06-Solar Resource Part 3

ECEGR 4530
Renewable Energy Systems

Overview

- Effect of the Atmosphere
- Clearness Index
- Irradiation
- Irradiance Algorithm
- Air Mass Ratio

What Influences Angle of Incidence?

- Declination (δ)
- Latitude (ϕ)
- Tilt (β)
- Time of day (hour angle) (ω)

What Influences Angle of Incidence?

- See Lecture 05-Solar Resource Part 2 for derivation

What Influences Angle of Incidence?

Extraterrestrial irradiance accounting for the tilt, latitude and declination of a surface at solar noon:

$$G_{0T} = G_{0n}\cos(\theta) = G_{0n}\cos(\phi-\delta-\beta)$$

$$= G_{0n}[\cos(\phi)\cos(\delta)\cos(\beta) - \cos(\phi)\sin(\delta)\sin(\beta) + \sin(\phi)\sin(\beta)\cos(\delta) + \sin(\phi)\cos(\beta)\sin(\delta)]$$

Important result

What Influences Angle of Incidence?

- Now also accounting for time of day
  - $\cos(\theta) = \sin(\delta)\sin(\phi)\cos(\beta)$

    $-\sin(\delta)\cos(\phi)\sin(\beta)$

    $+\cos(\delta)\cos(\phi)\cos(\beta)\cos(\omega)$

    $+\cos(\delta)\sin(\phi)\sin(\beta)\cos(\omega)$

Important result

Simplifications

- If $\beta = 0$ (no tilt), then $\theta_z = \theta$ and
  - $\cos(\theta) = \sin(\delta)\sin(\phi) + \cos(\delta)\cos(\phi)\cos(\omega)$

- For surfaces tilted at their latitude
  - $\cos(\theta) = \cos(\delta)\cos(\omega)$

- For surfaces at solar noon
  - $\cos(\theta) = \cos(\phi - \delta - \beta)$

What Influences Angle of Incidence?

- Try to maximize $\cos(\theta)$

$$G_{0T} = G_{0n}\cos(\theta) = G_{0n}\cos(\phi-\delta-\beta)$$

$$= G_{0n}[\cos(\phi)\cos(\delta)\cos(\beta) - \cos(\phi)\sin(\delta)\sin(\beta) + \sin(\phi)\sin(\beta)\cos(\delta) + \sin(\phi)\cos(\beta)\sin(\delta)]$$

Adjust tilt to minimize $\phi-\delta-\beta$

What Influences Angle of Incidence?

- Try to maximize $\cos(\theta)$

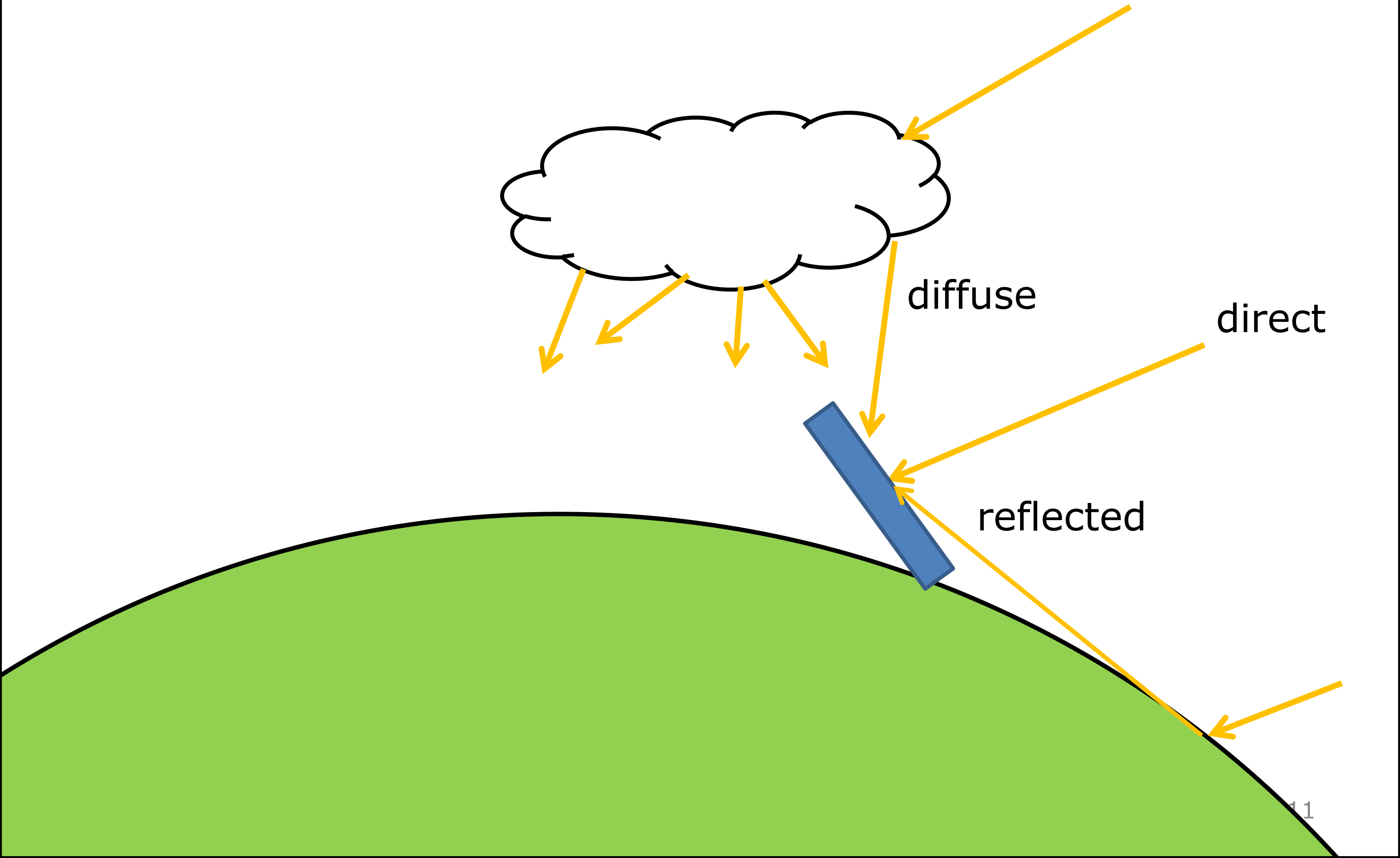
$$\cos(\theta) = \sin(\delta)\sin(\phi)\cos(\beta) - \sin(\delta)\cos(\phi)\sin(\beta) + \cos(\delta)\cos(\phi)\cos(\beta)\cos(\omega) + \cos(\delta)\sin(\phi)\sin(\beta)\cos(\omega)$$

Track sun east-west to minimize ω

Introduction

- Previous lectures have focused on extraterrestrial irradiance
- Now we examine irradiance on Earth's surface
  - Resolve GHI into beam and diffuse components
- Irradiance on a surface has 3 components:
  - A: Direct
  - B: Diffuse (from the sky)
  - C: Reflected (reflected from the ground)
  - $G_T = A + B + C$
- Empirical formulae

