05-Solar Resource Part 2

ECEGR 4530
Renewable Energy Systems

Overview

- Angle of Incidence Components
- Effect of Declination
- Effect of Latitude
- Effect of Tilt
- Effect of Hour Angle
- Hours of Day Light

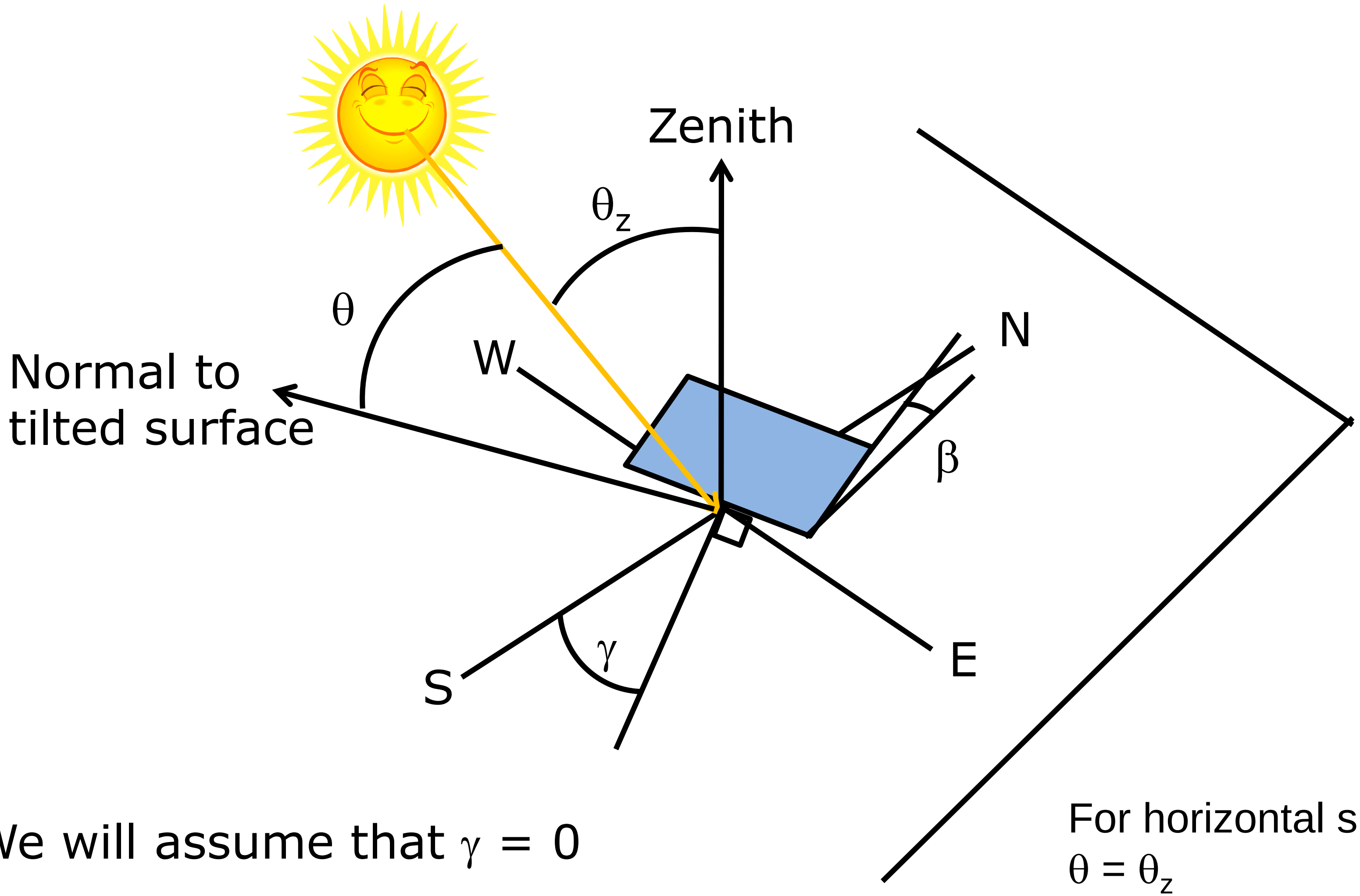
Introduction

- Last lecture we determined that the angle of incidence affects the irradiance received by a surface
- We now investigate the variables that affect the angle of incidence

Introduction

- Angle of incidence depends on many factors, including:
  - Tilt of the surface (already discussed)
  - Latitude (ϕ)
  - Declination angle (δ)
  - Surface azimuth angle (γ)
  - Hour angle (ω)

Introduction

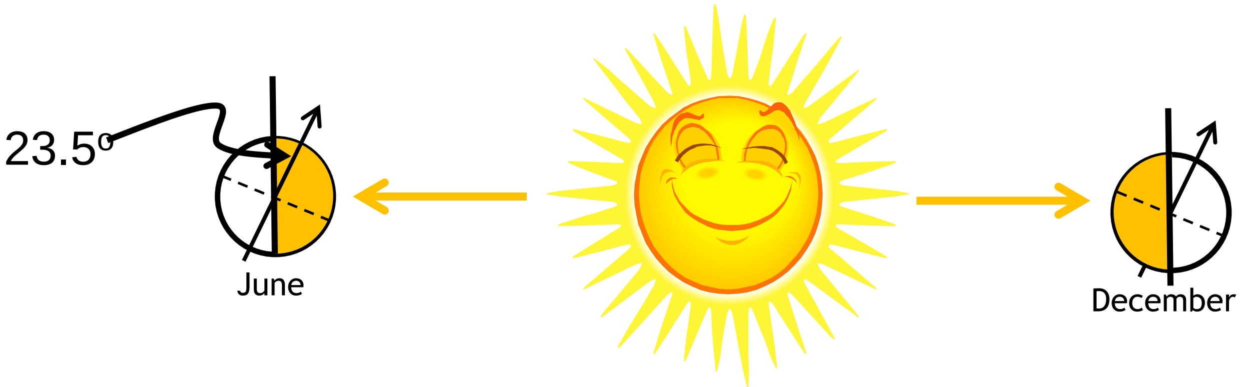


We will assume that $\gamma = 0$

For horizontal surfaces:
$\theta = \theta_z$

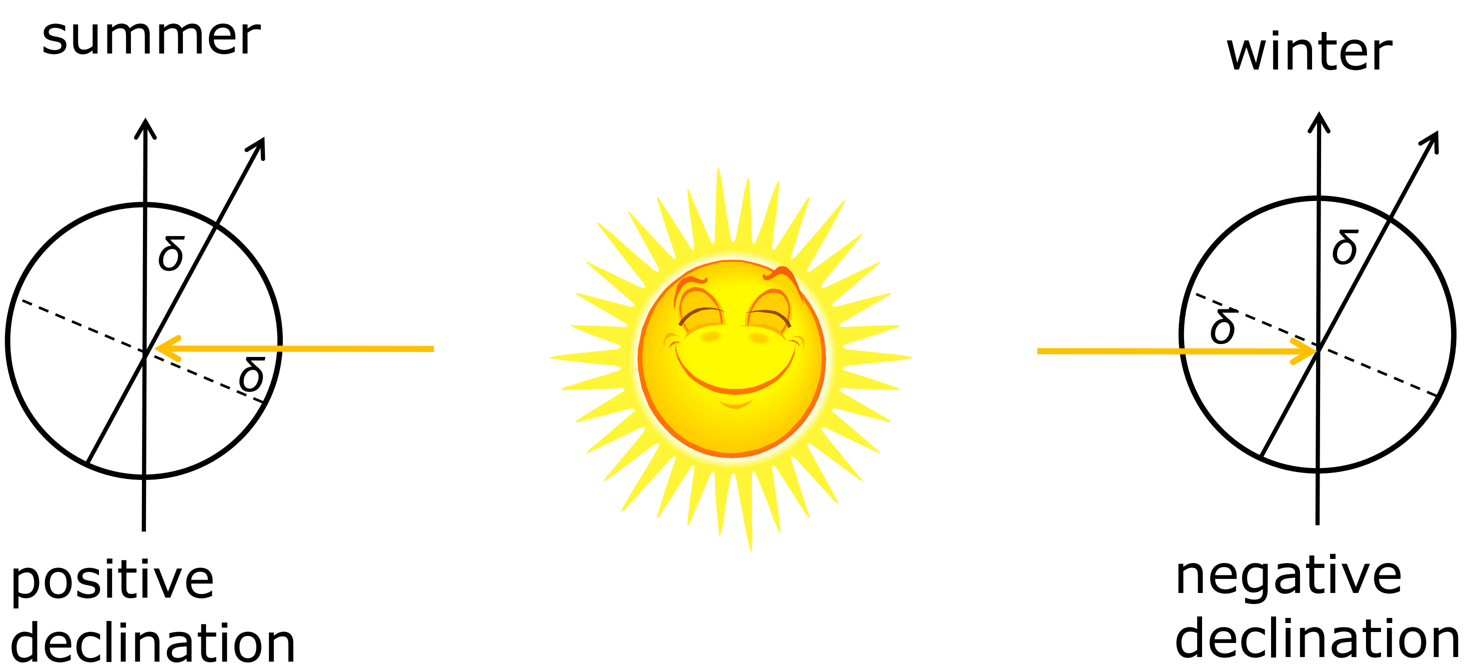
Effect of Declination Angle

- Earth is tilted on an axis, which causes seasons
- Axis is tilted at 23.5°
- Declination (δ): angular position of the sun at solar noon wrt the plane of the equator (degrees)



