06-Three Phase Circuits

Text: Chapter 8.2

ECEGR 3500

Electrical Energy Systems

Professor Henry Louie

Overview

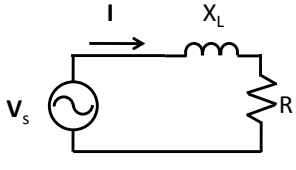
- Three Phase Voltage
- Delta, Wye Connections
- Load Connections

Questions

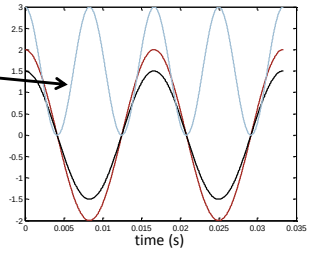
- How is three phase different from single phase?
- How can circuit elements be connected to make three phase systems?
- Why do some electrical panels say 208/120 or 480/277?

Single Phase

- We have analyzed single phase circuits
- Recall:
  - Power pulsates at twice the frequency of voltage, current
  - Two conductors are needed

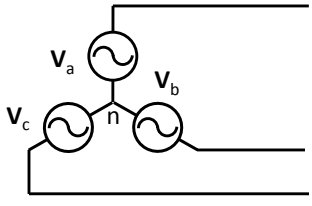


Instantaneous power



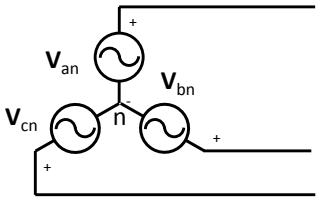
Three Phase

- Consider the connection of three, single phase voltage sources $\mathbf{V}_a$, $\mathbf{V}_b$, and $\mathbf{V}_c$
- Known as Three Phase

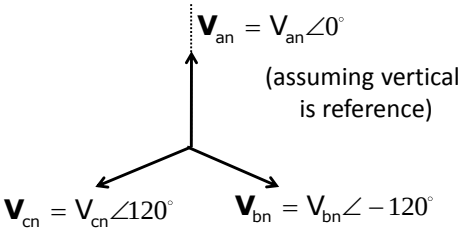


Three Phase

- Subscript n (neutral) notes that voltages are referenced to a common node
- Magnitude of phase voltage to neutral is equal for any phase

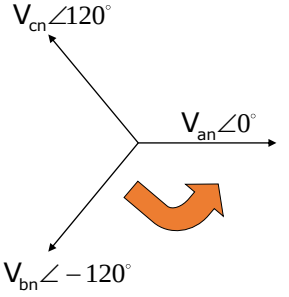


vector diagram

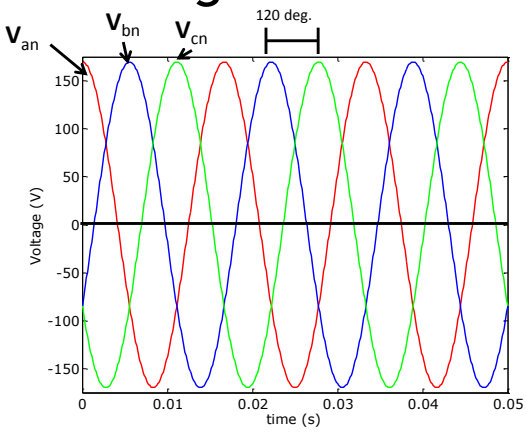


Three Phase

- Power systems use 3-phase
- We are concerned with balanced 3-phase
- Balanced circuit conditions:
  - impedances are equal for each phase
  - voltage source phasors have equal magnitude and have a 120 deg. phase shift
  - a, b, c phase rotation