Components of Irradiance



Atmospheric Effects

- Atmosphere: reflects, absorbs and scatters solar radiation
- Net result: reduction in irradiance at the Earth's surface and a non-uniform attenuation of the energy density spectrum
- On average, 30% of incident solar irradiance is reflected back into space

Atmospheric Effects

- Recall that the atmosphere also causes diffusion of irradiance
  - We have neglected G_d in all previous derivations
- G_d is difficult to compute, due to the complex geometry of clouds, etc
- Use empirical formula to determine the ratio of G_d and G_b for a given GHI and G_0

Clearness Index

- Basic idea: use ratio of irradiance on surface to irradiance at the top of the atmosphere as a proxy for how cloudy it is.
  - Clouds imply higher G_d component of G_{GHI}

- clear day: G_{GHI}/G_0 is closer to 1
  - Smaller G_d component of G_{GHI}
- cloudy day: G_{GHI}/G_0 is closer to 0
  - Larger G_d component of G_{GHI}

Clearness Index

- Clearness Index k_t : ratio of global irradiance received on a horizontal surface to the extraterrestrial irradiance (G_{GHI}/G_0)
  - Recall: G_o is the irradiance on a surface (with the same angle of incidence as the horizontal surface) without the atmosphere accounted for (extraterrestrial)
  - Average k_t values are available for many locations
- Usually, clearness index uses the ratio of radiations over an hour, day or month

Computing Diffuse Irradiance

- G_o can be computed (see previous lectures)
- Need to know either G_{GHI} or k_t
- A simple empirical formula is
  - $G_d/G_{GHI} = 1-1.13k_t$
  - Reasonably valid for $0.3 < k_t < 0.8$
- G_b can then be computed from
  - $G_b = G_{GHI} - G_d$

Computing Beam Irradiance

- Find G_b and G_d if $G_{GHI} = 800\ W/m^2$ and $G_o = 1350\ W/m^2$

Computing Beam Irradiance

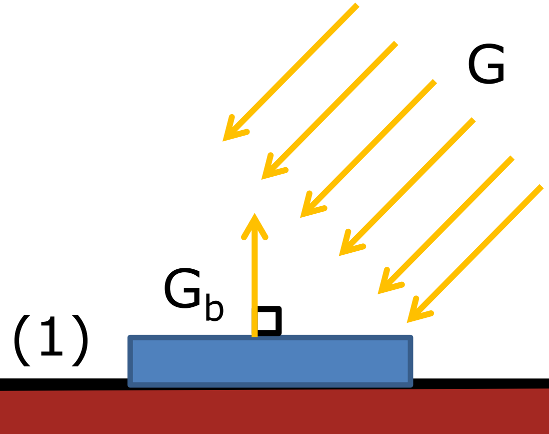
- Find G_b, G_d if $G_{GHI} = 800\ W/m^2$ and $G_o = 1350\ W/m^2$
  - $k_t = 800/1350 = 0.593$
  - $G_d/G_{GHI} = 1-1.13k_t$
    - $G_d = (1-1.13k_t)G_{GHI} = (0.3304)G_{GHI} = 264\ W/m^2$
  - $G_b = 800-264 = 536\ W/m^2$

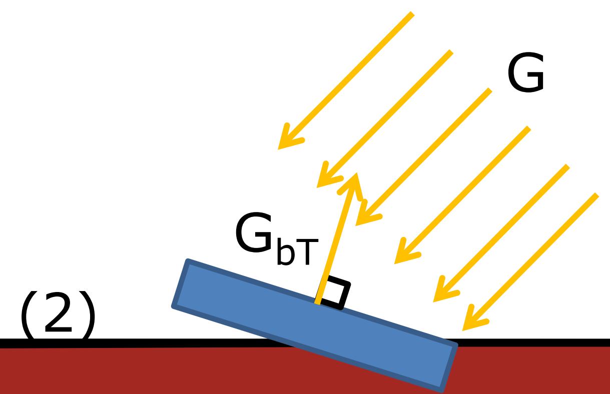
Computing Beam Irradiance

- Given G_b how do we find beam irradiance for a tilted surface, G_{bT}?
  - remember, G_b is for a horizontal surface
- $G_b = G\cos\theta_z$

we don't know G

Recall for horizontal surfaces, $\theta=\theta_z$

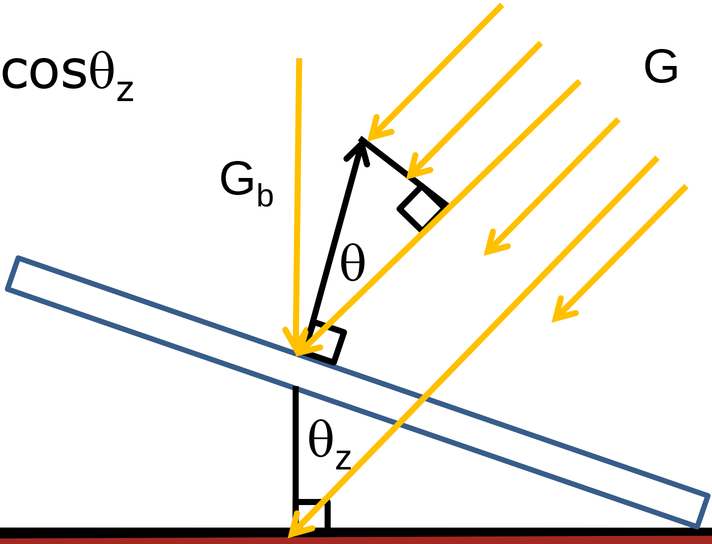




Computing Beam Irradiance

- $G_{bT} = G\cos\theta$
- $G_b = G\cos\theta_z$
  - $G = G_b(1/\cos\theta_z)$
- Then:
  - $G_{bT} = G_b R_b$
  - Where
    - $R_b = \cos\theta / \cos\theta_z$

Here we know G_b and can compute θ and θ_z, but we don't know G, so we need to eliminate it



Computing Beam Irradiance

- We now have solved for A
- $G_T = A + B + C$
  - $A = G_b R_b$

Exercise

- Compute the beam irradiance on a surface tilted at 15^o at 30^o N on April 15th, with a clearness index of 0.75 at solar noon.