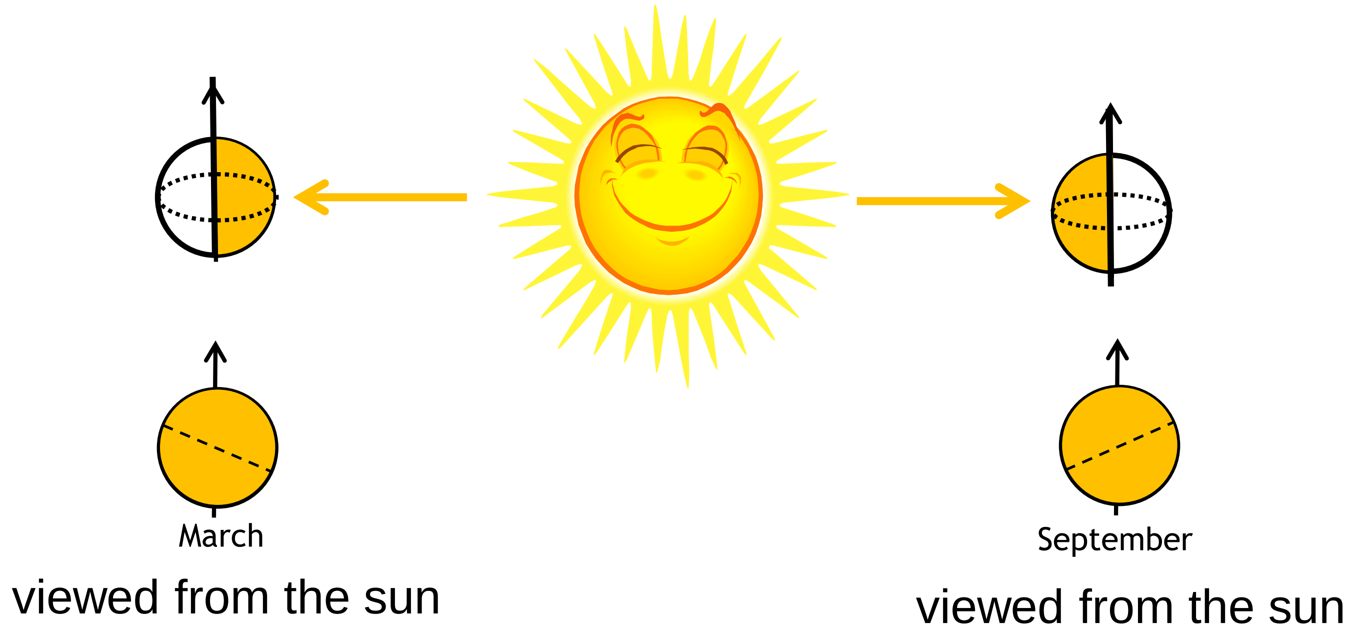
Effect of Declination Angle

For Northern Hemisphere



Effect of Declination Angle

- Declination angle is zero during the equinoxes



Effect of Declination Angle

- Declination is computed as:

$$\delta = \delta_0 \sin\left(\frac{360^\circ\left(284 + d\right)}{365}\right)$$

(where does the 284 come from?)

- where
  - $\delta_0 = 23.5^o$

Effect of Declination Angle

- Summer solstice: $\delta = \delta_0 = 23.5^o$
- Winter solstice: $\delta = -\delta_0 = -23.5^o$
- Spring equinox: $\delta = 0$
- Autumn equinox: $\delta = 0$

Effect of Declination Angle

- Northern Hemisphere: the axial tilt increases the daylight hours in March-September

- Southern Hemisphere: the axial tilt increases the daylight hours in the September-March

- More daylight hours means more daily insolation

Effect of Declination Angle

- Daylight on April 9th, 2012 at 13:57:25

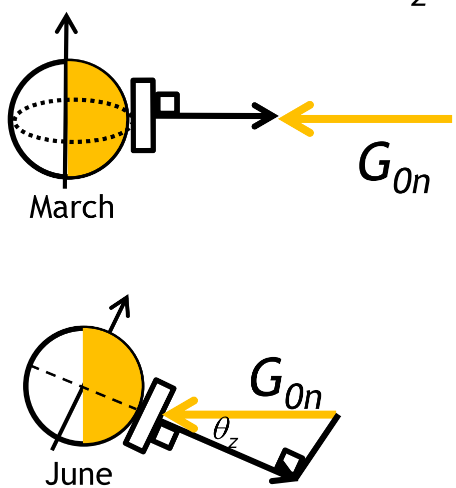
Source: time.gov

Effect of Declination Angle

- Daylight on January 17th, 2017 at 7:27:25

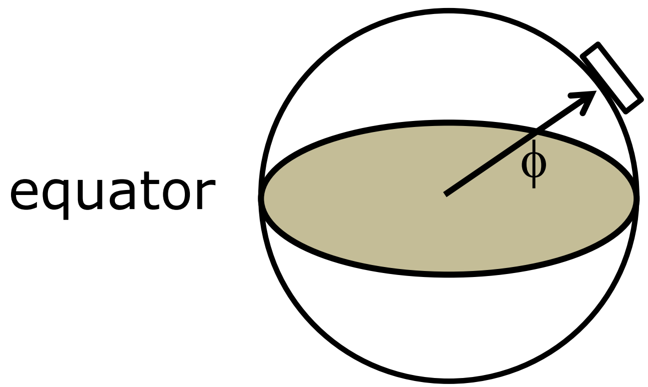
Effect of Declination Angle

- Declination affects zenith angle
- Assume solar noon (sun due south)
- Assume the surface is at the equator (latitude=0°)
  - spring and autumn equinox: $\theta_z = 0^o$
  - summer solstice: $\theta_z = 23.5^\circ$
  - winter solstice: $\theta_z = -23.5^\circ$



Effect of Latitude

- Let
  - ϕ: latitude of the surface (degrees)
    - Assume North is positive, South is negative
- $-90 \leq \phi \leq 90$

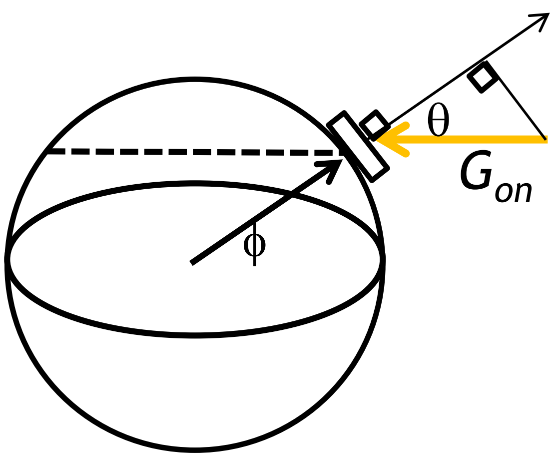


Effect of Latitude

- Assume:
  - declination = 0° (i.e. Spring/Autumn Equinox)
  - Sun directly due south (solar noon)
  - Horizontal surface is at latitude ϕ (degrees)
- It follows that $\theta = \theta_z = \phi$ and $G_0 = G_{0n}\cos(\phi)$

equator

Earth tilted out of the paper

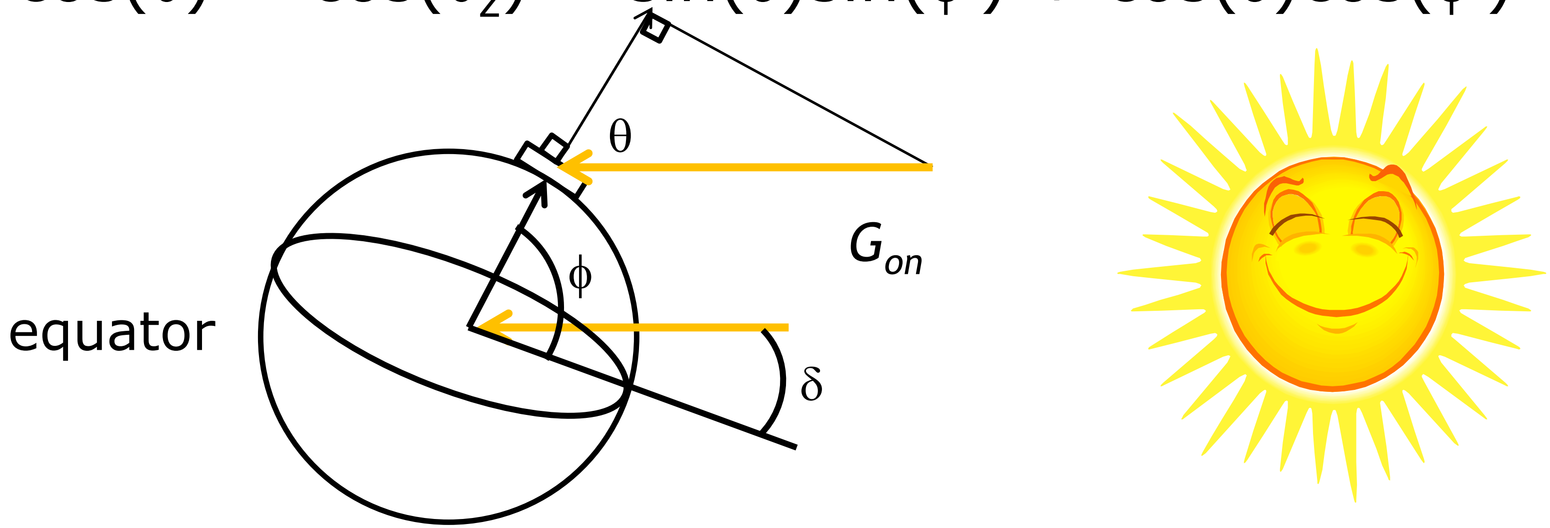


Effect of Latitude

- Combining the effects of declination and latitude
  - Assume solar noon (sun due south)
  - Assume the surface is horizontal ($\theta = \theta_z$)
- Using trigonometry:

$$\theta = \theta_z = \phi - \delta$$

$$\cos(\theta) = \cos(\theta_z) = \sin(\delta)\sin(\phi) + \cos(\delta)\cos(\phi)$$



Example

- What is the irradiance for a horizontal surface at the top of the atmosphere (extraterrestrial) above Seattle, Washington (latitude 47.6^0) on January 23 at solar noon? Account for intra-year irradiance variation.