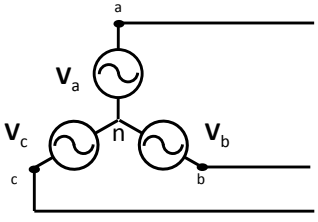
Three Phase Voltage



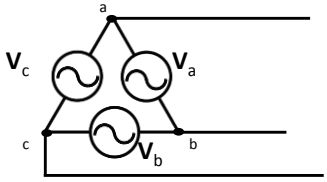
Note: instantaneous values sum to 0

Three Phase Voltage

Two common configuration of three phase voltage sources:



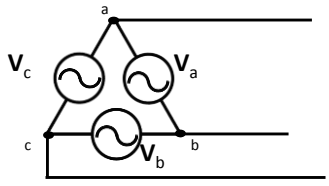
Y (Wye)

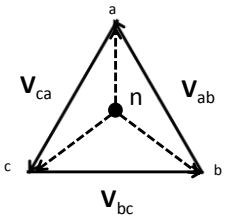


Delta

Three Phase Voltage

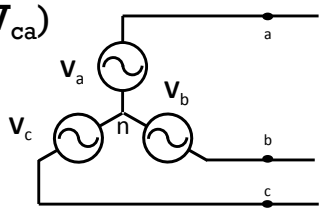
- Neutral point in Delta-connected sources
  - Equidistant between points a, b and c
  - Voltage magnitude between a-n, b-n, and c-n are equal (and less than V_{ab}, V_{bc} and V_{ca})



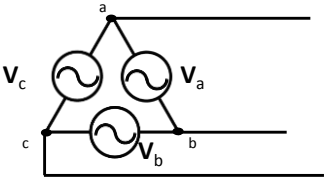


Three Phase Voltage

- Phase voltage: the voltage across each source (e.g.: $\mathbf{V}_a$, $\mathbf{V}_b$, and $\mathbf{V}_c$)
- Line voltage: the voltage between the lines of the conductors (also called "Line-to-Line voltage") for example (e.g.: $\mathbf{V}_{ab}$, $\mathbf{V}_{bc}$, and $\mathbf{V}_{ca}$)



Y



Delta

Three Phase Voltage

For Y-connected sources:

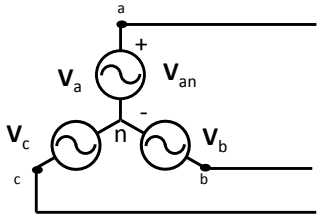
Phase Voltages

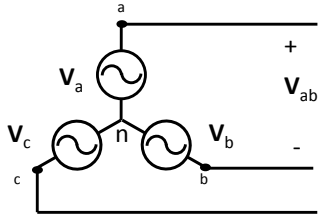
$$\mathbf{V}_{an} = \mathbf{V}_a$$
$$\mathbf{V}_{bn} = \mathbf{V}_b$$
$$\mathbf{V}_{cn} = \mathbf{V}_c$$

Line Voltages

by KVL

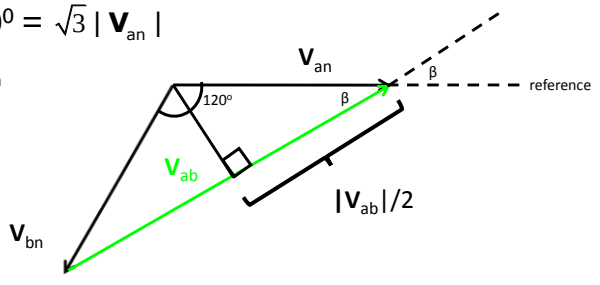
$$\mathbf{V}_{ab} = \mathbf{V}_{an} - \mathbf{V}_{bn} = \mathbf{V}_{an}(\sqrt{3}\angle 30^\circ)$$
$$\mathbf{V}_{bc} = \mathbf{V}_{bn} - \mathbf{V}_{cn} = \mathbf{V}_{bn}(\sqrt{3}\angle 30^\circ)$$
$$\mathbf{V}_{ca} = \mathbf{V}_{cn} - \mathbf{V}_{an} = \mathbf{V}_{cn}(\sqrt{3}\angle 30^\circ)$$