Exercise

- Compute the beam irradiance on a surface tilted at 15° at 30° N on April 15th, with a clearness index of 0.75 at solar noon.
  - $G_d/G_{GHI} = 1-1.13k_t$
- We have k_t and can compute G_0 to find G_{GHI} using $k_t = G_{GHI}/G_0$

Exercise

- Compute the beam irradiance on a surface tilted at 15° at 30° N on April 15th, with a clearness index of 0.75 at solar noon.

$$\phi = 30^\circ$$

$$\beta = 15^\circ$$

$$\omega = 0^\circ$$

$$\delta = \delta_0 \sin\left(\frac{360^\circ\left(284+d\right)}{365}\right) = 23.5^\circ \sin\left(\frac{360^\circ\left(284+105\right)}{365}\right) = 9.4^\circ$$

$$G_{on}\left(d\right) = G_{sc}\left[1+0.033\cos\left(2\pi\left(\frac{105}{365}\right)\right)\right] = 1356 \text{ W/m}^2$$

Exercise

- Now that we have the angles, we can compute

  $G_o = G_{on} \cos\theta_z$

- Solving for the cosine of the zenith angle (β = 0 for zenith angle)

  $\cos(\theta_z) = \sin(\delta)\sin(\phi) + \cos(\delta)\cos(\phi)\cos(\omega)$

  $= \sin(9.4^o)\sin(30^o)+\cos(9.4^o)\cos(30^o)\cos(0^o)$

  = 0.936 [or using $\cos(\theta_z) = \cos(\phi-\delta)$]

- Therefore, for a horizontal surface at the top of the atmosphere: $G_o = G_{on} \cos\theta_z = 1356 \times 0.936 = 1270$ W/m^2

Exercise

- Now, solving for GHI:
  - $k_t = G_{GHI}/G_0$
  - $G_{GHI} = 952.5\ W/m^2$
- Computing the Diffuse Irradiance:
  - $G_d/G_{GHI} = 1-1.13k_t$
  - $G_d = 145.25\ W/m^2$
- Solving for G_b
  - $G_b = G_{GHI} - G_d = 807.25\ W/m^2$

Exercise

- The G_b we computed is for a Horizontal surface, we need to find G_{bT}, the beam irradiance on a tilted surface
- We can relate G_b and G_{bT} by: $G_{bT} = G_b R_b$
- Where
  - $R_b = \cos\theta / \cos\theta_z$
- We already know $\cos\theta_z$, so we need to calculate $\cos\theta$

Exercise

- Solving for the cosine of the angle of incidence:

$$\cos(\theta) = \sin(\delta)\sin(\phi)\cos(\beta)$$
$$-\sin(\delta)\cos(\phi)\sin(\beta)$$
$$+\cos(\delta)\cos(\phi)\cos(\beta)\cos(\omega)$$
$$+\cos(\delta)\sin(\phi)\sin(\beta)\cos(\omega)$$

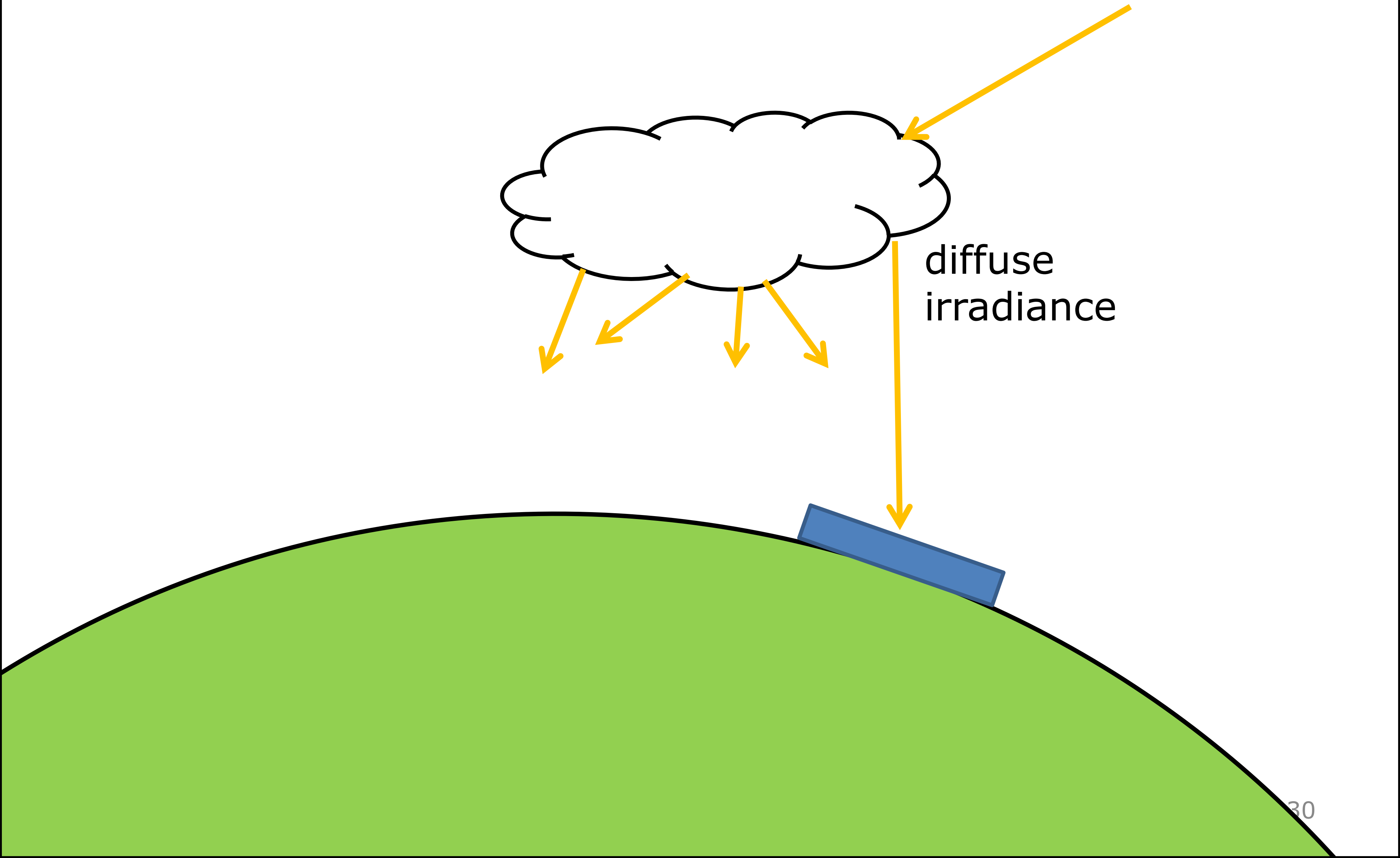
$$\cos(\theta) = \sin(9.4^\circ)\sin(30^\circ)\cos(15^\circ)$$
$$-\sin(9.4^\circ)\cos(30^\circ)\sin(15^\circ)$$
$$+\cos(9.4^\circ)\cos(30^\circ)\cos(15^\circ)\cos(0^\circ)$$
$$+\cos(9.4^\circ)\sin(30^\circ)\sin(15^\circ)\cos(0^\circ) = 0.995$$

[or using $\cos(\theta) = \cos(\phi-\delta-\beta)$]

