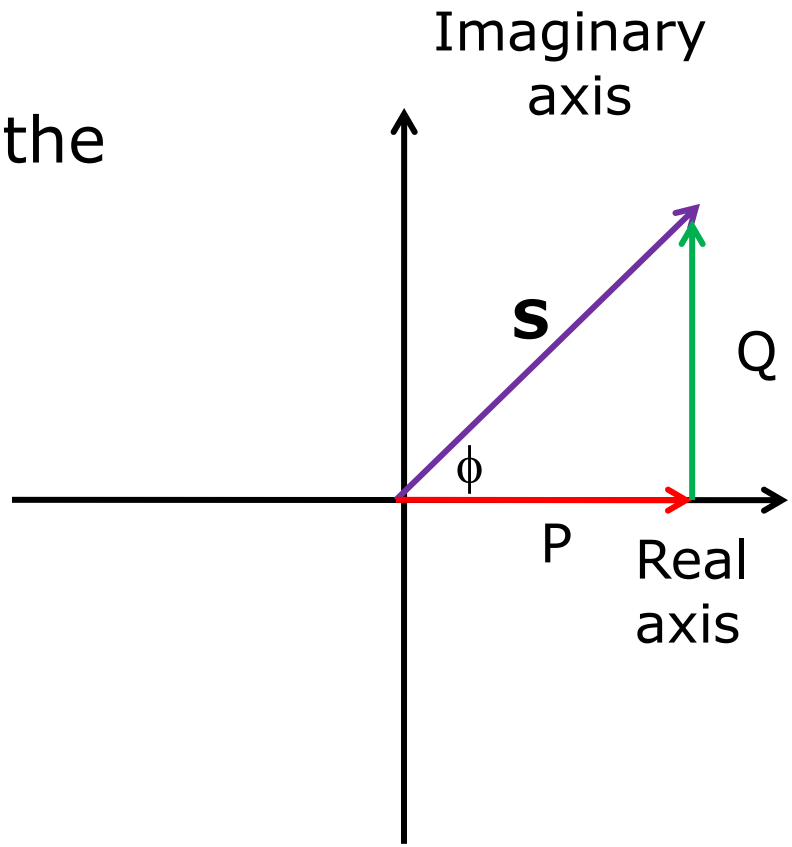
Power Triangle

- Consider $P = |\mathbf{V}||\mathbf{I}|\cos(\phi)$
- Since $|\mathbf{S}| = |\mathbf{VI}^*| = |\mathbf{VI}| = |\mathbf{V}||\mathbf{I}|$
- Then $P = |\mathbf{S}|\cos(\phi)$
  - P is the projection of S onto the real axis



Complex Power

- A similar result can be found for Q
  - P =|**S**|cos(ϕ)
  - Q =|**S**|sin(ϕ)
- Q is the projection of **S** onto the imaginary axis



Complex Power Cheat Sheet

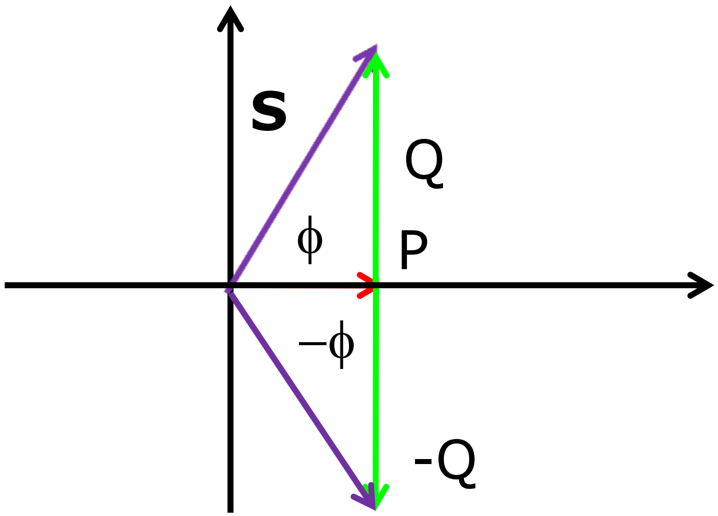
$$P = \text{Re}\{\mathbf{VI}^*\}$$
$$= \text{Re}\{\mathbf{IZI}^*\} = |\mathbf{I}|^2 \text{Re}\{\mathbf{Z}\}$$
$$P = |\mathbf{I}|^2 R$$
$$P = |\mathbf{V}||\mathbf{I}| \cos\phi$$
$$P = |\mathbf{I}|^2 |\mathbf{Z}| \cos\phi$$

$$Q = \text{Im}\{\mathbf{VI}^*\}$$
$$Q = |\mathbf{I}|^2 X$$
$$Q = |\mathbf{V}||\mathbf{I}| \sin\phi$$
$$Q = |\mathbf{I}|^2 |\mathbf{Z}| \sin\phi$$

$$\mathbf{S} = P + jQ$$
$$\phi = \tan^{-1}\left(Q/P\right)$$
$$\cos\phi = \frac{P}{\sqrt{P^2 + Q^2}}$$