Example

- What is the irradiance for a horizontal surface at the top of the atmosphere (extraterrestrial) above Seattle, Washington (latitude 47.6^0) on January 23 at solar noon?

$$d = 23$$

$$G_{on}(d) = G_{sc}\left[1 + 0.034\cos\left(2\pi\left(\frac{d}{365}\right)\right)\right] = 1408.6$$

$$\delta = \delta_0 \sin\left(\frac{360^\circ(284 + d)}{365}\right) = 23.5^\circ \sin\left(\frac{360^\circ(284 + 23)}{365}\right) = -19.75^\circ$$

Example

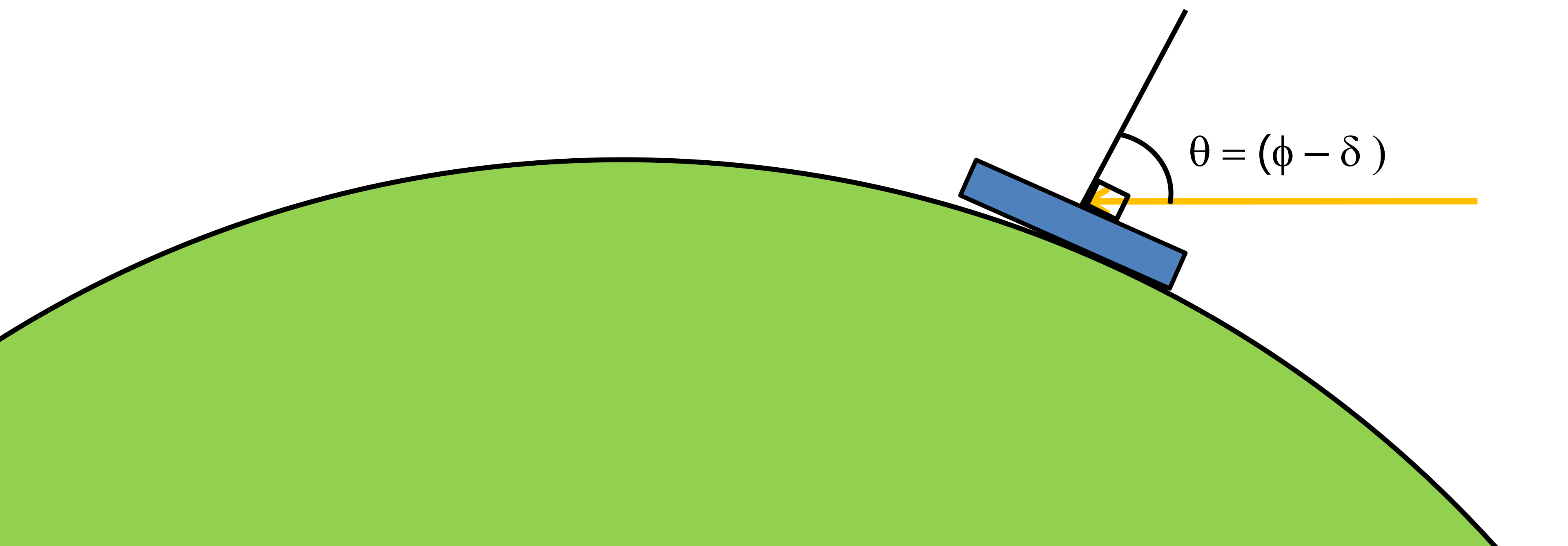
- What is the irradiance for a horizontal surface at the top of the atmosphere (extraterrestrial) above Seattle, Washington (latitude 47.6^0) on January 23 at solar noon?

$$\theta_z = 47.6 - -19.75 = 67.4^\circ$$

$$G = G_{0n} \cos\theta_z \Rightarrow 1408.6 \times 0.385 = 542 \, {}^{W}\!/_{m^2}$$

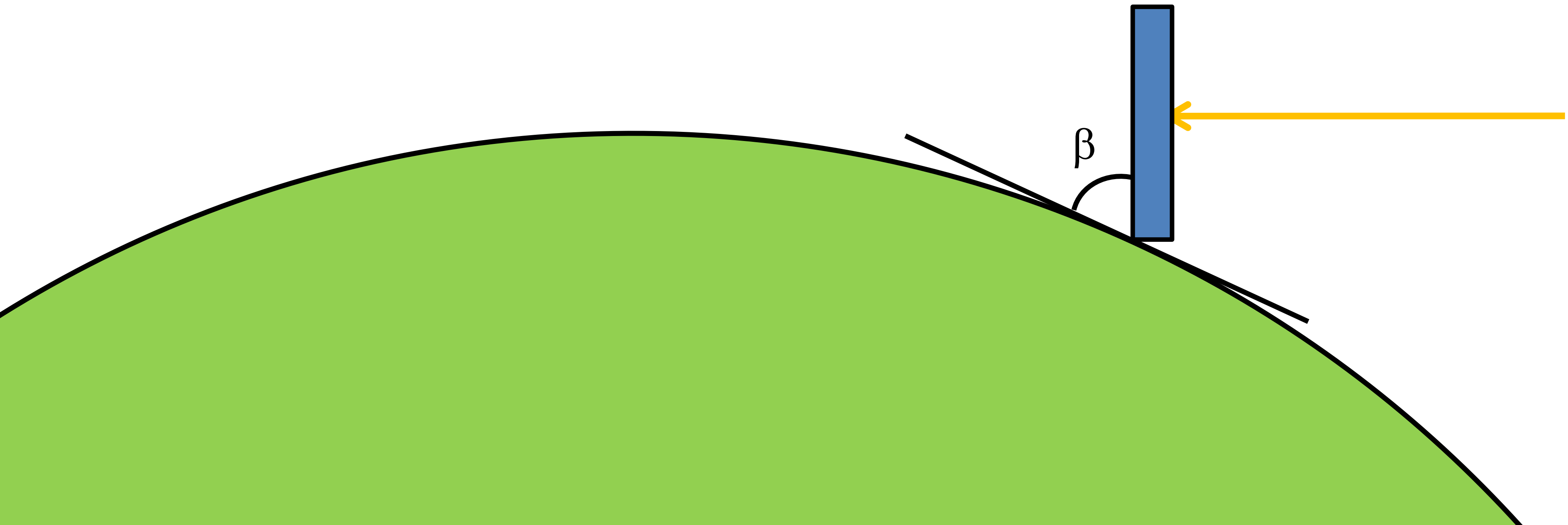
Effect of Declination Angle

- At large values of $(\phi - \delta)$, the angle of incidence is large (cosine effect is significant)
- How can we compensate for this?



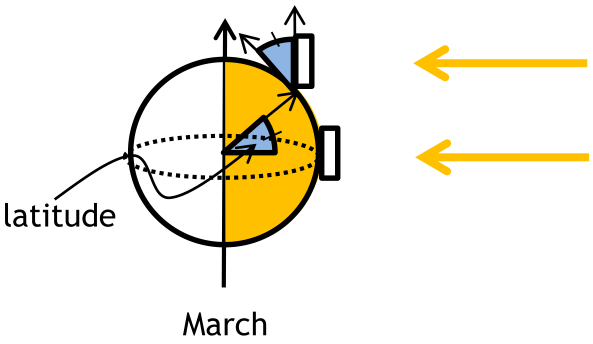
Surface Orientation

- Tilt the surface
- Want the surface to be normal to the irradiance
  - $\beta = (\phi - \delta)$ (Northern Hemisphere)
  - Want angle of incidence to be zero