Exercise

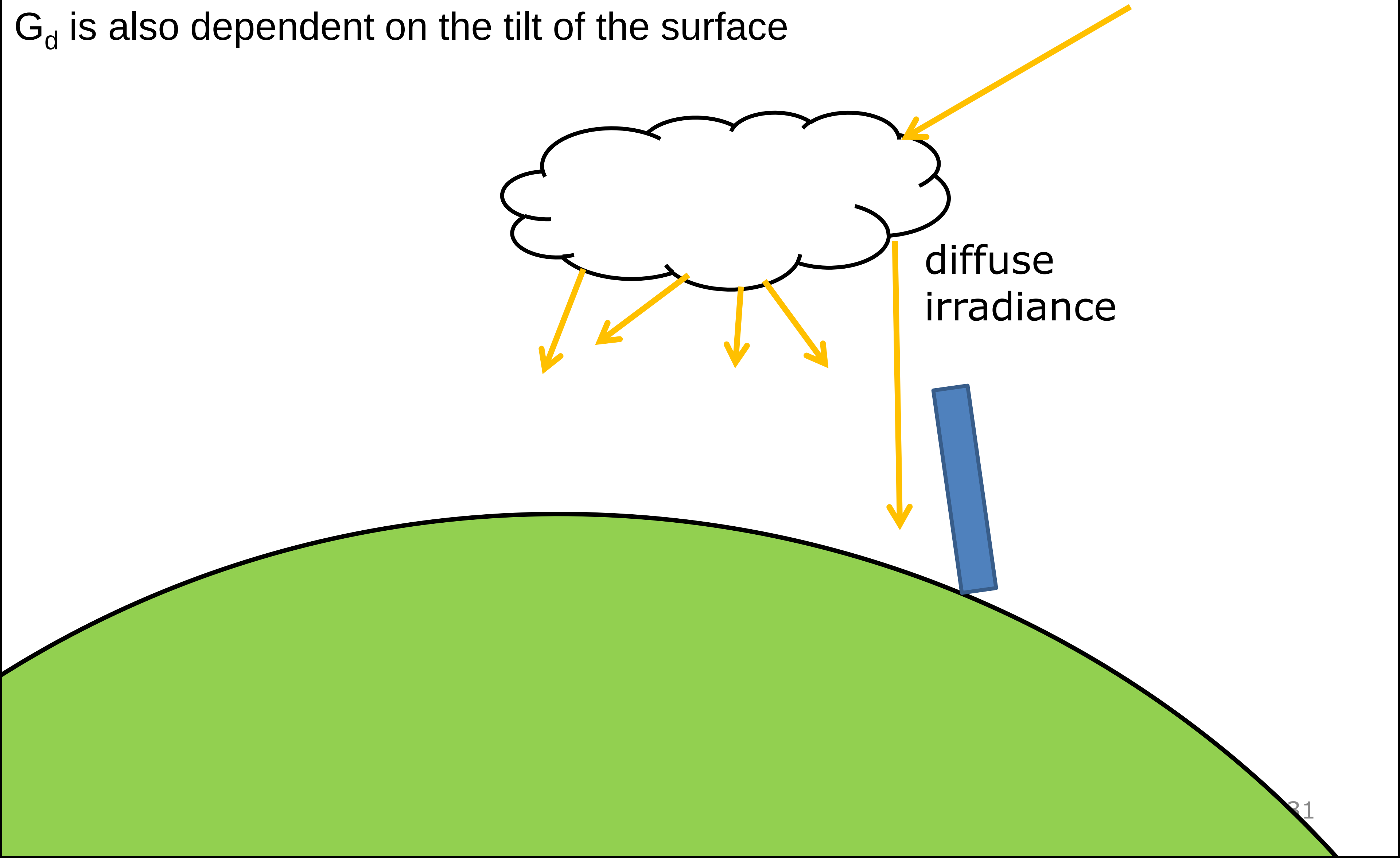
- $R_b = \cos\theta / \cos\theta_z = 1.06$
- So that the beam irradiance on the tilted surface is: $G_{bT} = G_b R_b = 858.1\ W/m^2$

Atmospheric Effects



Atmospheric Effects

G_d is also dependent on the tilt of the surface



Computing Diffuse Irradiance

- We already know G_d
- Now we need to be able to account for the tilting of a surface
  - When $\beta = 0^o$, all of G_d is received
  - When $\beta = 90^o$, half of G_d is received
  - When $\beta = 180^o$, no G_d is received (surface is facing the ground)
- $G_{dT} = ½G_d(1+\cos\beta)$
  - This assumes isotropic conditions