Power Factor

- Power factor is non-negative
- $\cos(\phi) = \cos(-\phi)$
- Need to distinguish between ϕ and $-\phi$



Power Factor

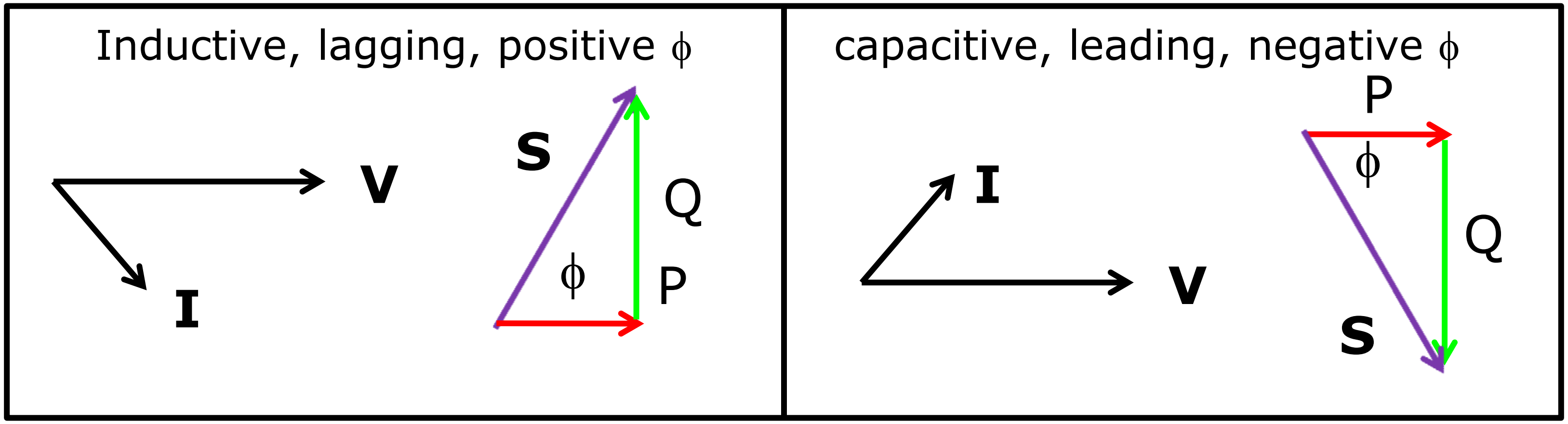
- For example let $\theta_v = 0^o$
- Case 1: $\theta_i = 30^o$
  - Capacitive circuit
  - PF = 0.866
- Case 2: $\theta_i = -30^o$
  - Inductive circuit
  - PF = 0.866

Same power factor

Leading/Lagging Power Factor

Must describe the PF value along with whether the current leads or lags voltage

- Lagging: current lags voltage (inductive)
- Leading: current leads voltage (capacitive)
- Useful mnemonic: ELI the ICE man



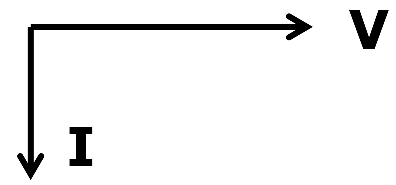
Why are Inductive Circuits Lagging?