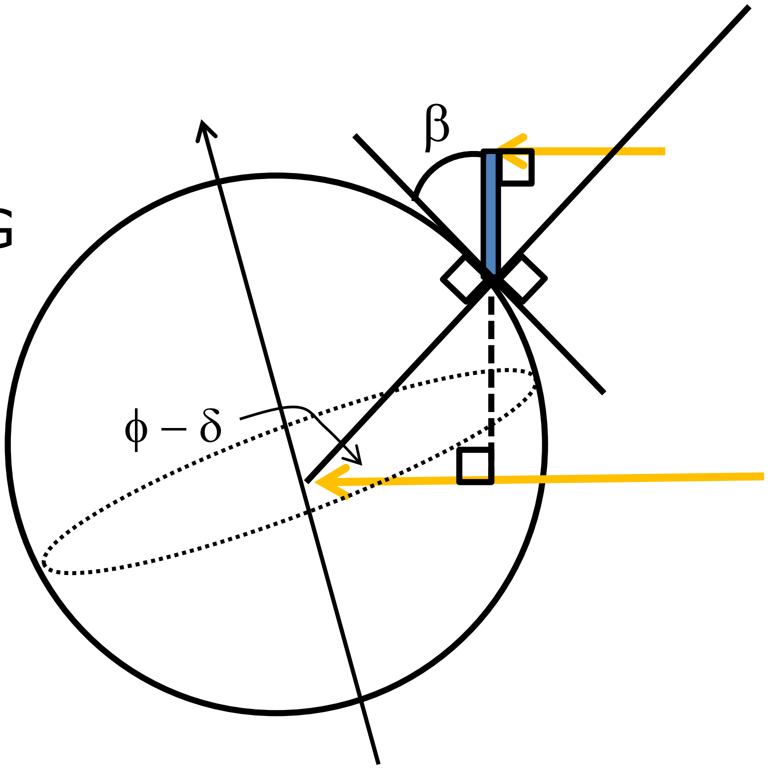
Surface Orientation

- Tilt should equal latitude during equinox
- As δ increases, less tilt needed
  - At solar noon: $\cos(\theta) = \cos(\phi-\delta-\beta)$
- In the southern hemisphere:
  - $\cos(\theta) = \cos(-\phi+\delta-\beta)$



Surface Orientation

Surface is normal to G
when $\beta = \phi - \delta$



Surface Orientation

- General rule of thumb: tilt a PV panel at the latitude
  - Normal to irradiance on equinoxes
  - Too much tilt in summer
  - Too little tilt in winter

Where in the world are these PV panels?

Ellensburg

Singapore

Snohomish

Surface Orientation

- $\cos(\theta) = \cos(\phi-\delta-\beta)$
  - Note: $\cos(w+z) = \cos(w)\cos(z) - \sin(w)\sin(z)$
  - Note: $\sin(w+z) = \sin(w)\cos(z)+\cos(w)\sin(z)$

- $\cos(\phi-\delta-\beta)= \cos(\theta + x)$ [set $x =-\delta-\beta$]

- $\cos(\phi + x)= \cos(\phi)\cos(x) - \sin(\phi)\sin(x)$

Surface Orientation

$\cos(\phi - \delta - \beta) = \cos(\phi)\cos(x) - \sin(\phi)\sin(x)$

[back substituting for the first $x = -\delta - \beta$]

$= \cos(\phi)\cos(-\delta - \beta) - \sin(\phi)\sin(x)$

[Now use $\cos(w + z) = \cos(w)\cos(z) - \sin(w)\sin(z)$]

$= \cos(\phi)[\cos(-\delta)\cos(-\beta) - \sin(-\delta)\sin(-\beta)] - \sin(\phi)\sin(x)$

[using $\cos(-u) = \cos(u)$ and $\sin(-u) = -\sin(u)$]

$= \cos(\phi)[\cos(\delta)\cos(\beta) - \sin(\delta)\sin(\beta)] - \sin(\phi)\sin(x)$

Surface Orientation

$= \cos(\phi)[\cos(\delta)\cos(\beta) - \sin(\delta)\sin(\beta)] - \sin(\phi)\sin(x)$

[back substituting for the remaining $x = -\delta - \beta$]

$= \cos(\phi)[\cos(\delta)\cos(\beta) - \sin(\delta)\sin(\beta)] - \sin(\phi)\sin(-\delta - \beta)$

[Using $\sin(w+z) = \sin(w)\cos(z) + \cos(w)\sin(z)$]

$= \cos(\phi)[\cos(\delta)\cos(\beta) - \sin(\delta)\sin(\beta)]$

$\quad - \sin(\phi)[\sin(-\beta)\cos(-\delta) + \cos(-\beta)\sin(-\delta)]$

Surface Orientation

$=\cos(\phi)[\cos(\delta)\cos(\beta) - \sin(\delta)\sin(\beta)]$

$\quad - \sin(\phi)[\sin(-\beta)\cos(-\delta)+\cos(-\beta)\sin(-\delta)]$

[multiplying out]

$=\cos(\phi)\cos(\delta)\cos(\beta) - \cos(\phi)\sin(\delta)\sin(\beta)$

$\quad - \sin(\phi)\sin(-\beta)\cos(-\delta) - \sin(\phi)\cos(-\beta)\sin(-\delta)$

[using $\cos(-u) = \cos(u)$ and $\sin(-u) = -\sin(u)$]

$\cos(\theta)=\cos(\phi)\cos(\delta)\cos(\beta) - \cos(\phi)\sin(\delta)\sin(\beta)$

$\quad + \sin(\phi)\sin(\beta)\cos(\delta) + \sin(\phi)\cos(\beta)\sin(\delta)$

Surface Orientation

- Extraterrestrial irradiance accounting for the tilt, latitude and declination of a surface at solar noon:

$$
\begin{aligned}
G_{0T} = G_{0n}\cos(\theta) &= G_{0n}\cos(\phi-\delta-\beta) \\
&= G_{0n}[\cos(\phi)\cos(\delta)\cos(\beta) \\
&\quad - \cos(\phi)\sin(\delta)\sin(\beta) \\
&\quad + \sin(\phi)\sin(\beta)\cos(\delta) \\
&\quad + \sin(\phi)\cos(\beta)\sin(\delta)]
\end{aligned}
$$

Important result

Surface Orientation

- Compute the extraterrestrial irradiance on a vertical surface above 30° N on April 15 at solar noon.
  - Hint: April 15 is the 105th day of the year

Surface Orientation

- Compute the extraterrestrial irradiance on a vertical surface above 30° N on April 15 at solar noon.
  - Hint: April 15 is the 105th day of the year

$$\phi = 30^\circ$$

$$\beta = 90^\circ$$

Surface Orientation

- Compute the extraterrestrial irradiance on a vertical surface above 30° N on April 15 at solar noon.
  - Hint: April 15 is the 105th day of the year

$$\phi = 30^\circ$$

$$\beta = 90^\circ$$

$$G_{on}(d) = G_{sc}\left[1 + 0.033\cos\left(2\pi\left(\frac{105}{365}\right)\right)\right] = 1356.4 \text{ W/m}^2$$

$$\delta = \delta_0 \sin\left(\frac{360^\circ(284+d)}{365}\right) = 23.5^\circ \sin\left(\frac{360^\circ(284+105)}{365}\right) = 9.4^\circ$$

Surface Orientation

$$G_{0T} = G_{0n}[\cos(\phi)\cos(\delta)\cos(\beta) - \cos(\phi)\sin(\delta)\sin(\beta) + \sin(\phi)\sin(\beta)\cos(\delta) + \sin(\phi)\cos(\beta)\sin(\delta)] = 476\ \mathrm{W/m^2}$$