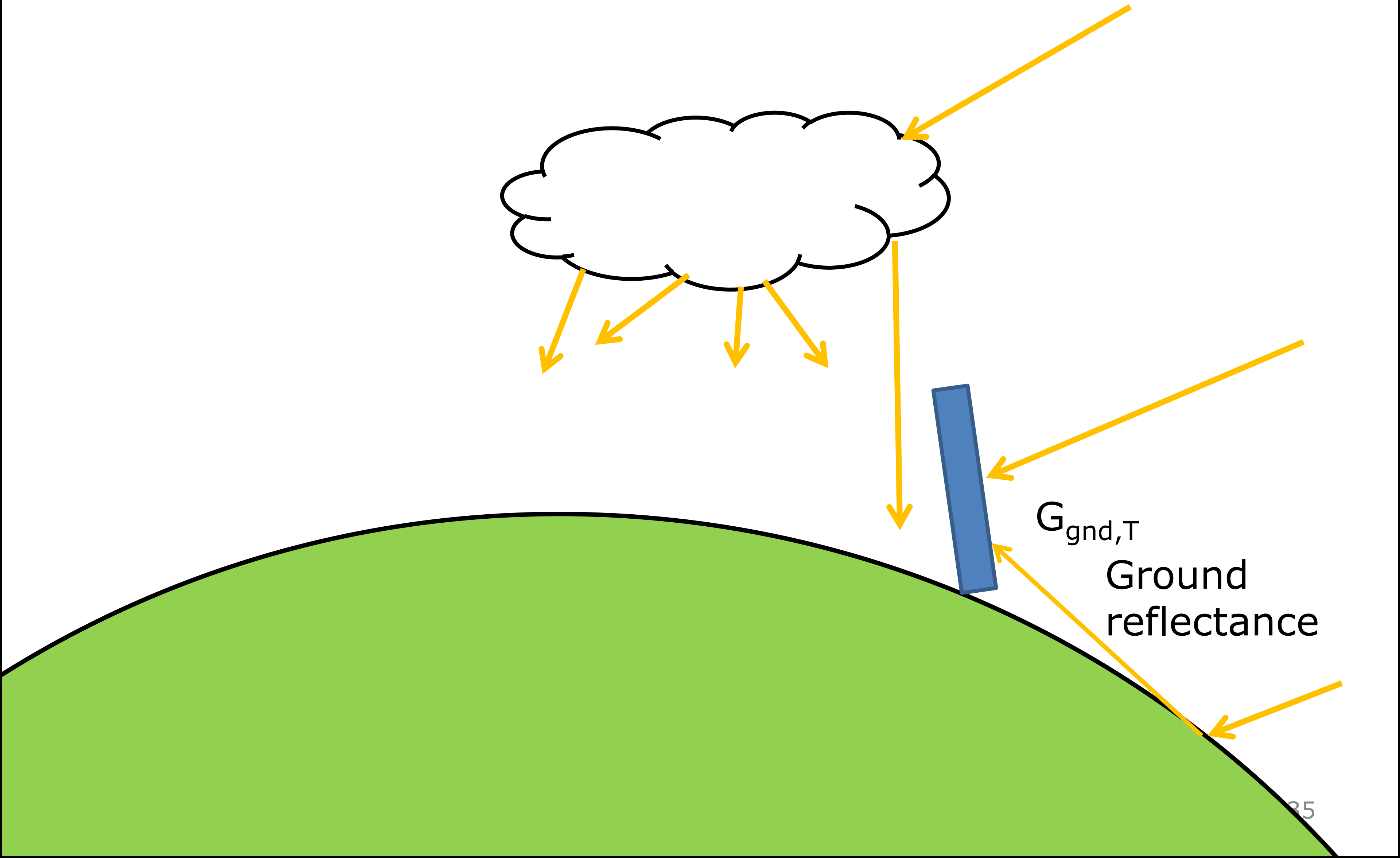
Computing Beam Irradiance

- We now have solved for A and B
- $G_T = A + B + C$
  - $A = G_b R_b$
  - $B = \frac{1}{2} G_d (1 + \cos\beta)$

Ground Reflectance

- Final component is the diffuse irradiance that reflects off the ground
  - Ground albedo (ρ)
    - Usually 0.2, but can be up to 0.8 for snow, ice
- We assume that this is proportional to the G_{GHI}
  - $G_{gnd} = \rho G_{GHI}$
- Amount received depends on the tilt of the surface

Ground Reflectance



Ground Reflectance

- Accounting for the tilting of a surface
  - When $\beta = 0^o$, no of G_{gnd} is received
  - When $\beta = 90^o$, half of G_{gnd} is received
  - When $\beta = 180^o$, all G_{gnd} is received (surface is facing the ground)
- $G_{gnd,T} = \frac{1}{2} \rho G_{GHI}(1-\cos\beta)$

Computing Beam Irradiance

- We now have solved for A, B and C
- $G_T = A + B + C$
  - $G_{bT} = G_b R_b$
  - $G_{dT} = ½G_d(1+\cos\beta)$
  - $G_{gnd,T} = ½\ \rho G_{GHI}(1-\cos\beta)$

Note: B depends on G_d, whereas C depends on G_{GHI}

Exercise

- Consider a surface that is tilted at 30°. The measured GHI is 250 W/m^2, and G_0 = 404 W/m^2. Compute G_b, G_d and G_T. Let $\cos(\theta) = 0.693$ and $\cos(\theta_z) = 0.286$. Assume $\rho = 0.2$.

Exercise

- Consider a surface that is tilted at 30°. The measured GHI is 250 W/m^2, and G_0 = 404 W/m^2. Compute G_b, G_d and G_T. Let $\cos(\theta) = 0.693$ and $\cos(\theta_z) = 0.286$. Assume $\rho = 0.2$.
  - $k_t = 250/404 = 0.622$

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  - $k_t = 250/404 = 0.622$
  - $G_d/G_{GHI} = 1-1.13k_t$
    - $G_d = (1-1.13k_t)G_{GHI} = (0.297)G_{GHI} = 74.3\ W/m^2$

Exercise

- Consider a surface that is tilted at 30°. The measured GHI is 250 W/m^2, and G_0 = 404 W/m^2. Compute G_b, G_d and G_T. Let $\cos(\theta) = 0.693$ and $\cos(\theta_z) = 0.286$. Assume $\rho = 0.2$.
  - $k_t = 250/404 = 0.622$
  - G_d = 74.3 W/m^2
  - G_b = 250 – 74.3 = 175.7 W/m^2

