

# 18-Geothermal Systems

ECEGR 4530

Renewable Energy Systems



# Overview

- Introduction
- State of the Geothermal Industry
- Geothermal Resource
- Energy Extraction
- Geothermal Plant Operation
- Environmental Concerns



# Introduction

- Geothermal energy is one of three sources of renewable energy (solar, gravitational)
- Electricity generation from geothermal sources dates back to 1904 (just 22 years after Edison's power plant in New York)
- Geothermal energy can be used for direct heating applications as well as electricity generation
- We focus specifically on geothermal for electricity generation



## Is it renewable?

- Classifying geothermal energy as being renewable is a subject of debate
- Current technologies extract energy at a faster rate than is naturally replenished
  - “energy mining”

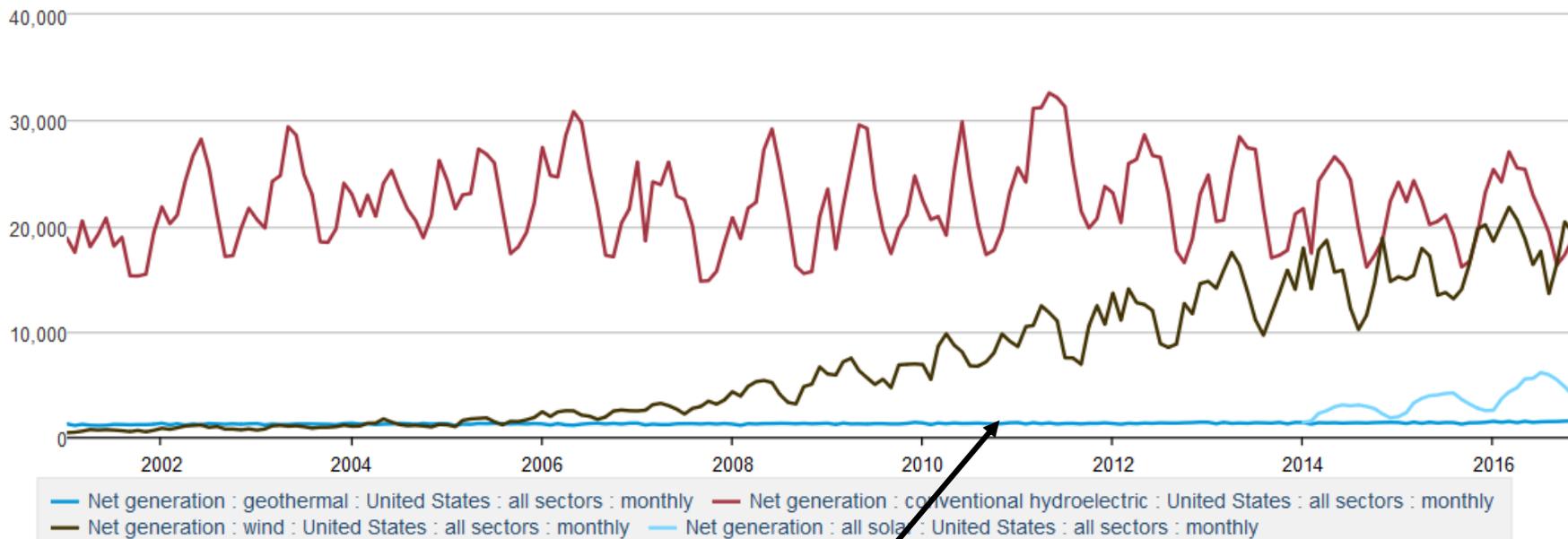


# Geothermal in the U.S.

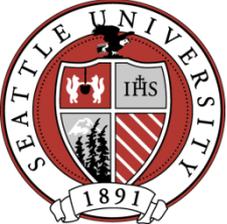
- Total Capacity (2015): 3,811 MW
  - Twice as much as solar thermal
  - One-third of solar PV
- 197 total units, located primarily in the Western/Southwestern states
- Average rating per unit: 19.3 MW (note: some plants consist of several units)



thousand megawatthours



Geothermal output  
has remained steady



## Exercise

In the U.S. in 2015, the total geothermal electricity production was 15,900 GWh. The installed capacity was 3.811 GW.

Compute the capacity factor.

How does this compare to the capacity factor of wind and solar?



## Exercise

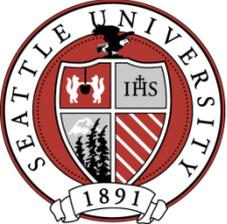
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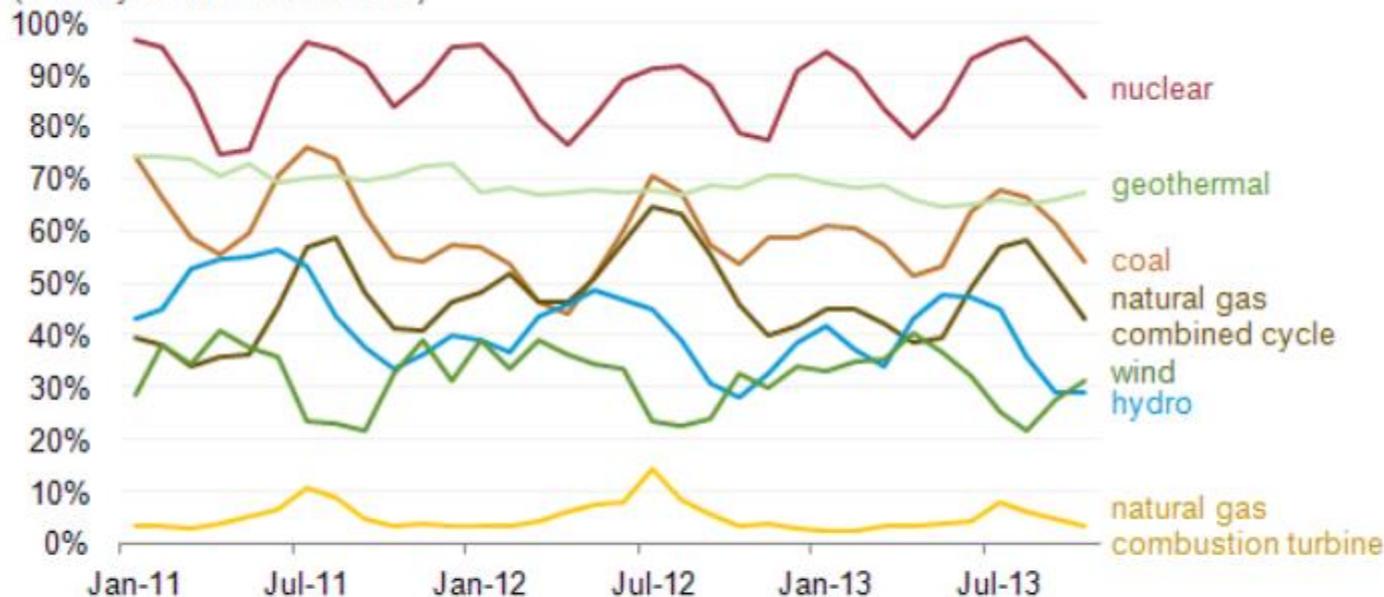
$$CF = 15,900 / (3.8 \times 8760) = 47\%$$

CF for geothermal is greater than that for wind and solar. Some geothermal plants have capacity factors of 90%.



# Why does the EIA report a CF >70%?

Monthly capacity factors for select fuels and technologies  
(January 2011-October 2013)



Source: U.S. Energy Information Administration, Electric Power Monthly, Tables 6.7a and 6.7b

Capacity factors are an important measure of electric generator usage. In December 2013, EIA began publishing tables of monthly capacity factors for 16 different fossil and non-fossil fuel and technology combinations in the [Electric Power Monthly](#).