Recall

$$\mathbf{V} = jX_L\mathbf{I}$$

$$\mathbf{I} = \mathbf{V}/(jX_L) = -j\mathbf{V}/X_L$$

$$\mathbf{S} = \mathbf{VI}^*$$

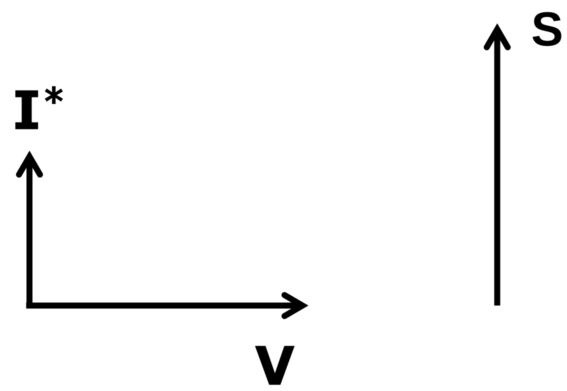


therefore

$$\mathbf{I}^* = j\mathbf{V}^*/X_L$$

$$\mathbf{S} = j\mathbf{VV}^*/X_L = j|\mathbf{V}|^2/X_L$$

angle is always +90 degrees
for purely inductive circuits

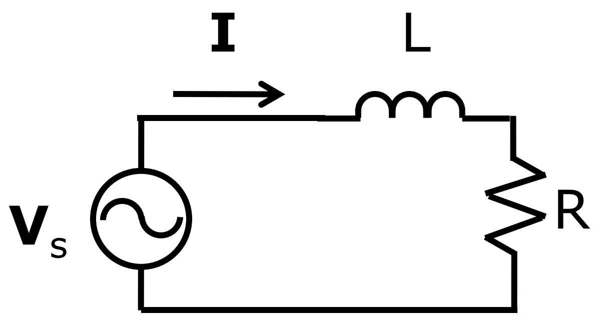


Leading/Lagging Power Factor

- Similar result for capacitive circuits
  - $\mathbf{S} = -j|\mathbf{V}|^2/X_C$
- Note:
  - the presence of resistance does not affect whether or not a circuit is leading or lagging, but it does affect the magnitude of the power factor
  - circuits with L and C (or L, C and R) must be analyzed before leading or lagging can be determined

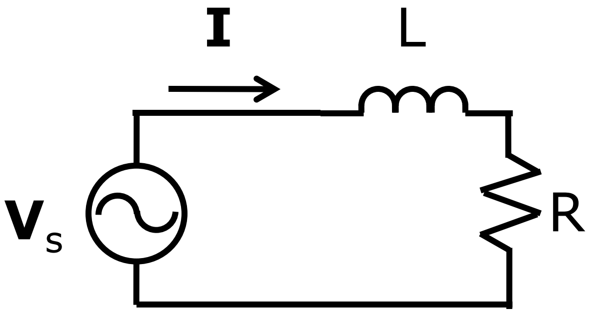
Phasor Analysis

- Find the current out of the source, the power out of the source, and power consumed by the resistor assuming:
  - V_s = 120 Volts at 60 Hz
  - L = 0.01 Henry
  - R = 10 Ohms



Phasor Analysis

- V_s = 120 Volts (RMS) at 60 Hz
- L = 0.01 Henry
  - $jX_L = j\omega L = j(60 \times 2\pi) x.01 = j3.77$
- R = 10 Ohms



Phasor Analysis

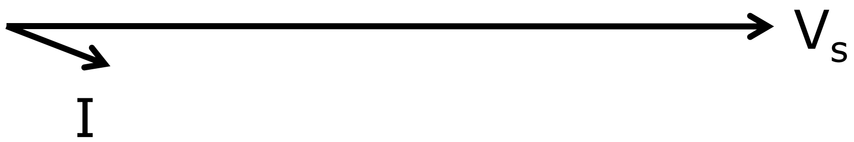
$$\mathbf{Z} = (3.77j + 10) = \sqrt{3.77^2 + 10^2}\angle \tan^{-1}\left(\frac{3.77}{10}\right)$$

$$\mathbf{Z} = 10.69\angle 20.7^\circ\,\Omega$$

$$\mathbf{V}_s = \mathbf{I}\mathbf{Z}$$

$$\mathbf{I} = \frac{120\angle 0^\circ}{10.69\angle 20.7^\circ} = 11.22\angle -20.7^\circ\,\text{A}$$

phasor diagram



Phasor Analysis

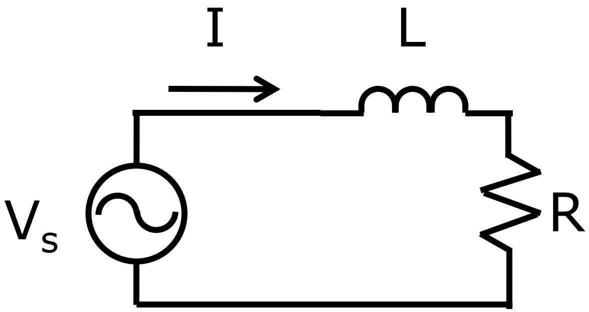
- Power from the source

$$P = |\mathbf{V}_s||\mathbf{I}|\cos(\phi) = (120)(11.22)\cos(20.7^\circ) = 1.26\text{kW}$$

- Power consumed by the load resistor

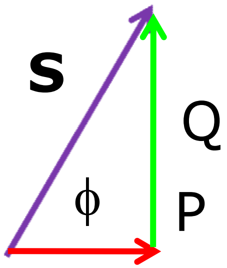
$$P = |\mathbf{I}|^2 R = 11.22^2 \times 10 = 1.26\text{kW}$$

- The inductor does not consume any power P



Summary

- P, Q, **S** related by power triangle



- P has a physical interpretation, Q and **S** do not
- **S** is a vector, Q and P are scalars
- Resistors associated with P; inductors/capacitors associated with Q