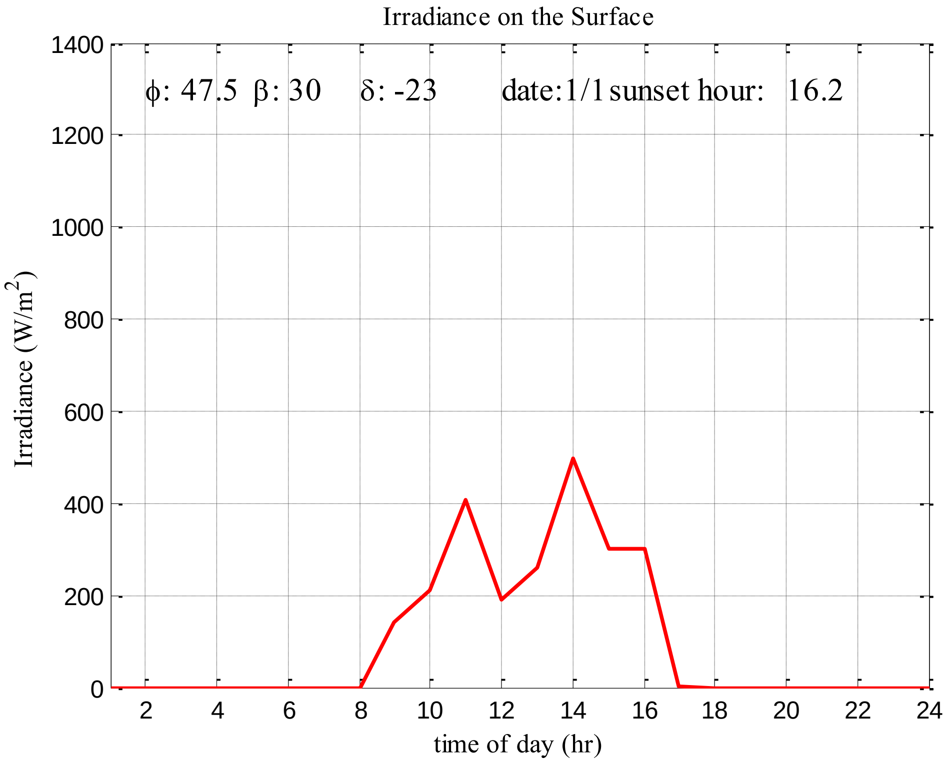
Exercise

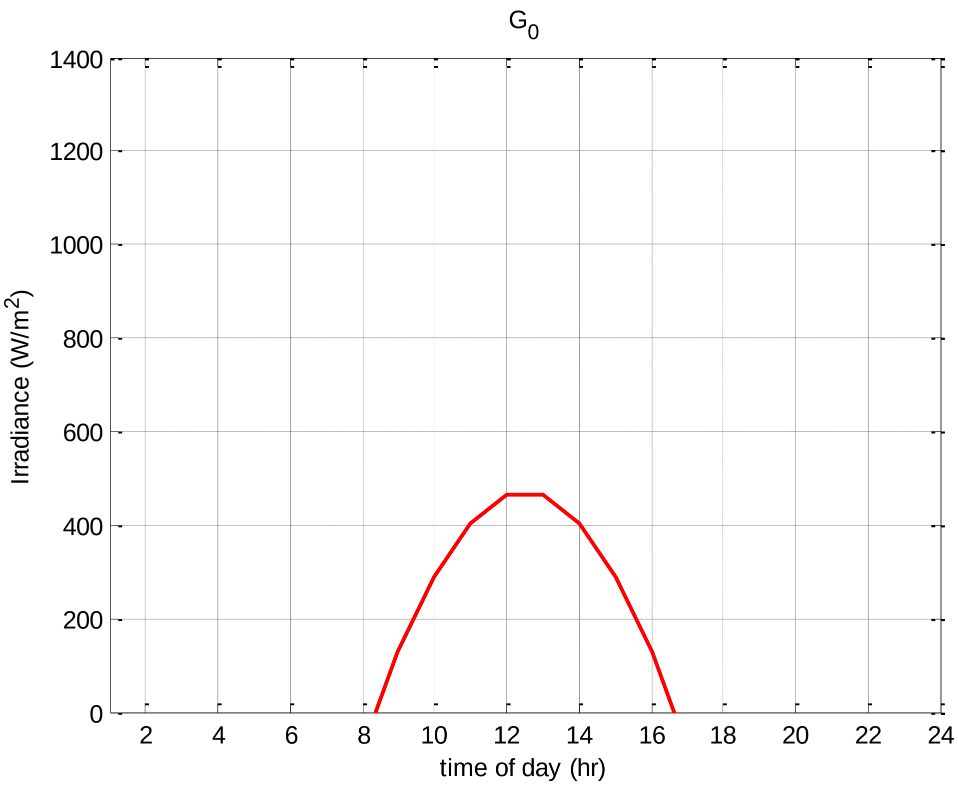
- Consider a surface that is tilted at 30°. The measured GHI is 250 W/m^2, and G_0 = 404 W/m^2. Compute G_b, G_d and G_T. Let $\cos(\theta) = 0.693$ and $\cos(\theta_z) = 0.286$. Assume $\rho = 0.2$.
  - $k_t = 250/404 = 0.622$
  - G_d = 74.3 W/m^2
  - G_b = 175.7 W/m^2
  - $R_b = \cos\theta / \cos\theta_z = 2.42$

Exercise

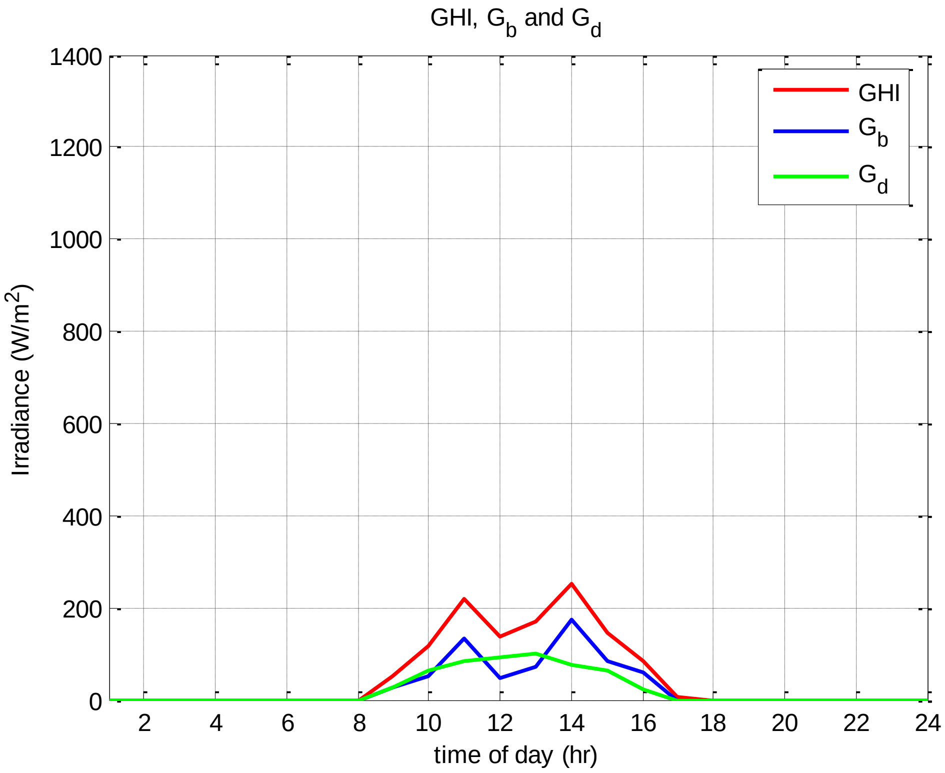
- Consider a surface that is tilted at 30°. The measured GHI is 250 W/m^2, and G_0 = 404 W/m^2. Compute G_b, G_d and G_T. Let cos(θ) = 0.693 and cos (θ_z) =0.286. Assume ρ = 0.2.
  - k_t = 250/404 = 0.622
  - G_d = 74.3 W/m^2
  - G_b = 175.7 W/m^2
  - $R_b = \cos\theta / \cos\theta_z = 2.42$
  - $G_T = G_b R_b + ½G_d(1+\cos\beta) + ½ \rho G_{GHI}(1-\cos\beta)$
  - 175.7(2.42) + ½(74.3)(1+cos30)+½ (0.2)(250)(1-cos30)
  - G_T = 425.6 + 69.3 + 3.3 = 498.3 W/m^2