# Why does the EIA report a CF > 70%?

Energy Source	Facility Type	Number of Generators	Generator Nameplate Capacity	Net Summer Capacity	Net Winter Capacity
Coal	Utility Scale	968	304,789.8	279,719.9	281,105.8
Petroleum	Utility Scale	3,550	42,321.3	36,830.3	40,372.6
Natural Gas	Utility Scale	5,774	503,936.9	439,425.4	472,495.2
Other Gases	Utility Scale	100	2,824.0	2,500.4	2,490.7
Nuclear	Utility Scale	99	103,860.4	98,672.0	101,001.4
Geothermal	Utility Scale	197	3,811.8	2,541.5	2,799.3
Wind	Utility Scale	1,008	73,303.2	72,573.4	72,675.8
Solar Photovoltaic	Utility Scale	1,633	11,983.7	11,905.4	11,795.2
Solar Thermal	Utility Scale	19	1,774.6	1,757.9	1,631.8

Source: [http://www.eia.gov/electricity/annual/html/epa\\_04\\_03.html](http://www.eia.gov/electricity/annual/html/epa_04_03.html)



## Why does the EIA report a CF >70%?

- EIA uses the “summer net capacity” for geothermal CF calculation
- Capacity of geothermal plants varies more substantially than other power plants
  - Coal: 304.8 GW nameplate / 279.7 GW summer
  - Geothermal: 3.8 GW nameplate/ 2.5 GW summer

# Geothermal Facilities in the US





# World Geothermal Generation

- Approximately 13 GW worldwide in 2015
  - 7,972 MW in 2000
- Top countries:
  1. U.S. (3.8 GW)
  2. Philippines (1.87 GW)
  3. Indonesia (1.34 GW)
  4. Mexico (1.02 GW)
  5. New Zealand (1.01 GW)

Also notable: Kenya, Iceland



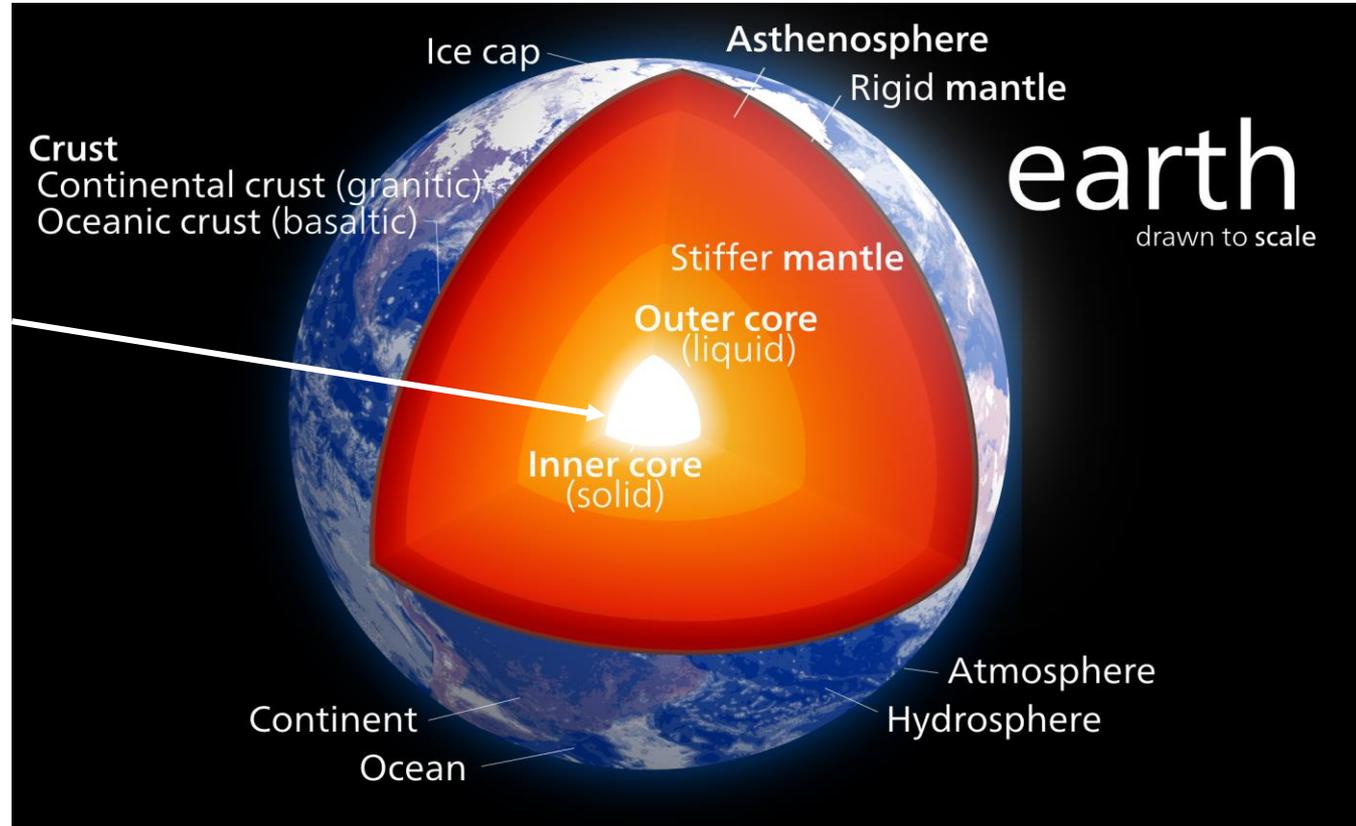
# Geothermal Resource

- Basic idea: collect energy from hot material underground and use it to drive a thermodynamic cycle
- Geothermal energy source:
  - Residual energy from formation of the earth (friction)
  - Decay of radioactive materials in earth's crust
  - Naturally occurring chemical reactions



# Geothermal Resource

Inner core temperature  
~4000° C



By Kelvinsong - Own work, CC BY-SA 3.0,  
<https://commons.wikimedia.org/w/index.php?curid=23966175>



# Geothermal Resource

- Heat flows from core to the surface, primarily by conduction by also convection (e.g. molten magma and water)
- As heat flows from the core, it becomes less concentrated (similar to irradiance as it travels from the surface of the sun)
- Average power density at the surface:  $60 \text{ mW/m}^2$  (compare this to the density of solar irradiance and wind)
- Average thermal gradient:  $30 \text{ }^\circ\text{C/km}$



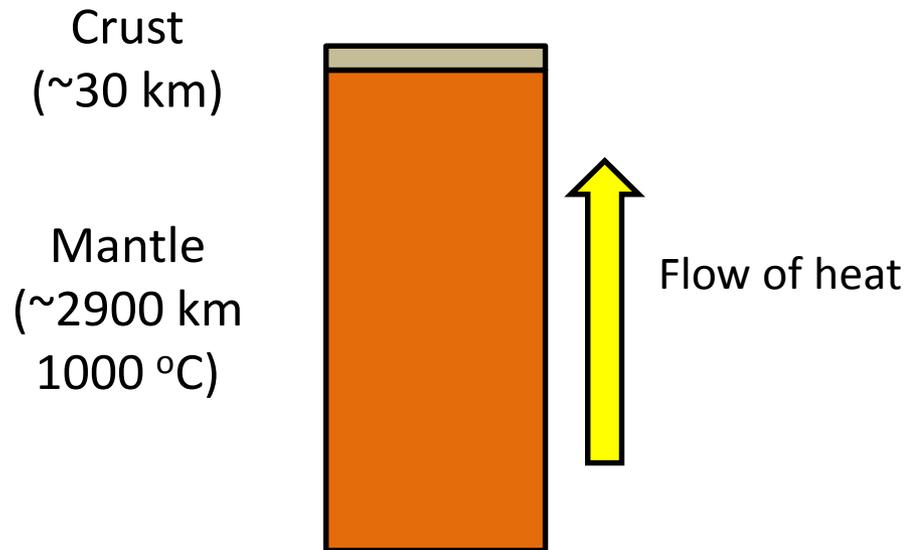
# Geothermal Resource

- Low average power density at the surface means we need to look for regions that are outliers (much greater power density) and/or extract energy deep below the surface
- Recall that high temperature is needed for high efficiency thermodynamic processes
  - Rankine:  $\sim 500$  °C
  - Brayton: up to 1400 °C

} Conventional electric power plants
- Rule of thumb: use geothermal for direct heating if the temperature is  $< 150$  °C