Three Phase Voltage

- Generically:
  - $\beta = 30^\circ$
  - $|\mathbf{V}_{ab}| = 2|\mathbf{V}_{an}|\cos 30^0 = \sqrt{3}\,|\mathbf{V}_{an}|$
- So that $\mathbf{V}_{ab} = (\sqrt{3}\angle 30^\circ)\mathbf{V}_{an}$



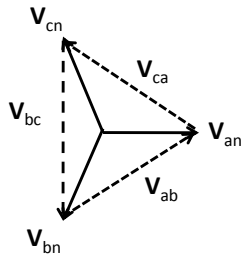
Three Phase Voltage

$$\mathbf{V}_{an} = \mathbf{V}_{bn}(1\angle 120^\circ)$$
$$\mathbf{V}_{bn} = \mathbf{V}_{cn}(1\angle 120^\circ)$$
$$\mathbf{V}_{cn} = \mathbf{V}_{an}(1\angle 120^\circ)$$

$$\mathbf{V}_{ab} = \mathbf{V}_{bc}(1\angle 120^\circ)$$
$$\mathbf{V}_{bc} = \mathbf{V}_{ca}(1\angle 120^\circ)$$
$$\mathbf{V}_{ca} = \mathbf{V}_{ab}(1\angle 120^\circ)$$

Balanced sets

Vector Diagram



Three Phase Voltage

For Delta-connected sources:

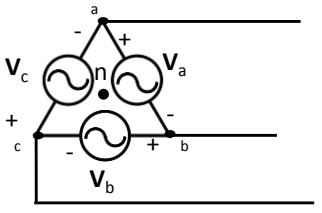
Phase Voltages

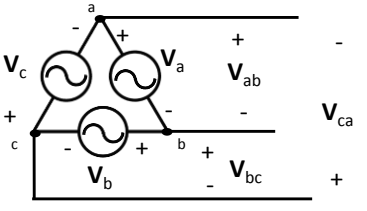
$$\mathbf{V}_a$$
$$\mathbf{V}_b$$
$$\mathbf{V}_c$$

Phase voltage and line voltages are the same in Delta connections

Line Voltages

$$\mathbf{V}_{ab} = \mathbf{V}_a$$
$$\mathbf{V}_{bc} = \mathbf{V}_b$$
$$\mathbf{V}_{ca} = \mathbf{V}_c$$





Three Phase Voltage

- We can convert to and from line-neutral and line-line voltages as:

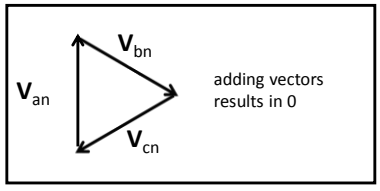
$$\mathbf{V}_{an} = \frac{\mathbf{V}_{ab}}{\sqrt{3}} \angle -30°$$
$$\mathbf{V}_{bn} = \frac{\mathbf{V}_{bc}}{\sqrt{3}} \angle -30°$$
$$\mathbf{V}_{cn} = \frac{\mathbf{V}_{ca}}{\sqrt{3}} \angle -30°$$

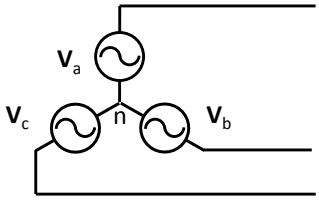
$$\mathbf{V}_{ab} = \mathbf{V}_{an}(\sqrt{3}\angle 30^\circ)$$
$$\mathbf{V}_{bc} = \mathbf{V}_{bn}(\sqrt{3}\angle 30^\circ)$$
$$\mathbf{V}_{ca} = \mathbf{V}_{cn}(\sqrt{3}\angle 30^\circ)$$