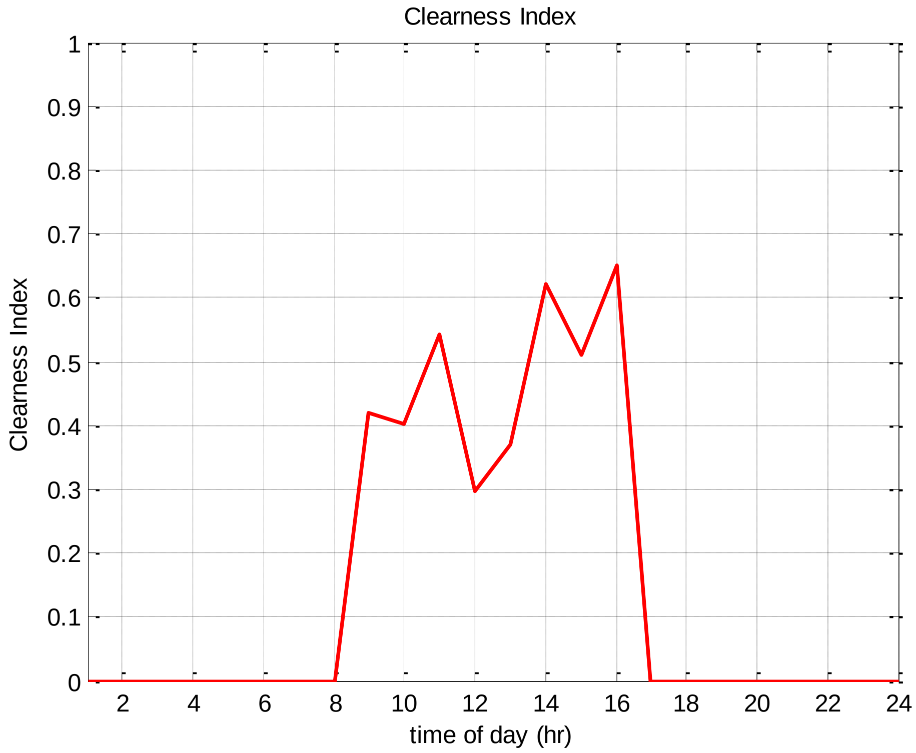
Calculating Irradiance





Calculating Irradiance





Irradiation

- Irradiation: irradiance received per unit area
- Computed by integrating G over time
  - I: irradiance received over one hour, Wh/m^2
  - H: irradiance received over one day, Wh/m^2
- Multiply irradiation by the surface area to compute radiation

Irradiation

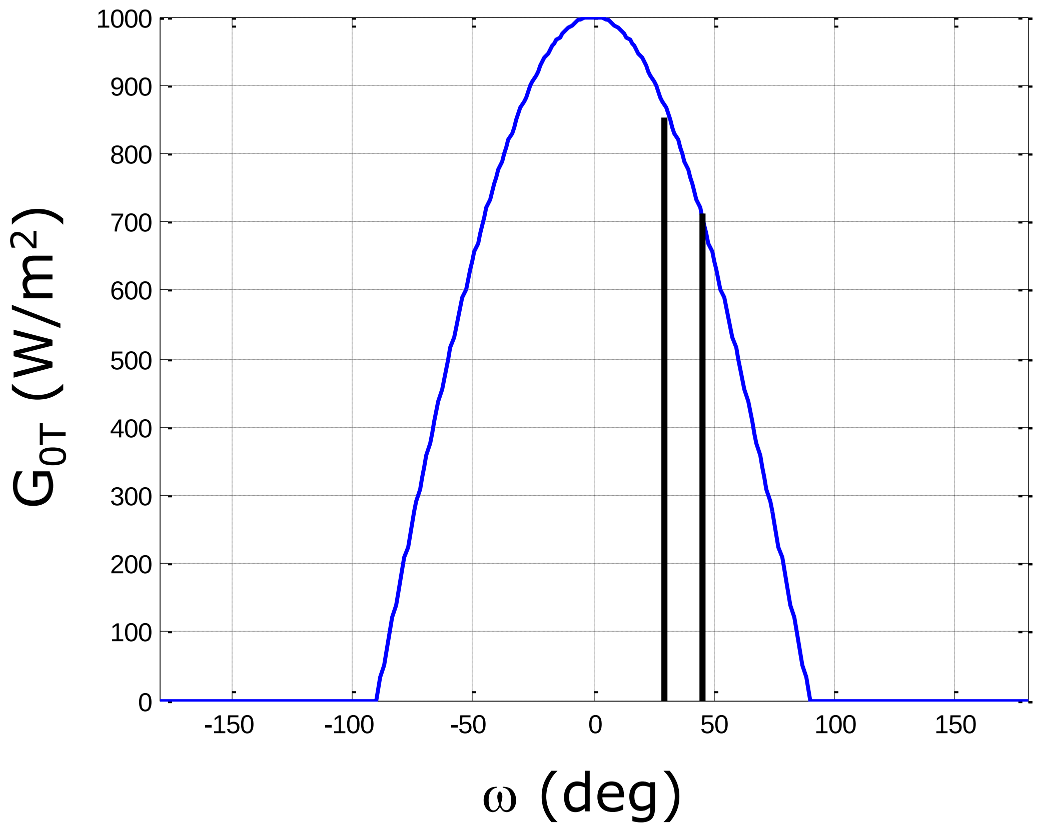
- Consider a surface tilted at its latitude
  - $\cos(\theta) = \cos(\delta)\cos(\omega)$
- What is the extraterrestrial irradiation from 2 pm to 3 pm?
  - Let $G_{0n}\cos(\delta) = 1000\ W/m^2$
- Integrating:

$$I_{0T} = 1000\int_{30^\circ}^{45^\circ} \cos(\omega)d\omega$$

$$I_{0T} = 1000\left(\sin\left(\frac{\pi}{6}\right) - \sin\left(\frac{\pi}{4}\right)\right) = 1000(0.207) = 207$$

What are the units?

Irradiation



Irradiation

- Clearly the I_{0T} does not equal 207 Wh
- We performed the integration using radians
  - Units: (W/m^2) x radians
- Expressed in Wh or kWh is more useful
  - 24 hours = 2π radians
  - 1 rad = $24/(2\pi)$ = $12/\pi$ hours
- Therefore:

$$207\left(\frac{12}{\pi}\right) = 790.7 \text{ Wh/m}^2$$

Irradiation

- GHI values are usually given as time-average values
  - 10 minutes
  - 1 Hour
  - 1 Day
  - 1 Month
- To compute the clearness index, average values of G_0 must be used

Irradiation

- Let $\bar{G}_0$ be the hourly averaged value of G_0 (average irradiance on an extraterrestrial horizontal surface $\beta = 0$)

$$\bar{G}_0 = G_{0n}\int_{\omega}^{\omega+15^\circ} \cos(\delta)\cos(\phi)\cos(\omega) + \sin(\delta)\sin(\phi)d\omega$$

$$\bar{G}_0 = G_{0n}\int_{\omega}^{\omega+15^\circ} \cos(\delta)\cos(\phi)\cos(\omega)d\omega + G_{0n}\int_{\omega}^{\omega+15^\circ} \sin(\delta)\sin(\phi)d\omega$$