# Geothermal Resource





# Temperature Gradient

- The temperature difference across the crust is approximate 30 °C/km
- For meaningful power generation, well depths of 5km to 6km are needed
- Drilling costs increase exponentially with depth, so deeper depths are typically not economically feasible



# Specific Heat Capacity

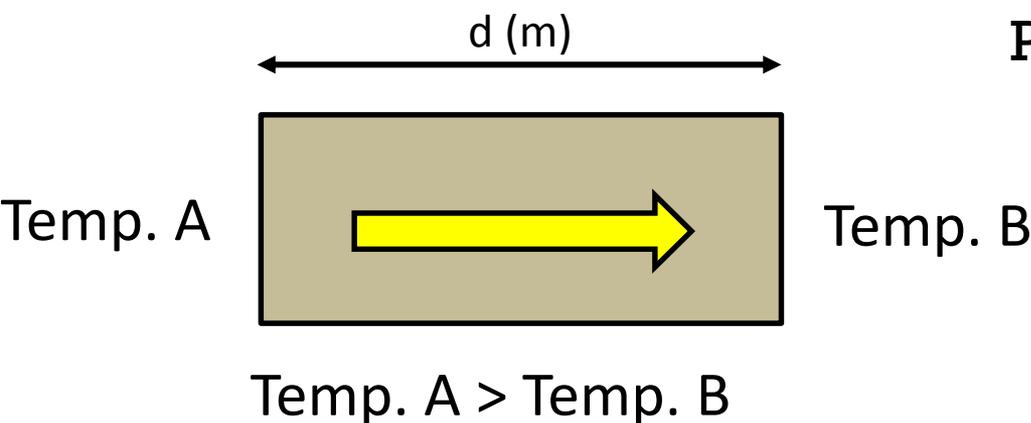
- Recall specific heat capacity: energy required to increase the temperature of 1kg of a material by 1 degree K (or C)
- Average solid crust heat capacity: 1000 J/(kgK)
- Average solid crust density: 2700 kg/m<sup>3</sup>



# Thermal Conductivity

- Thermal conductivity ( $\kappa$ ): thermal power that conducts through a material per degree of temperature difference
- Thermal conductivity of solid crust material:  
2 W/(m-K) (**crust material is a poor thermal conductor**)

Meter-Kelvin



$$P_{\text{therm}} = \frac{\kappa (\text{Temp}_A - \text{Temp}_B) A}{d}$$

Surface area



# Geothermal Power Flow

- The power density flowing through the crust is therefore:

$$P_{\text{therm}} = \frac{2 \times (1000 - 0) \times 1}{30,000} = 0.067 \text{ W}$$

Note: the units are a power density.  
We've assumed the surface temperature is 0 degrees for simplicity (also note the bottom of the ocean is very cold).

Crust  
(~30 km)

Mantle  
(~2900 km  
1000 °C)



Flow of heat



# Energy in the Crust

What is the energy stored in the crust beneath a 1 m<sup>2</sup> patch of land?

First compute the volume of the crust beneath the patch

Then compute the mass of the crust

Compute the energy assuming the average temperature is 500 °C.



## Energy in the Crust

- What is the energy stored in the crust beneath a 1 m<sup>2</sup> patch of land?

$$\text{Volume} = 1 \times 1 \times 30,000 = 30,000\text{m}^3$$

$$\text{mass} = 2700 \times 30,000 = 81\text{e}6 \text{ kg}$$

$$E = 81\text{e}6 \times 1000 \times 500 \approx 4\text{e}13 \text{ J} = 11,250 \text{ MWh}$$



Note: we are using Celsius here under the assumption that for the energy to be useful it must be at least the surface temperature (assumed to be zero). The total energy calculation would involve using Kelvin instead of Celsius.



# Geothermal Resource

- Although the replenish rate is very low, there is a tremendous amount of energy stored in the crust
- “Energy mining” appears to be an appropriate term



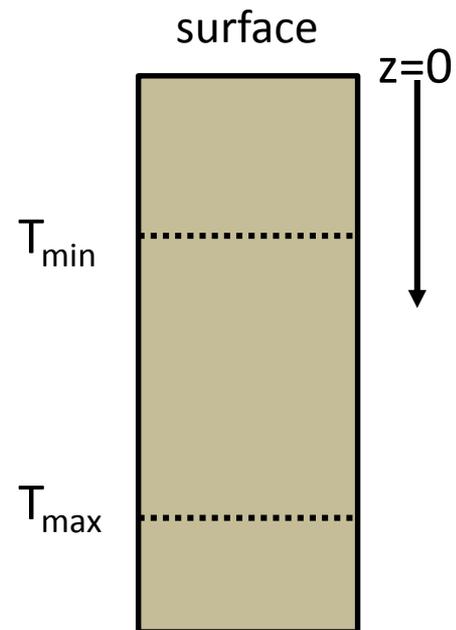
# Geothermal Energy Extraction

- Let the temperature  $T$  (K) at depth  $z$  be modeled as:

$$T = T_0 + z \frac{dT}{dz}$$

Known as the "temperature gradient"

- where  $T_0$  is the temperature at the surface (ground)
- Let  $T_{\min}$  be the minimum temperature for useful heat
- Let  $T_{\max}$  be the temperature at maximum depth (of the well)





# Exercise

- Let the temperature at the surface be  $10\text{ }^{\circ}\text{C}$ . If the temperature gradient is  $35\text{ }^{\circ}\text{C}/\text{km}$ , what is the temperature at a depth of  $6\text{ km}$ ?



## Exercise

- Let the temperature at the surface be 10 °C. If the temperature gradient is 35 °C/km, what is the temperature at a depth of 6km?

$$T = T_0 + z \frac{dT}{dz}$$

$$T = 10 + 6(35) = 220 \text{ °C}$$



# Geothermal Energy Extraction

- For dry rock, energy is extracted by pumping water underground into the hot rock and reclaiming it at a higher temperature
- As energy is extracted from the rock, the temperature will decrease
- We are interested in knowing the power that can be extracted, as well as how this changes over time



# Geothermal Energy Extraction

- Let
  - $\rho_r$ : density of the rock (kg/m)
  - $\kappa_r$ : specific heat of the rock (J/kg-K)
  - $A$ : cross-sectional area (m<sup>2</sup>)
- The useful energy stored in a slice of rock with height  $d$  (meters) at depth  $z$  is:

$$\delta E = \kappa_r \underbrace{(\rho_r A d)}_{\text{mass}} \underbrace{(T - T_1)}_{\text{subtract}} = \kappa_r (\rho_r A d) \underbrace{\frac{dT}{dz} (z - z_1)}_{\text{Via substitution}}$$

$T_1$  since  $T_1$  is the minimum useful temp

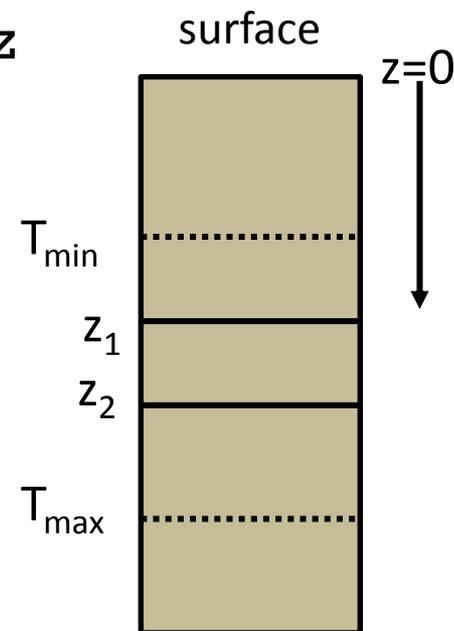


# Geothermal Energy Extraction

- The energy in the column of dry rock between depth  $z_1$  and  $z_2$  is therefore:

$$E_0 = \int_{z_1}^{z_2} \rho_r A \kappa_r \frac{dT}{dz} (z - z_1) dz = \rho_r A \kappa_r \frac{dT}{dz} \int_{z_1}^{z_2} (z - z_1) dz$$

$$= \rho_r A \kappa_r \frac{dT}{dz} \frac{(z_2 - z_1)^2}{2}$$





## Exercise

- What is the energy stored in 2km of hot dry rock below a 1m<sup>2</sup> patch with thermal gradient of 40 °C/km between a depth of 5km and 7km, if  $\rho_r = 2700 \text{ kg/m}^3$ ,  $\kappa_r = 820 \text{ J/(kg-K)}$ .