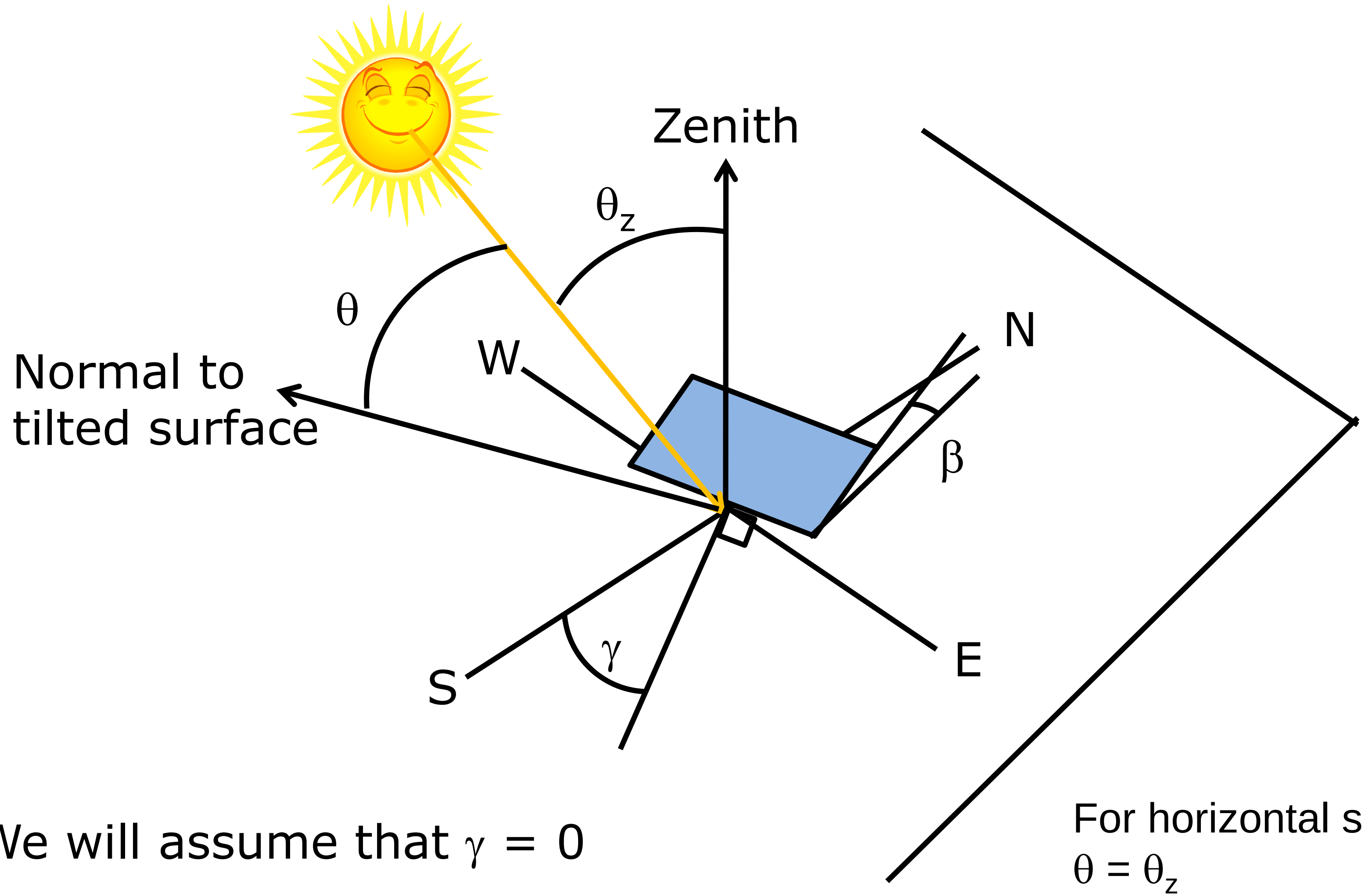
or

- $G_{0T} = G_{0n}\cos(\phi-\delta-\beta) = 476\ \mathrm{W/m^2}$

Effect of Hour Angle

- We want to relate this angle to time
- How many degrees does the Earth rotate each hour?

$$\frac{360^\circ}{24} = 15^\circ$$



We will assume that $\gamma = 0$

For horizontal surfaces:
$\theta = \theta_z$

Effect of Hour Angle

- We define the hour angle, ω, as:

  $$\omega = 15^\circ (h - 12) - (\lambda - \lambda_{zone})$$

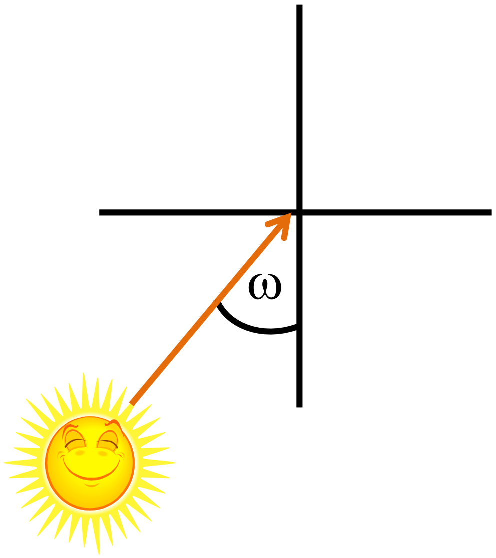
  - h local civil time (hours)
  - λ longitude (degrees)
  - λ_{zone} longitude of the meridian defining the local time (degrees)
- ω: angle that the Earth has rotated since solar noon

Effect of Hour Angle

- UTC (Coordinated Universal Time) is defined at 0^o longitude
- Seattle is 8 hours behind UTC during standard time
  - λ_{zone} is then $8 \times 15^o = 120^o$ W
- During Day Light Savings Time (roughly March – Nov) we are 7 hours behind UTC
  - λ_{zone} is then $7 \times 15^o = 105^o$ W
- For a more accurate calculation use the Equation of Time
- We will assume that solar time = civil time
  - $(\lambda - \lambda_{zone} = 0)$

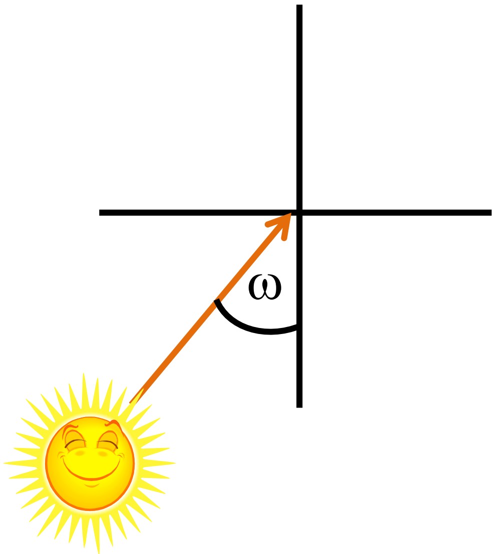
Effect of Hour Angle

- Hour Angle is:
  - negative in the morning (before solar noon)
  - positive in the evening (after solar noon)



Effect of Hour Angle

- If $\phi = \delta = 0$ and $\beta = 0$, then
  - $\cos(\theta) = \cos(\omega)$



Angle of Incidence

- Derivation of the angle of incidence is more difficult, so the result is provided
  - $\cos(\theta) = \sin(\delta)\sin(\phi)\cos(\beta)$

    $-\sin(\delta)\cos(\phi)\sin(\beta)$

    $+\cos(\delta)\cos(\phi)\cos(\beta)\cos(\omega)$

    $+\cos(\delta)\sin(\phi)\sin(\beta)\cos(\omega)$

Important result

Simplifications

- If $\beta = 0$ (no tilt), then $\theta_z = \theta$ and
  - $\cos(\theta) = \sin(\delta)\sin(\phi)+\cos(\delta)\cos(\phi)\cos(\omega)$

- For surfaces tilted at their latitude
  - $\cos(\theta) = \cos(\delta)\cos(\omega)$

- For surfaces at solar noon
  - $\cos(\theta) = \cos(\phi-\delta-\beta)$

Angle of Incidence

- Note: $\cos(\theta)$ must be greater than or equal to 0, otherwise the sun is shining on the rear of the surface (set the value to 0)
- Note: angle of incidence equations do not account for the Earth blocking the sun's irradiance
  - Try: ω =180, β = 90, ϕ =0 and d=1 (sunny at midnight!)
- Only use the angle of incidence for daylight hours

Astronomy Trivia

- How many hours of daylight are there in Seattle during the spring equinox?
  A. 6
  B. 10
  C. 12
  D. 14
  E. 16
  F. 18

Astronomy Trivia

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  A. 6
  B. 10
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  D. 14
  E. 16
  F. 18

Hours of Day Light

- Daylight hours vary depending on latitude and declination
- For a horizontal surface the sun sets ($G = 0$) when $\theta = 90^o$
- Find ω such that:
  - $\cos(\theta) = \sin(\delta)\sin(\phi)+\cos(\delta)\cos(\phi)\cos(\omega) = 0$
- Solving yields:
  - $\cos(\omega_s) = -\tan(\delta)\tan(\phi)$
  - ω_s: sunset angle

Hours of Day Light

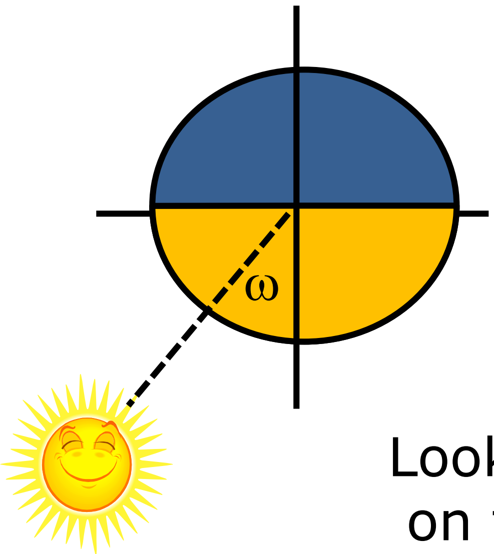
- Since every 15^0 is one hour:
  - Hours of daylight is:

$$N = \frac{2}{15}\cos^{-1}\left(-\left(\tan\delta\right)\times\left(\tan\phi\right)\right)$$

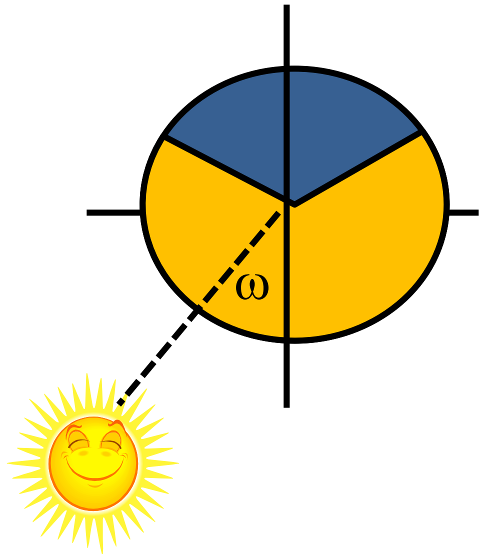
Effect of Hour Angle

- Visualization

During Equinox,
Sunrise at -90^o
Sunset 90^o



In Summer:
Sunrise $<-90^o$
Sunset $>90^o$



Looking down
on the North pole

Hours of Sunlight on a Surface

- If a surface is tilted, it may receive fewer hours of sunlight than the number of daylight hours
- To compute the sunset hour angle for a titled panel, set $\cos(\theta) = 0$ and solve for ω