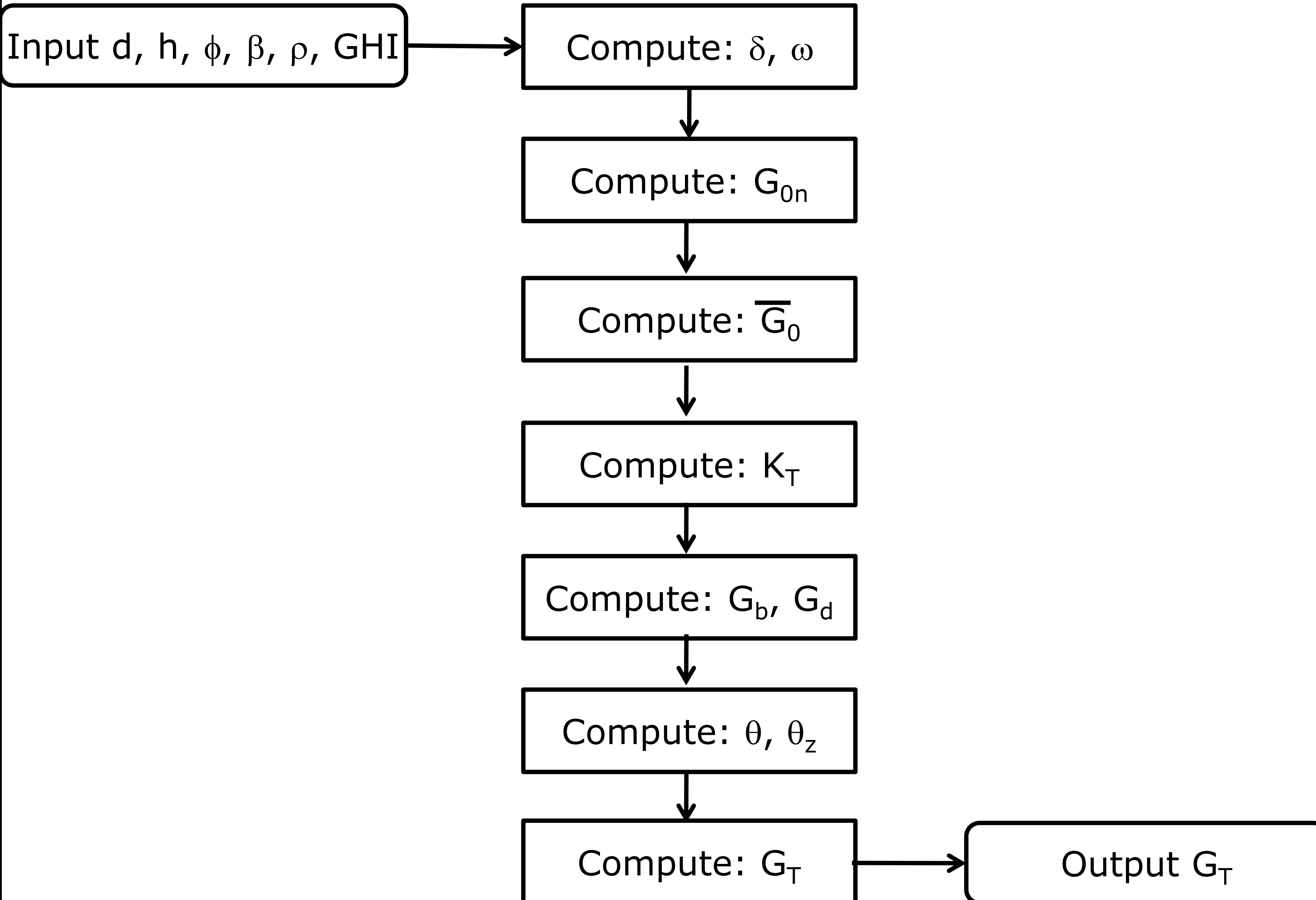
$$\bar{G}_0 = \frac{12}{\pi}G_{0n}\left[\cos(\delta)\cos(\phi)(\sin(\omega+15^\circ) - \sin(\omega)) + \frac{\pi 15^\circ}{180^\circ}\sin(\delta)\sin(\phi)\right]$$

$$K_T = \frac{\bar{G}_{GHI}}{\bar{G}_0}$$

- Clearness index using hourly-averaged values is

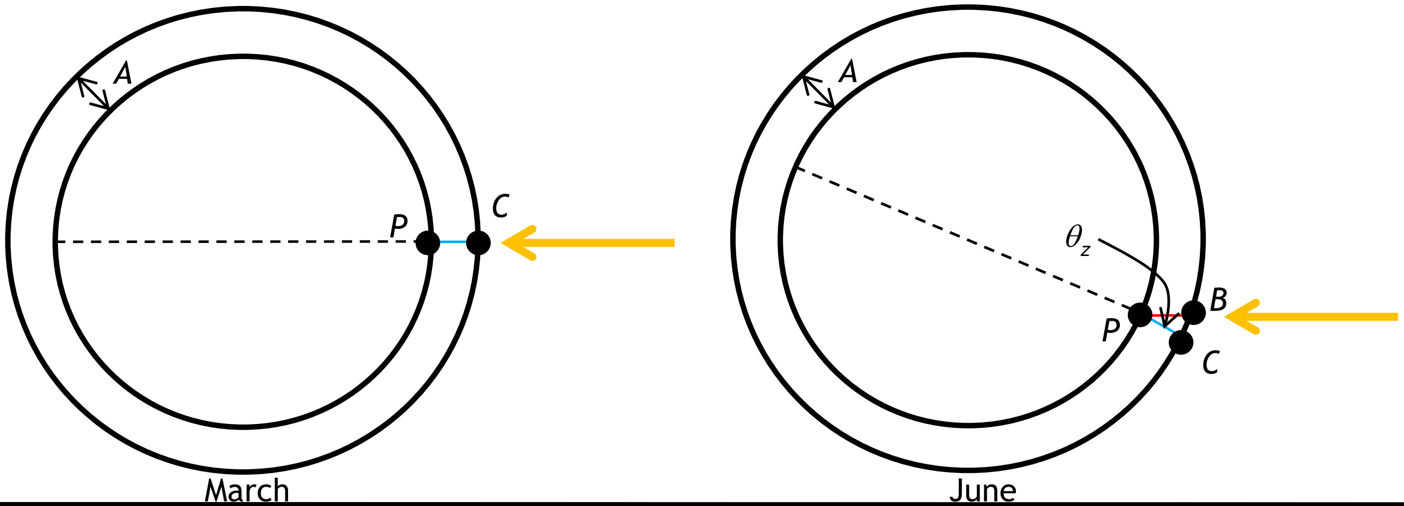
Irradiance Algorithm

- Computer programs such as Homer and RETScreen compute irradiance for a surface under specified conditions
- Algorithms differ in detail, but in approach they are very similar
- Once irradiance is known, power output of any arrangement of PV panels can be computed


Input d, h, ϕ, β, ρ, GHI
Compute: δ, ω
Compute: G_{0n}
Compute: $\overline{G}_0$
Compute: K_T
Compute: G_b, G_d
Compute: θ, θ_z
Compute: G_T
Output G_T

Air Mass

- Effects of absorption depend on the mass of air the radiation travels through
- The mass of air that solar radiation travels before reaching the surface varies with zenith angle
- This mass is smallest at solar noon with the sun directly overhead



Atmospheric Absorption

- Air Mass Ratio (AM) = PC/PB
- for $\theta_z < 70^o$, this approximates to

$$AM \approx \frac{1}{\cos\theta_z}$$

- AM0 is at the top of the atmosphere
- AM is sometimes divided by direct irradiance (AMD XX) and global irradiance (AMG XX) at air mass XX
- AMG 1.5 ($\theta_z = 48^o$)is the PV industry standard for the spectrum distribution, with 1000 W/m^2 irradiance
- How common is G = 1000 W/m^2 ?

Viewing

- The Power of the Sun - The Science of the Silicon Solar Cell, http://www.youtube.com/watch?v=u0hckM8TKY0