## Exercise

- What is the energy stored in 2km of hot dry rock below a 1m<sup>2</sup> patch with thermal gradient of 40 °C/km between a depth of 5km and 7km, if  $\rho_r = 2700 \text{ kg/m}^3$ ,  $\kappa_r = 820 \text{ J/(kg-K)}$ .

$$E_0 = \rho_r A \kappa_r \frac{dT}{dz} \frac{(z_2 - z_1)^2}{2} = 2700 \times 1 \times 820 \times \frac{40}{1000} \frac{(7000 - 5000)^2}{2}$$
$$= 177.12 \text{ GJ} = 49.2 \text{ MWh}$$

Note: be careful with your units!  
Some are in km, some are in m.

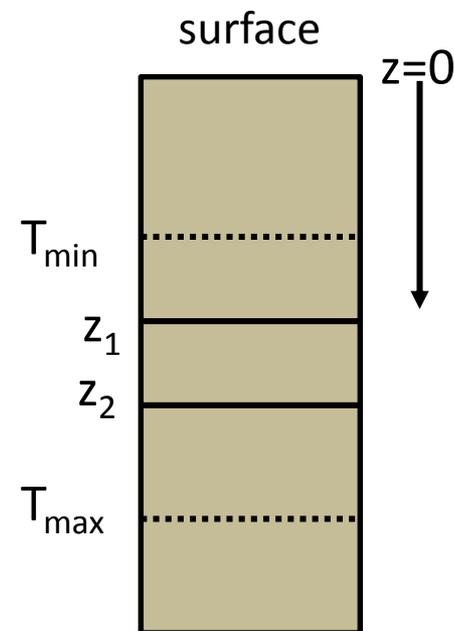


# Geothermal Energy Extraction

- Note that the average temperature of the rock that is greater than  $T_1$  is:

$$\theta = \frac{T_2 - T_1}{2} = \frac{dT}{dz} \frac{(z_2 - z_1)}{2}$$

- For example, if  $T_1 = 100$  °C, and  $T_2 = 150$  °C, then  $\theta = 25$  °C





# Geothermal Energy Extraction

- Note that the “thermal capacity” of the rock is

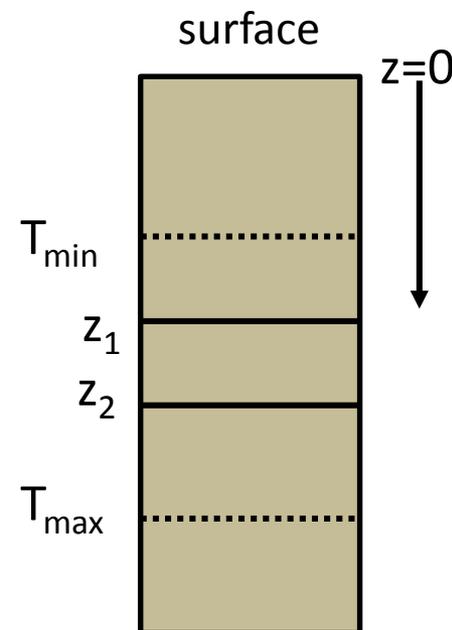
$$C_r = \rho_r A k_r (z_2 - z_1)$$

- It tells us how much energy  $E_0$  can be stored when the rock is at a given (average) temperature

$$E_0 = C_r \theta$$

- Note that the change in energy from the rock due to a temperature change is:

$$P = \frac{E_0}{dt} = C_r \frac{d\theta}{dt}$$





# Geothermal Energy Extraction

- Assume water is pumped into and extracted from the rocks where:
  - $\dot{V}$  : Flow rate of volume of water ( $\text{m}^3/\text{s}$ )
  - $\rho_w$  : Density of water ( $\text{kg}/\text{m}^3$ )
  - $\kappa_w$ : specific heat of water
- The mass of water injected each second is:

$$m_w = \dot{V}\rho_w$$

- The mass of water injected over a period of t seconds is:

$$m_w = \dot{V}\rho_w t$$

- The water will heat to an average useful temperature  $\theta$ , leading to an increase in its useful energy



# Geothermal Energy Extraction

- The useful energy in the water  $E_w$  due to its increase in temperature is:

$$E_w = \dot{V} \rho_w t \kappa_w \theta$$

- The power extracted is

$$P_w = \dot{V} \rho_w \kappa_w \theta$$

← This assumes that  $\theta$  is constant



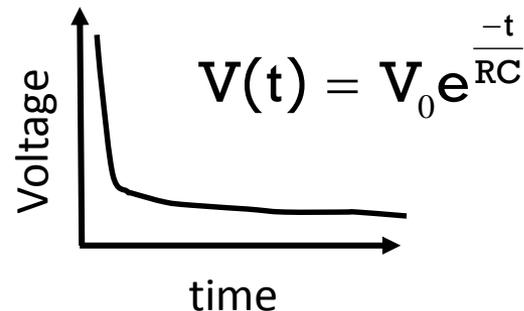
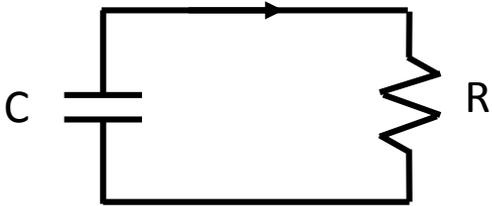
# Geothermal Energy Extraction

- As the water extracts heat, the temperature of the rocks will decrease
- As the temperature decreases, the average temperature above minimum  $\theta$ , will also decrease
- Because the energy extracted by the water is dependent on the temperature,  $E_w$  will decrease as time goes on
- In other words, the heat extraction rate by the water is not constant and will decrease with time



# Geothermal Energy Extraction

- This is analogous to a charged capacitor that discharges through a resistor
- The current through the resistor depends on the voltage of the capacitor
- As the capacitor discharges, the current through the resistor also decreases, slowing the rate of discharge
- Determining the capacitor voltage requires solving a first order differential equation





# Geothermal Energy Extraction

- The thermal power extracted by the water must equal the thermal power leaving the rocks

$$\dot{V} \rho_w \kappa_w \theta = - \underbrace{\rho_r A \kappa_r (z_2 - z_1)}_{\text{Thermal capacity of the rock}} \frac{d\theta}{dt} = -C_r \frac{d\theta}{dt}$$

solving

$$\theta = \theta_0 e^{-\frac{t}{\tau}}$$

where

$$\tau = \frac{C_r}{\dot{V} \rho_w \kappa_w} = \frac{\rho_r A \kappa_r (z_2 - z_1)}{\dot{V} \rho_w \kappa_w}$$



# Geothermal Energy Extraction

- The useful heat content is

$$E = \rho_r A \kappa_r (z_2 - z_1) \theta$$

so that

$$E = E_0 e^{-\frac{t}{\tau}}$$

and the power transfer is

$$P = \frac{dE}{dt} = \frac{E_0}{\tau} e^{-\frac{t}{\tau}} \quad \leftarrow \text{Thermal power extracted by the water}$$



## Example

- Consider a  $1 \text{ km}^2$  patch of land. Water is injected and extracted at a rate of  $1 \text{ m}^3/\text{s}$ . The thermal gradient is  $40 \text{ }^\circ\text{C}/\text{km}$ . Let the minimum useful temperature be  $200 \text{ }^\circ\text{C}$  and the maximum well depth is  $7\text{km}$ . The rock is granite with:  $\rho_r = 2700 \text{ kg}/\text{m}^3$ ,  $\kappa_r = 820 \text{ J}/(\text{kg}\text{-K})$ .
- Note: for water  $\rho_w = 1000 \text{ kg}/\text{m}^3$ ,  $\kappa_w = 4200 \text{ J}/(\text{kg}\text{-K})$