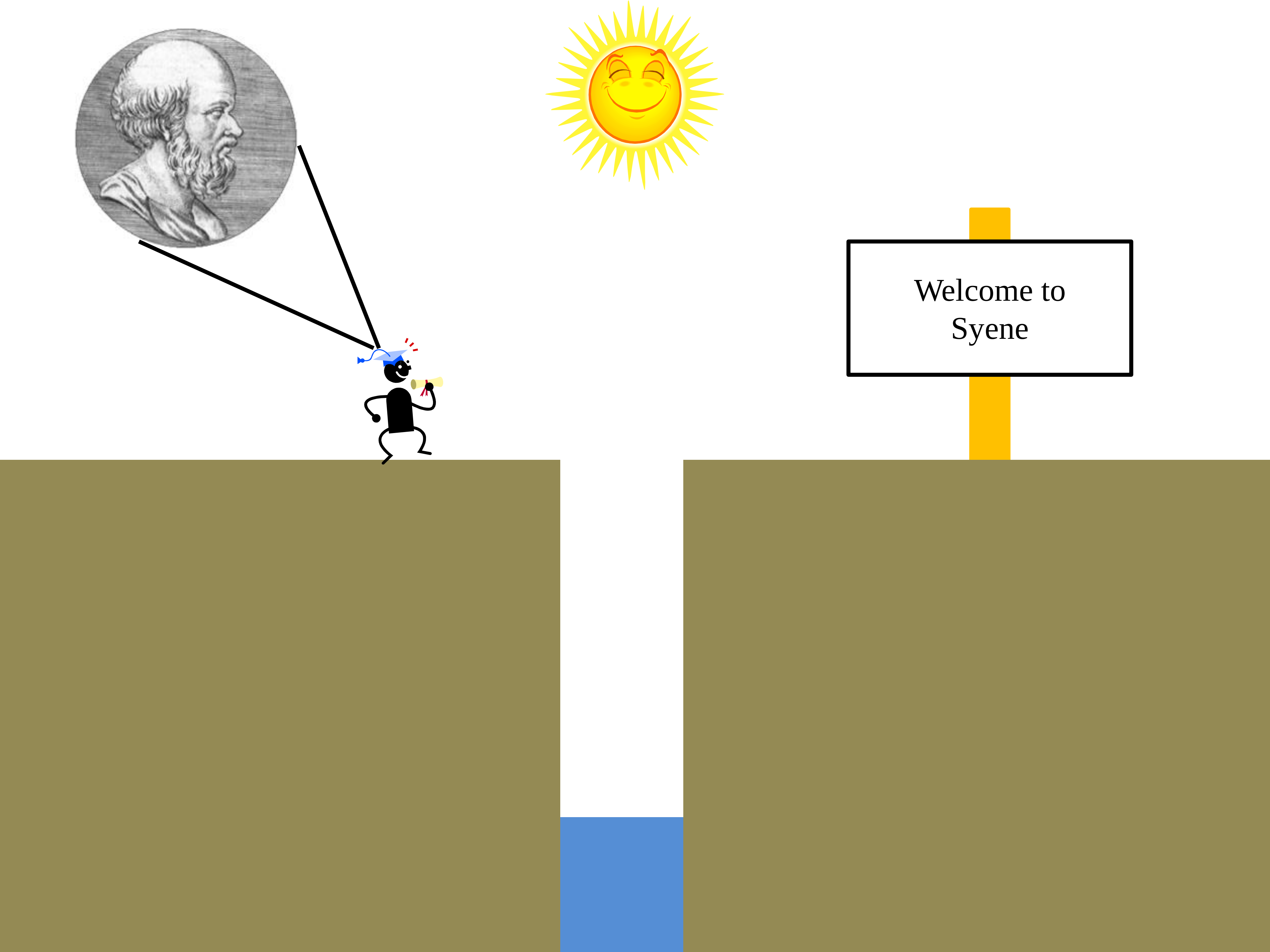
$$\begin{aligned}\cos(\theta) = 0 = &\sin(\delta)\sin(\phi)\cos(\beta) \\ &-\sin(\delta)\cos(\phi)\sin(\beta) \\ &+\cos(\delta)\cos(\phi)\cos(\beta)\cos(\omega) \\ &+\cos(\delta)\sin(\phi)\sin(\beta)\cos(\omega)\end{aligned}$$

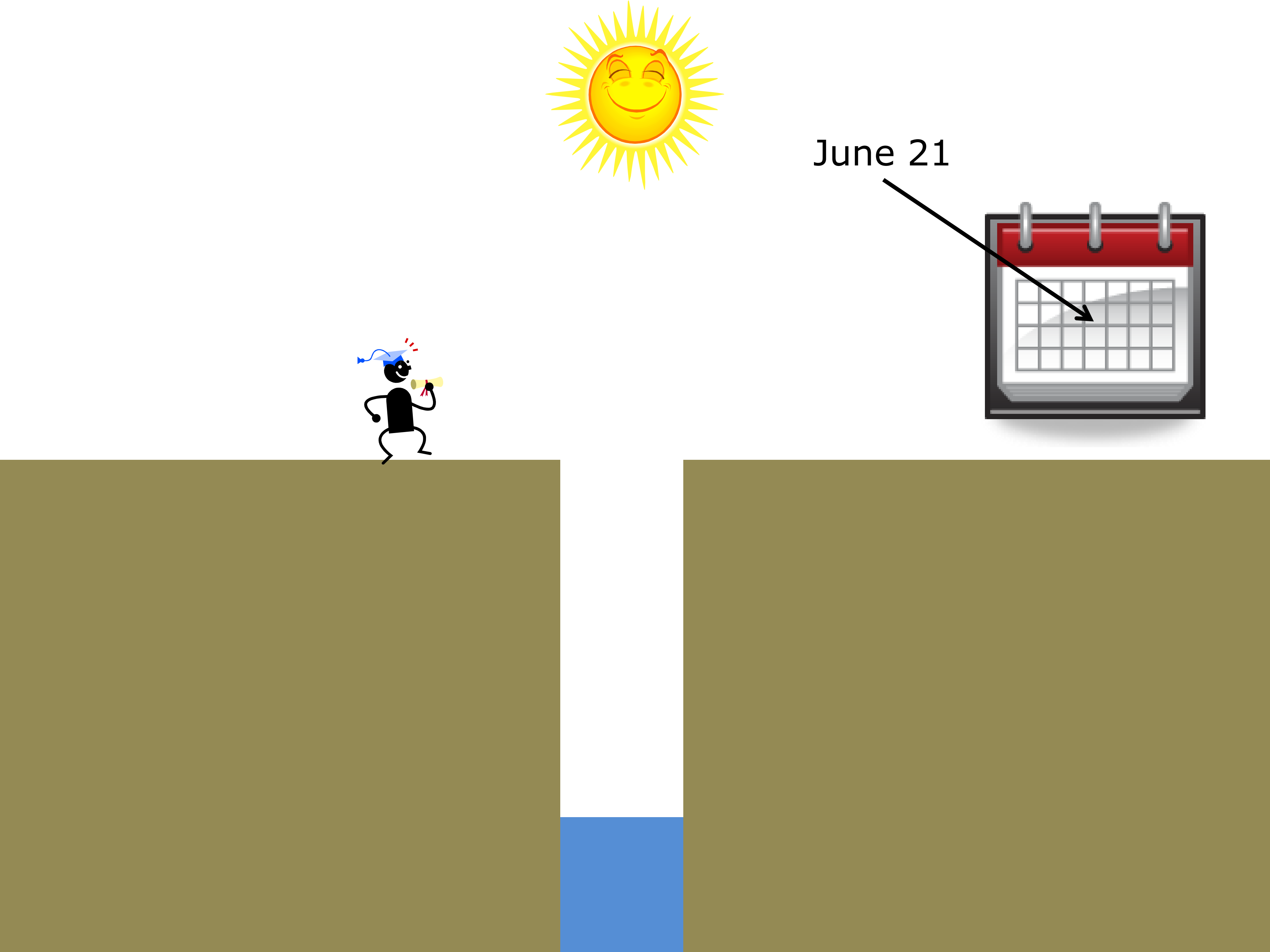
Hours of Sunlight on a Surface

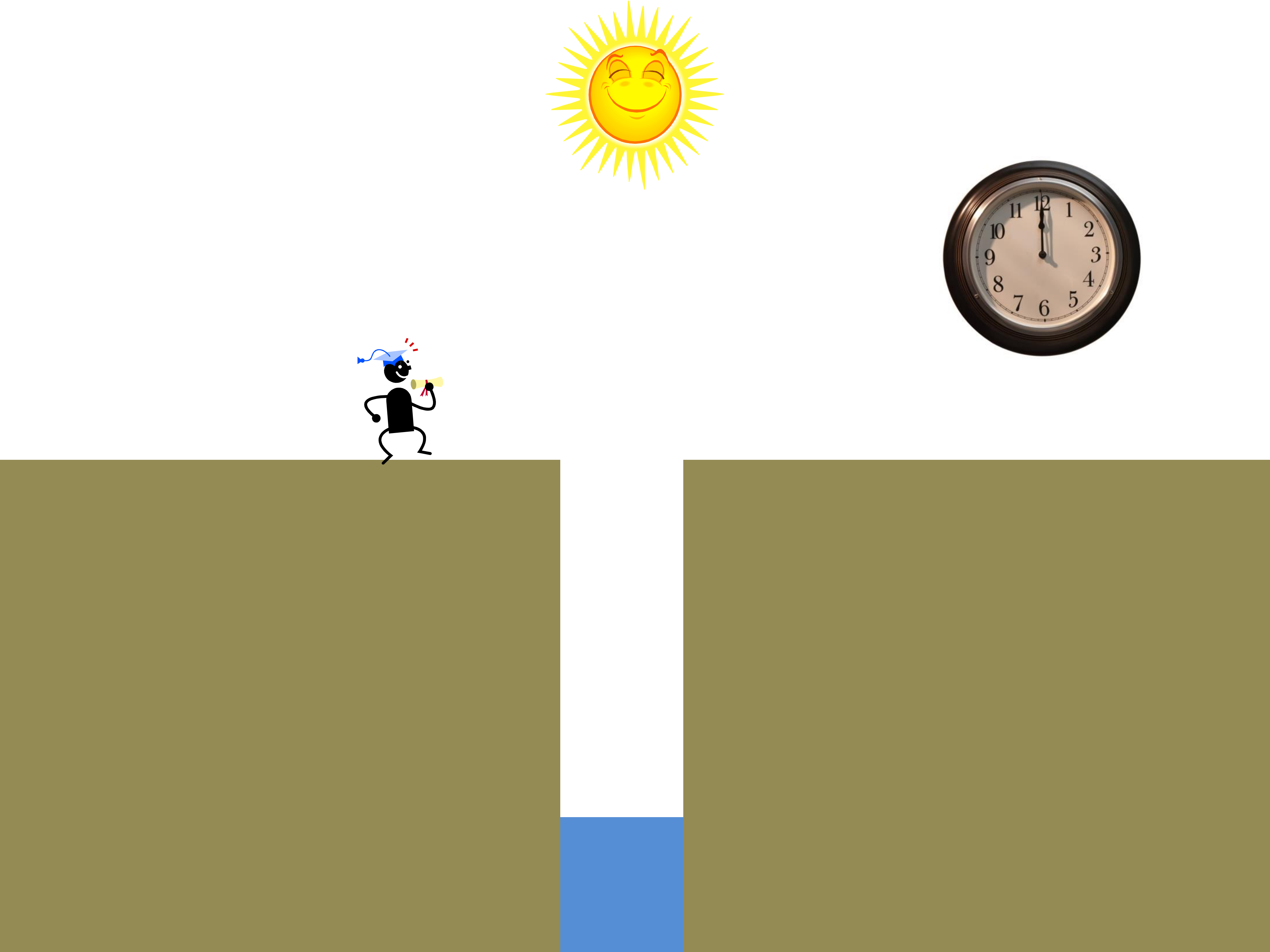
- Note: by definition, the sun stops/starts shining on a surface whenever $\cos(\theta) = 0$
- Whenever $\cos(\theta) < 0$, the sun is not shining on the panel (but the sun perhaps has not yet set)
- For example, the sun will stop shining on a vertical surface facing south in the summer before the sun sets (i.e. the sun is "behind" the face of the surface)

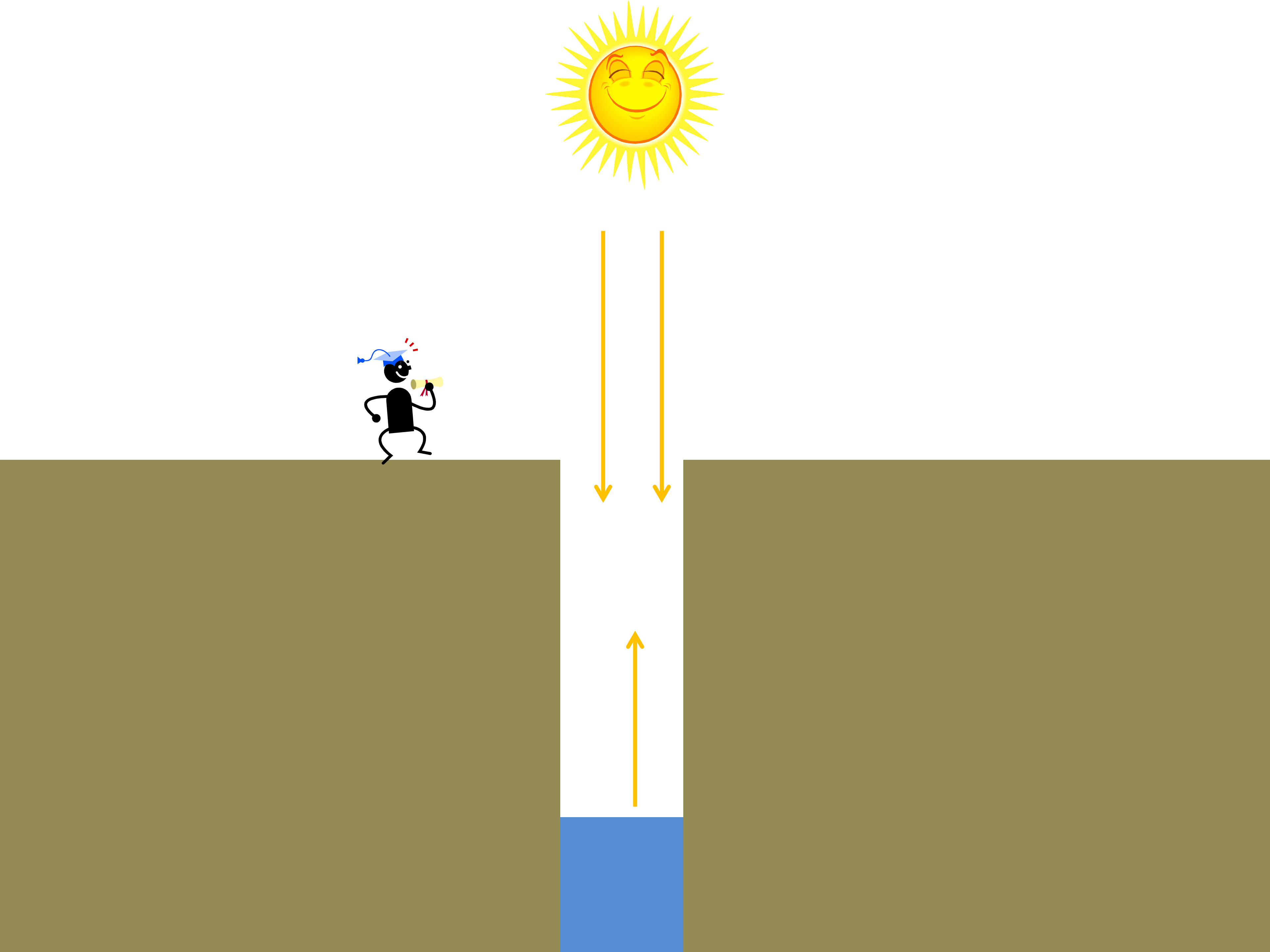
Side Note

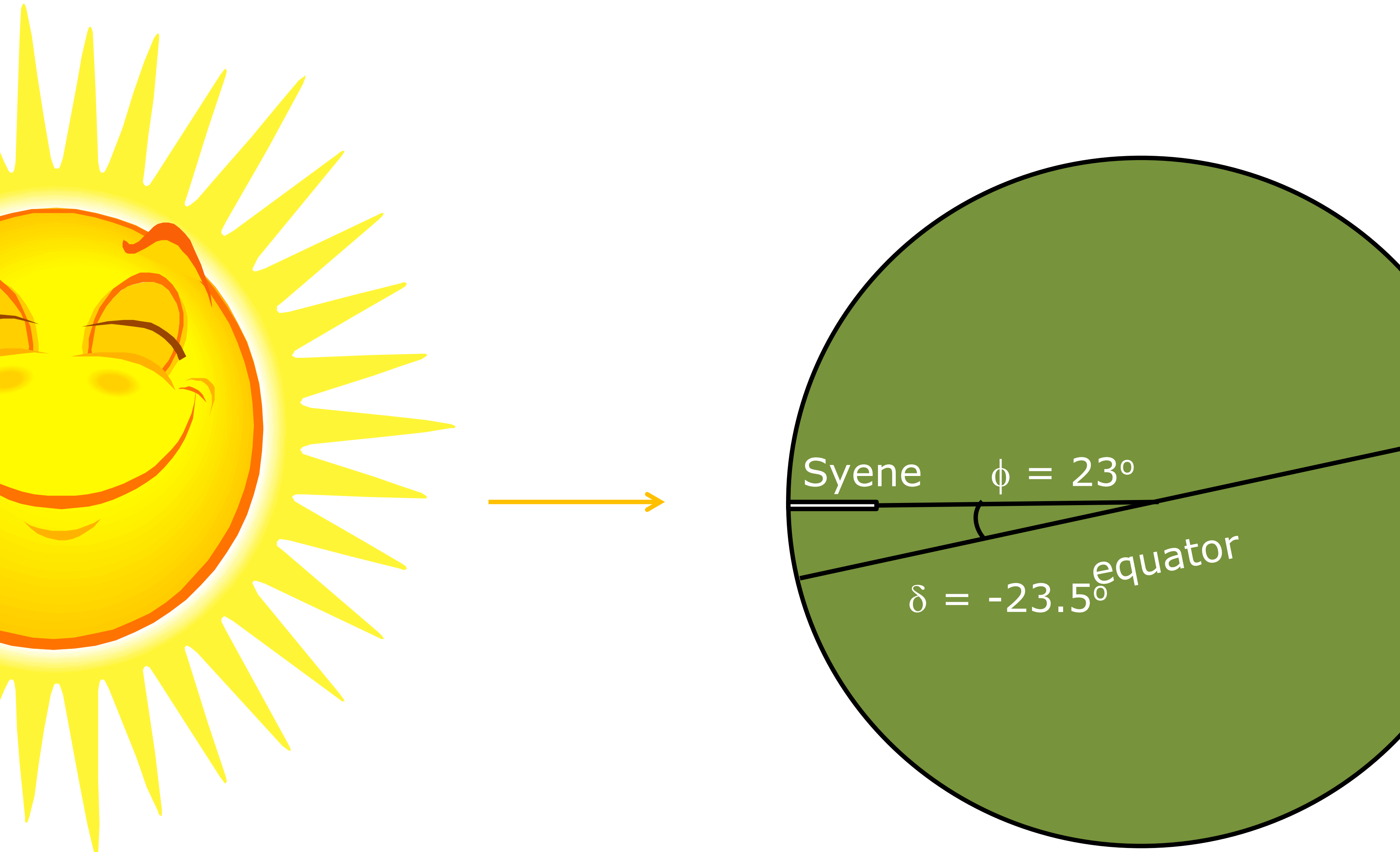
How did Eratosthenes estimate the circumference in the third century BCE?