# Example

- The initial useful energy is:

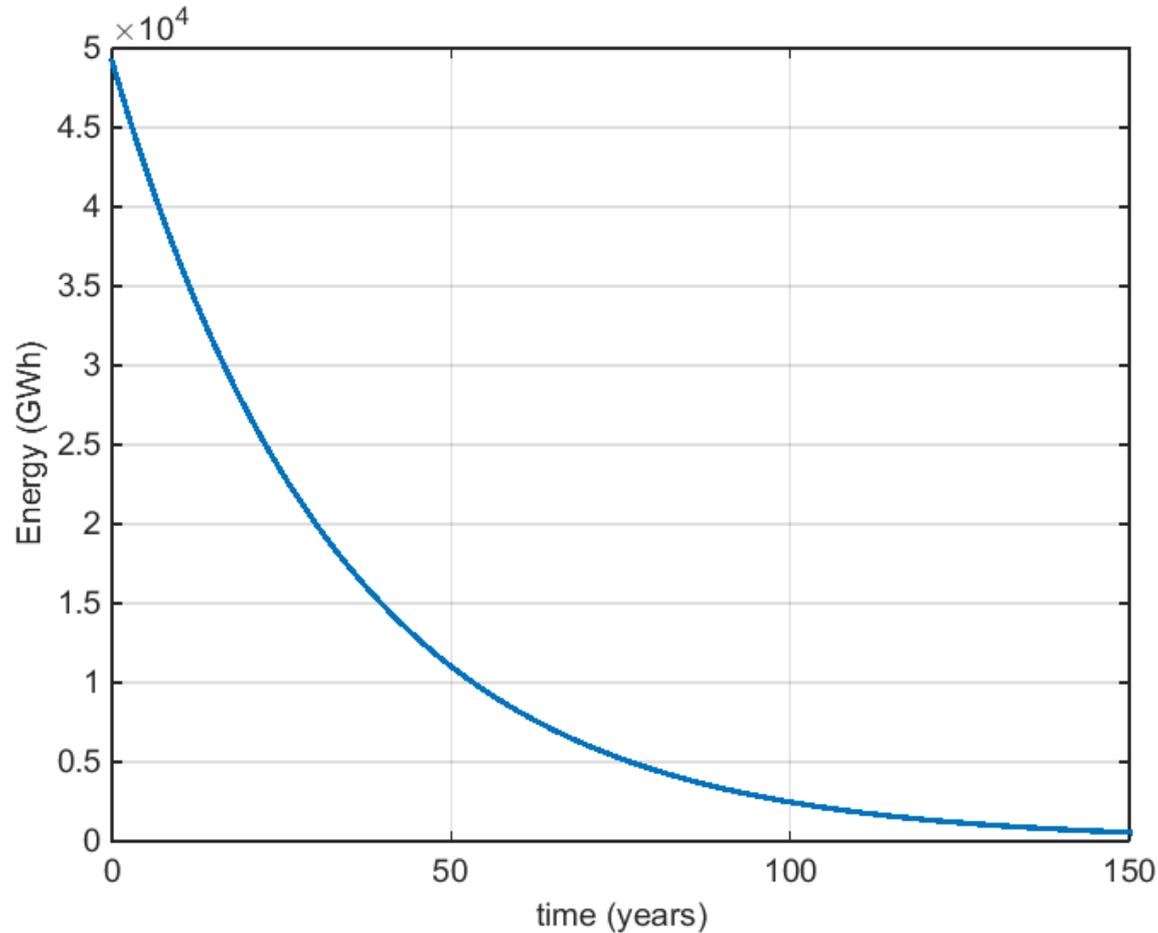
$$E_0 = \rho_r A \kappa_r \frac{dT}{dz} \frac{(z_2 - z_1)^2}{2} = 1.77 \times 10^{17} \text{ J} = 49,200 \text{ GWh}$$

- The time constant is:

$$\tau = \frac{C_r}{\dot{V} \rho_w \kappa_w} = 33.4 \text{ years}$$



# Energy Remaining in the Rock

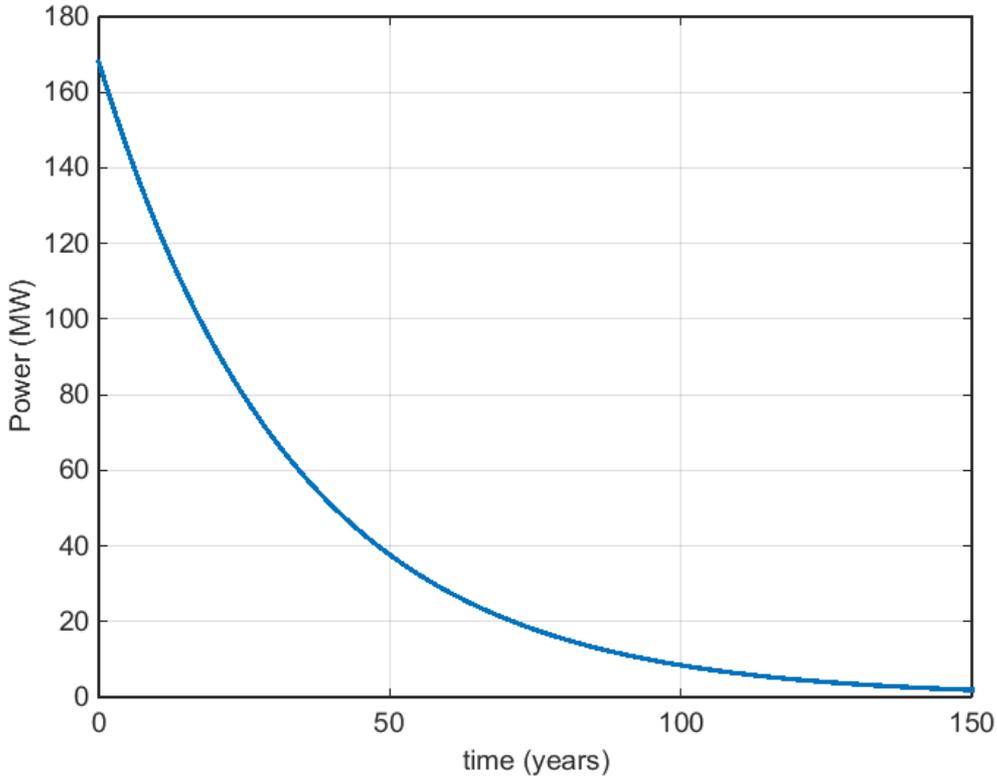


$$E = E_0 e^{\frac{-t}{\tau}}$$

Note: this ignores replenishment from natural flow of heat



# Power Extracted from the Rock

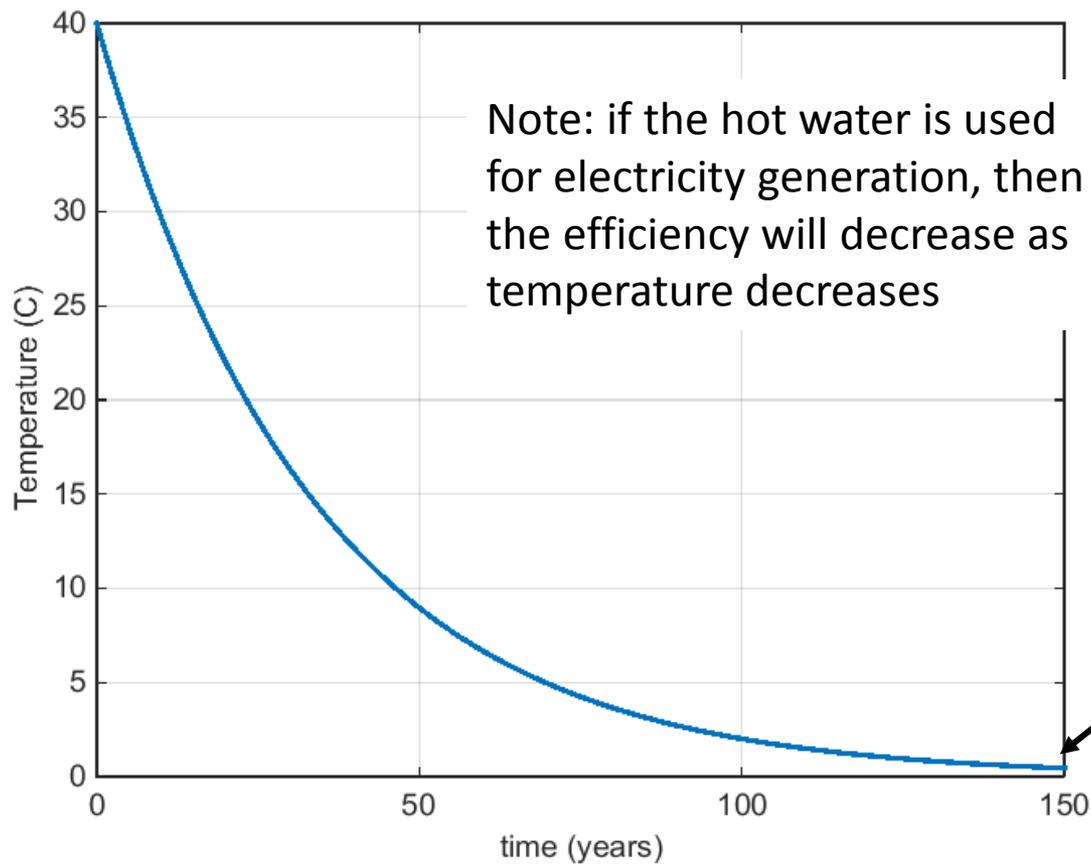


$$P = \frac{dE}{dt} = \frac{E_0}{\tau} e^{-\frac{t}{\tau}}$$

Thermal power extraction  
decreases as time increases



# Temp. Above minimum



Approaches minimum useable temperature



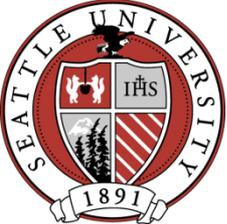
# Energy Extraction

- For geothermal resources with water (hot aquifer), follow a similar derivation as before, but the account for the specific heat, volume, density of hot water within the rock

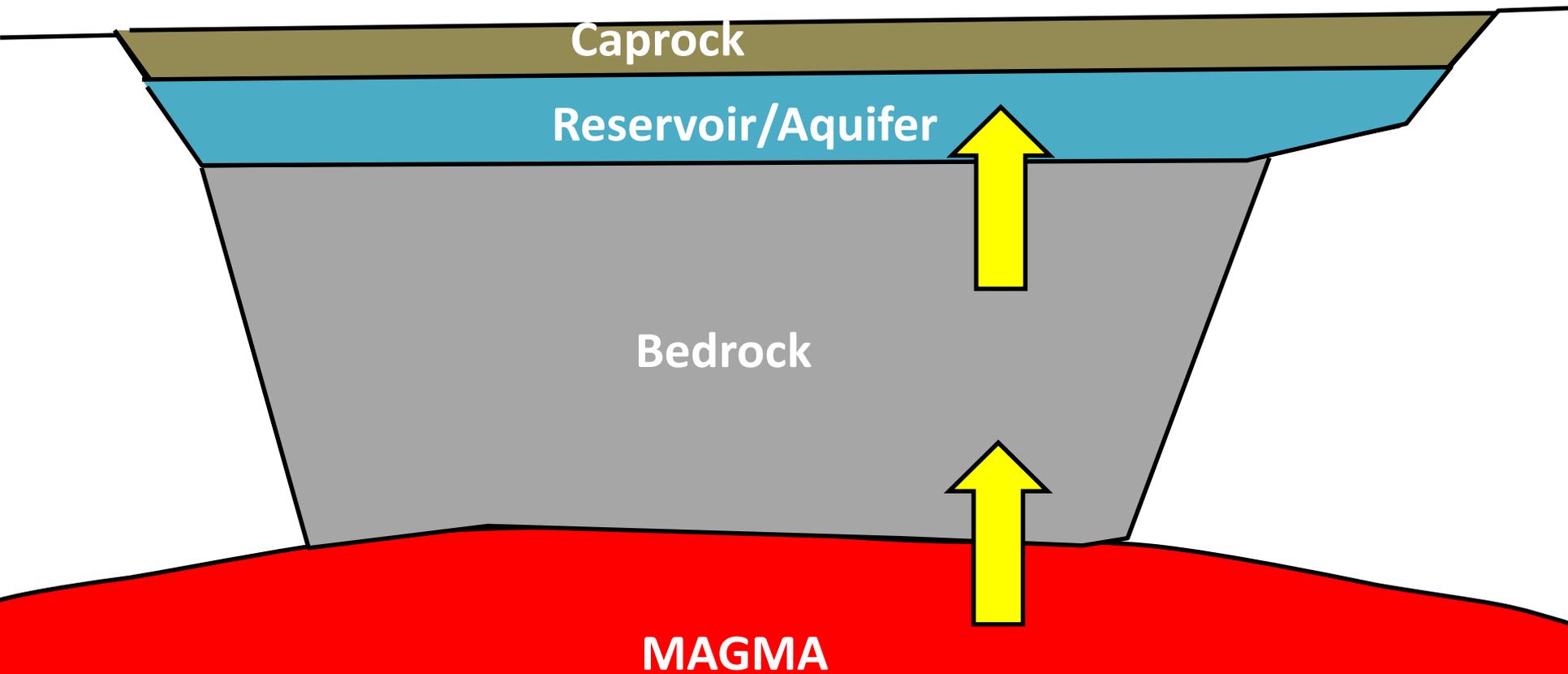


# Types of Resources

- Hot Dry Rock (HDR): underground hot dry (relatively) rock; relatively abundant and at high temperatures ( $\sim 200$  °C). Despite high temperatures, HDR fields are not yet commercially viable as the heat exchange is difficult (low thermal conductivity)
- Hot Aquifer: hot water ( $\sim 200$  °C) under pressure in an aquifer below the surface; requires more specific geology; used in commercial geothermal plants



# Hot Aquifer



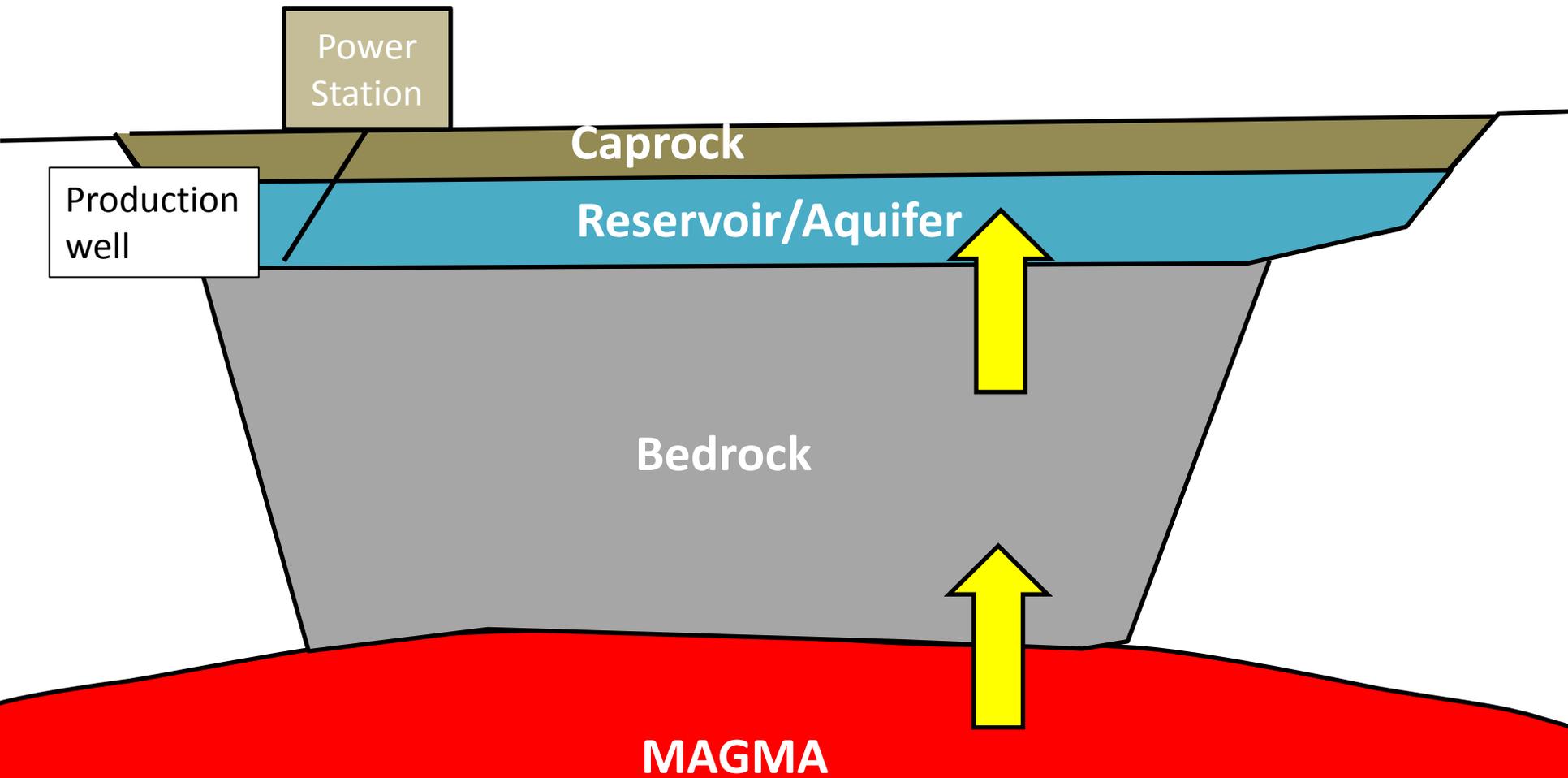


# Hot Aquifer

- For the location to be attractive for geothermal power production, the rocks in the aquifer must have high permeability and porosity
  - Porosity: cavities in the rock (granite has low porosity; clay and gravel have high porosity)
  - Permeability: ability for water to flow through rock (clay has low permeability; gravel has high permeability)
- Note: hot aquifer water often contains a high amount of dissolved solids



# Basic Operation





# Basic Operation

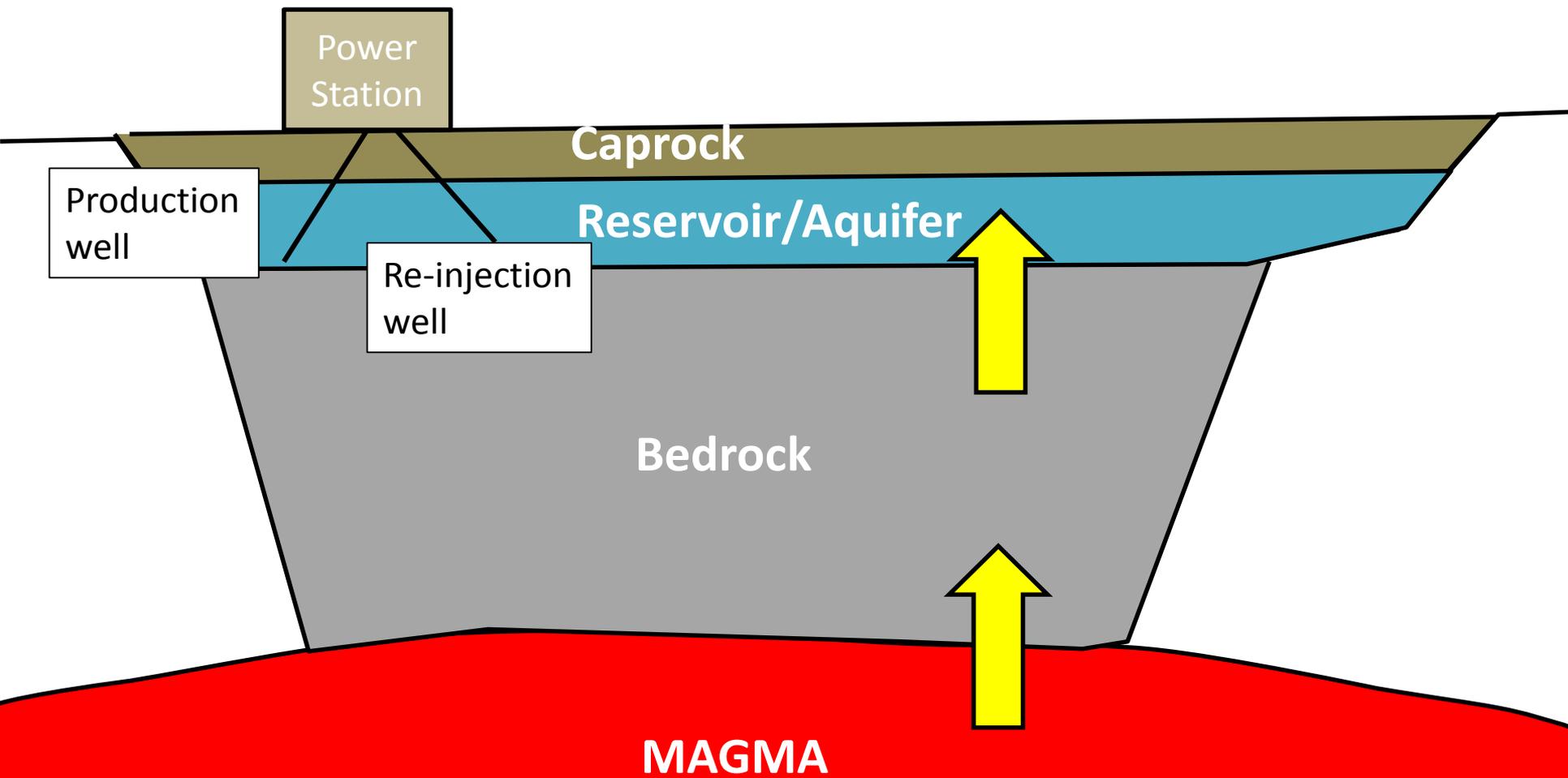
1. Drill well(s) into hot aquifer
2. Aquifer water/steam is under pressure and naturally rises to the surface (pressure can be 4-8 MPa)
3. Exchange heat through heat engine
4. Generate electricity
5. Condense back into water
6. Re-inject into aquifer (through another well)

Step 5 is optional—it is possible to vent steam directly into atmosphere, but this lowers efficiency and can have environmental concerns.