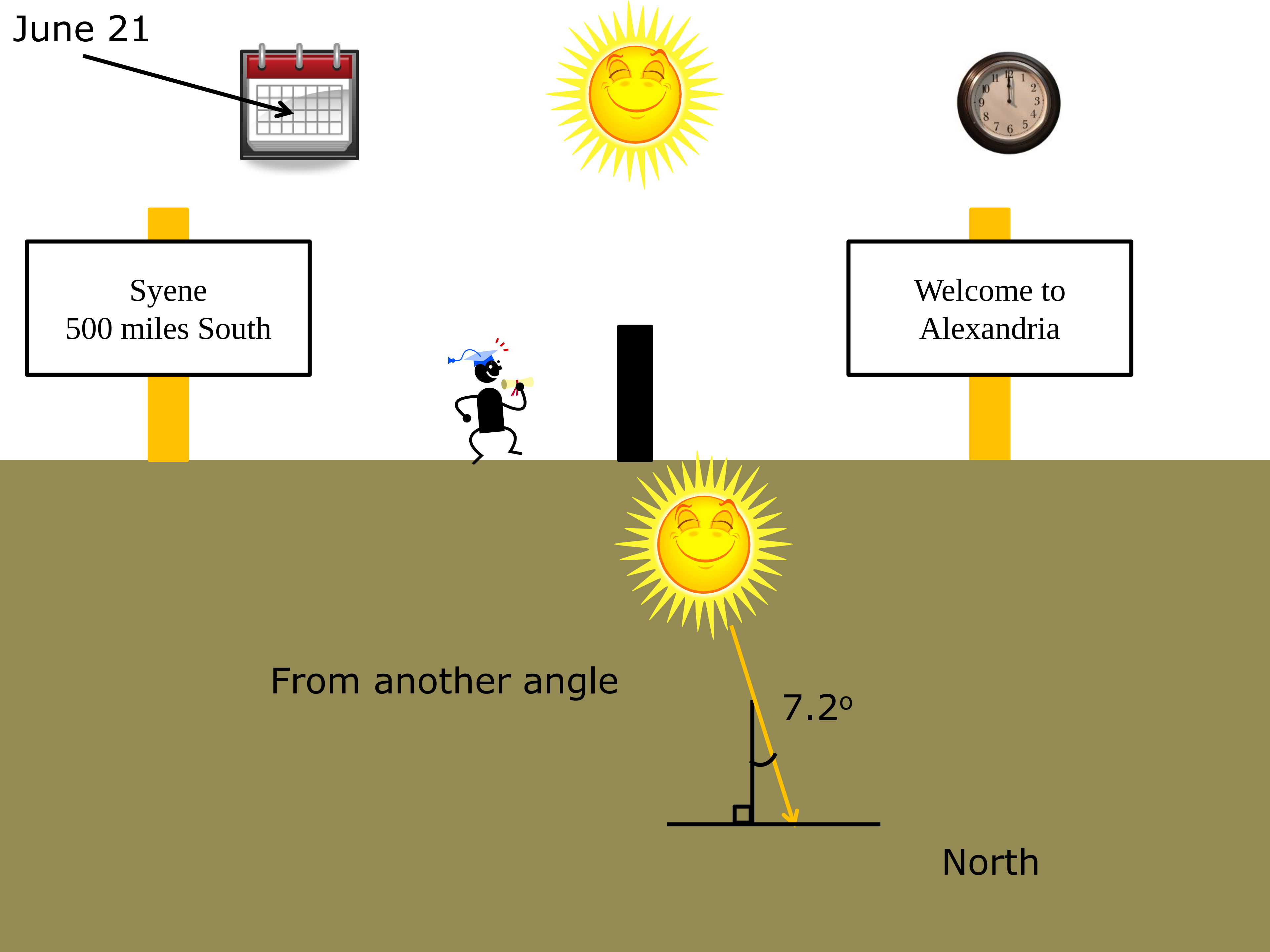

Welcome to
Syene

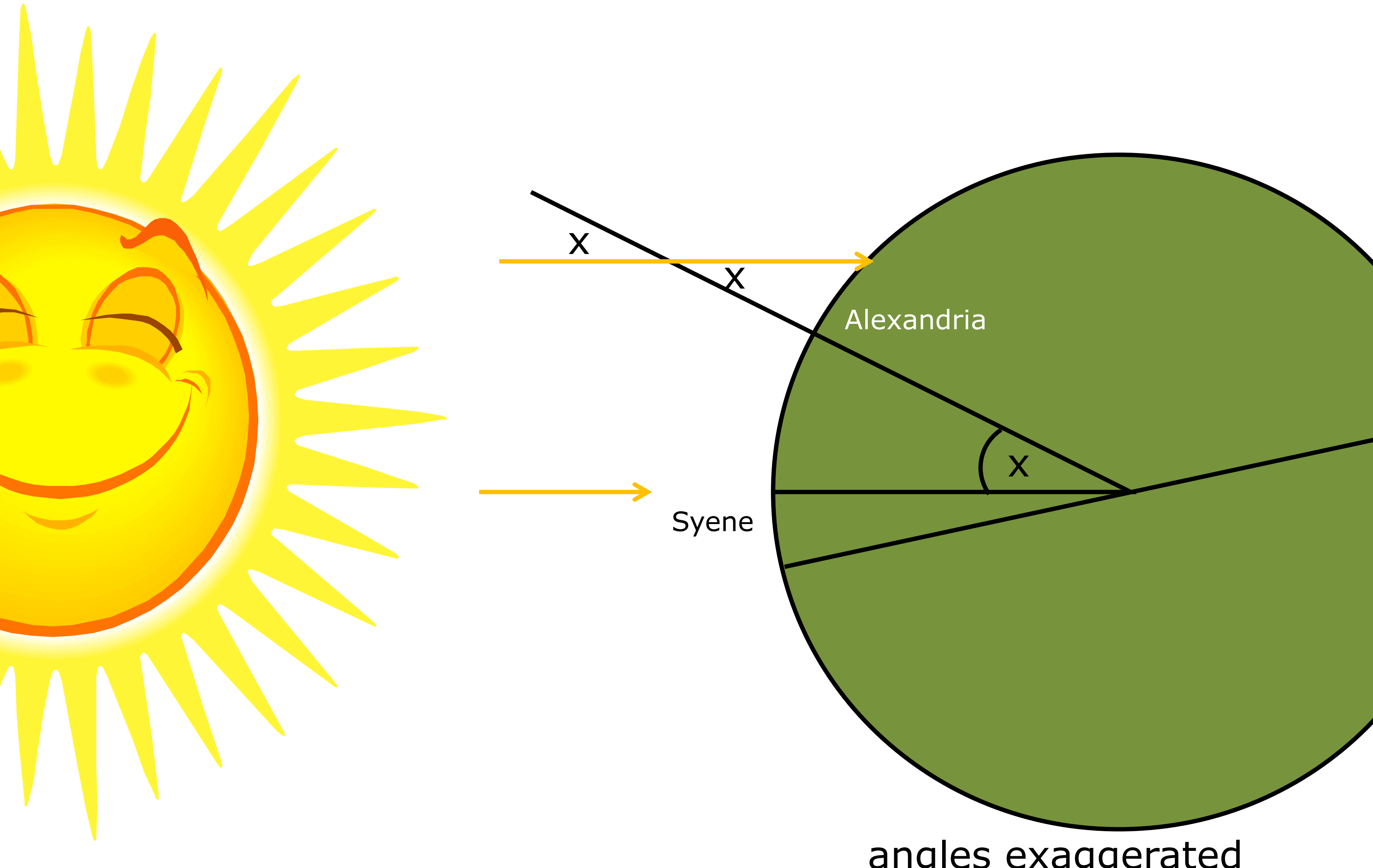

June 21






Syene
ϕ = 23°
equator
δ = -23.5°


June 21
Syene
500 miles South
Welcome to
Alexandria
From another angle
7.2°
North


x
x
Alexandria
x
Syene
angles exaggerated

- Therefore, Syene and Alexandria are 7.2° of latitude apart
  - Syene: 24° N, 33° E
  - Alexandria: 31° N, 30° E
- Distance between Syene and Alexandria: 500 miles
- (7.2/360)C = 500 miles
  - => C = 25,000
  - Actual circumference: ~24,900 miles