Step 6 is also optional, but it prolongs the lifespan of the well



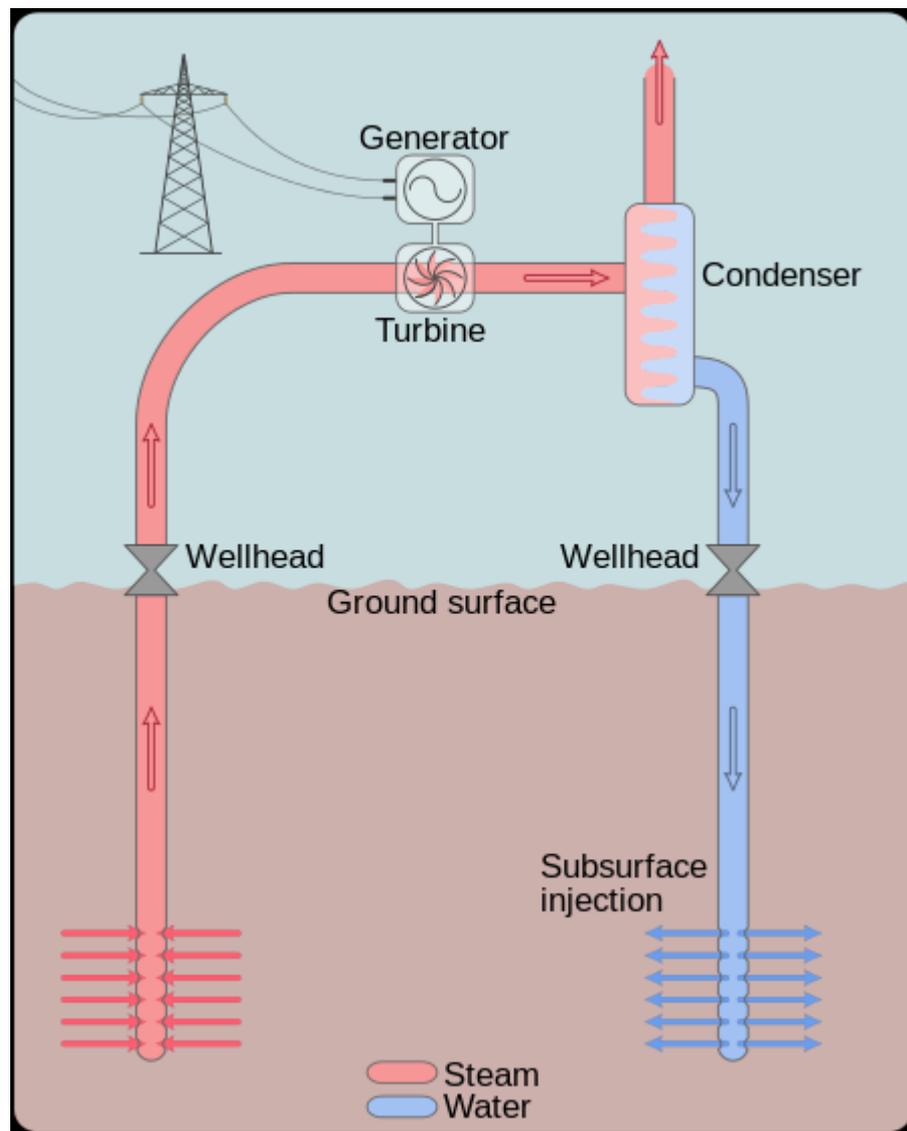
# Basic Operation





# Dry Steam Geothermal

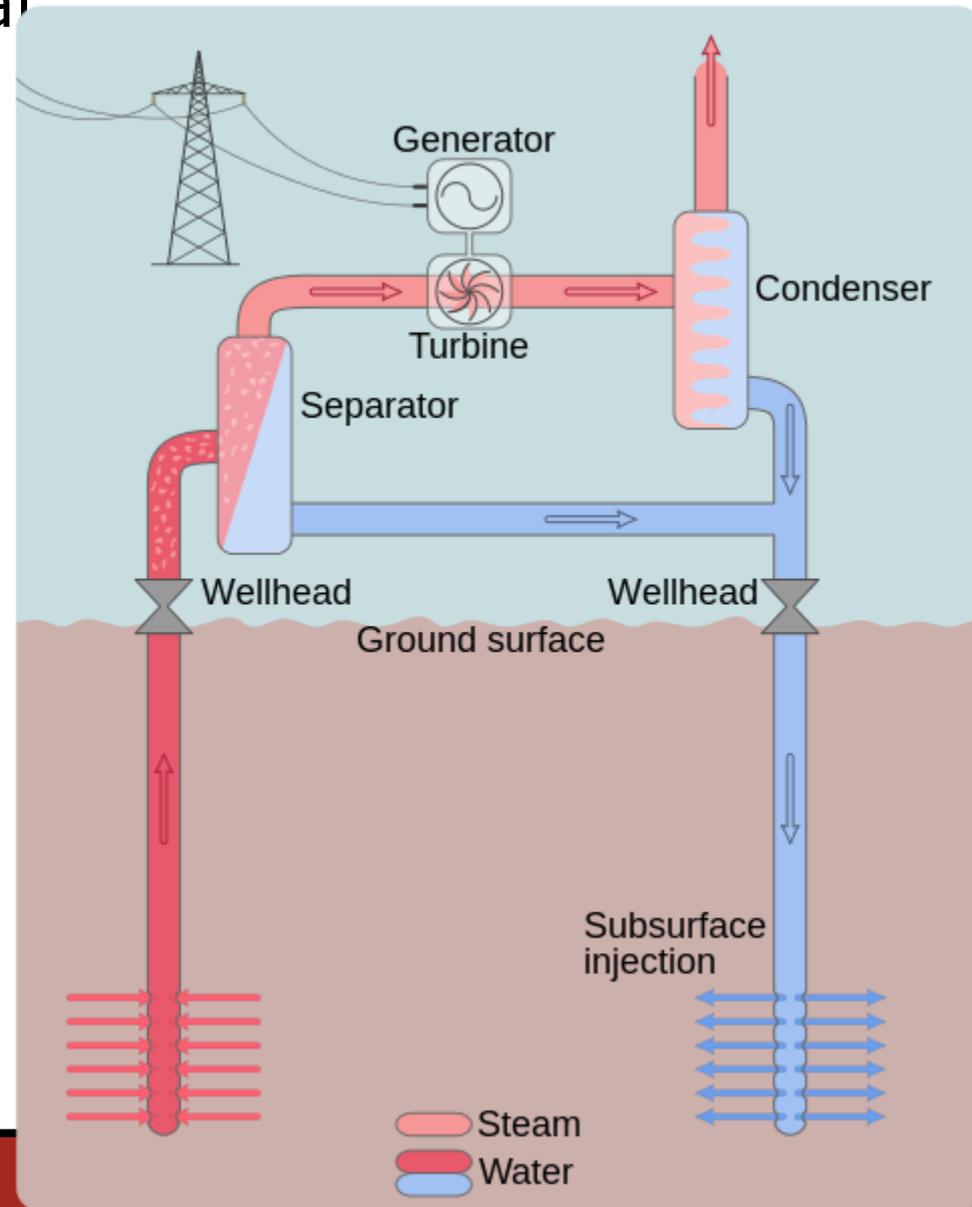
- Used when the source is vapor-dominated
- Most commercially viable and desirable
- Steam can be directly run through a steam turbine (Rankine cycle)
- Efficiencies (<20%) are lower than traditional steam turbines, due to the lower input temperature





# Single Flash Geothermal

- Fluid reaching surface may be vapor (hot water turning into steam is called “flashing”) or hot water at high pressure
- Separator is needed to protect the turbine
- Hot, high temperature water is flashed into steam and run through turbine
- Unflashed water and condensed steam are re-injected





# Binary Cycle Geothermal

- Used primarily as an alternative to single-flash plants
- Working fluid in steam turbine has a lower boiling point than water (e.g. butane, pentane)
- Hot geothermal fluid is passed through a heat exchanger with the working fluid before being pumped into the ground (close loop system)
- Also known as "Organic Rankine Cycle"



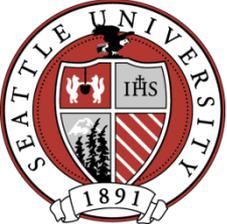
# Hot Dry Rock

- HDR sites are not yet commercially viable
- Low thermal conductivity of crust rock makes it difficult to extract the heat
- Basic idea: inject water into the HDR first to fracture it, then to be heated and extracted
- Challenges: more expensive than other methods; water loss; fracturing is not well controlled



# Geothermal Plants

- Geothermal power plants are relatively modest in size (<100 MW, but 50MW is more typical) compared to coal, and nuclear
- Piping steam/hot water from several wells far apart results in high losses and expenses
- Possible to rotate production across geothermal field over many years, allowing a depleted field to recharge from natural influx of heat



# Environmental Considerations

- Noise: prospecting for geothermal sites (drilling, release of steam) causes noise pollution
- Seismic: some evidence that geothermal plants cause microseismic activity (but geothermal plants tend to be located in seismically active places)
- Emissions: certain dissolved gases are not condensed and re-injected—CO<sub>2</sub>, hydrogen sulphide, Sulphur dioxide, hydrogen, methane and nitrogen; also present can be heavy metals



## Reading

- K.H. Williamson, R.P. Gunderson, G.M. Hamblin, D.L. Gallup and K. Kitz, "Geothermal Power Technology," Proceedings of the IEEE, vol. 89, no. 12, 1783-1792, Aug. 2002 (available on IEEE Xplore through the SU library)