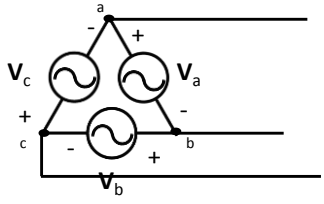
Three Phase Voltage

Voltages (line or phase) sum to zero

$$\mathbf{V}_a + \mathbf{V}_b + \mathbf{V}_c = 0$$
$$\mathbf{V}_{an} + \mathbf{V}_{bn} + \mathbf{V}_{cn} = 0$$
$$\mathbf{V}_{ab} + \mathbf{V}_{bc} + \mathbf{V}_{ca} = 0$$

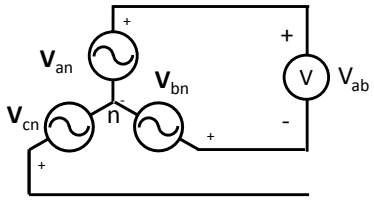






Three Phase Voltage

- Consider a voltmeter placed as shown
- If $|\mathbf{V}_{an}| = 120$ V, then what value is displayed on the voltmeter?



Three Phase Voltage

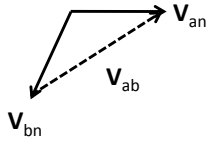
- Analytically:

$$\mathbf{V}_{ab} = \mathbf{V}_{an} - \mathbf{V}_{bn} = 120\angle 0^\circ - 120\angle -120^\circ$$

$$\mathbf{V}_{ab} = (120 + j0) - (-60 - j103.92) = 180 + j103.92 = 120\angle -120^\circ = 208\angle 30^\circ \text{ V}$$

- 208V is displayed and the phasor leads $\mathbf{V}_{an}$ by 30 degrees
- By inspection:

$$|\mathbf{V}_{ab}| = 2V_{an}\cos 30^o = \sqrt{3}\,|\mathbf{V}_{an}|$$

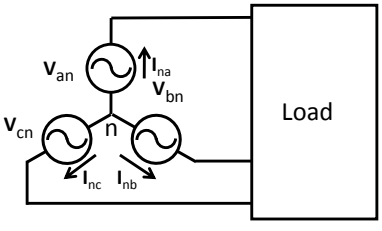


Three Phase Current

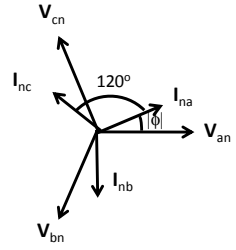
- Phase current: current flowing through the voltage sources in a three-phase circuit
- Line current: current flowing from a three-phase source to a load
- Each set of phase currents and sets of line currents are balanced:
  - Sum to zero
  - Equal in magnitude
  - Displaced by 120 degrees

Three Phase Current

Phase currents in Y-connected sources:



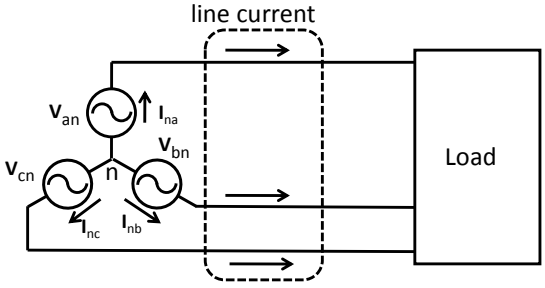
Vector Diagram



V_{an} is used as reference

Three Phase Current

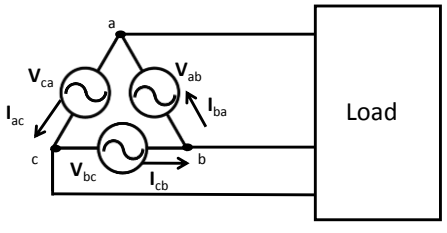
Phase currents in Y-connected sources:



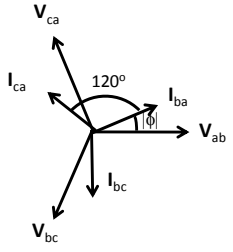
Y-connections:
phase current = line current

Three Phase Current

Phase currents in Delta-connected sources:



Vector Diagram



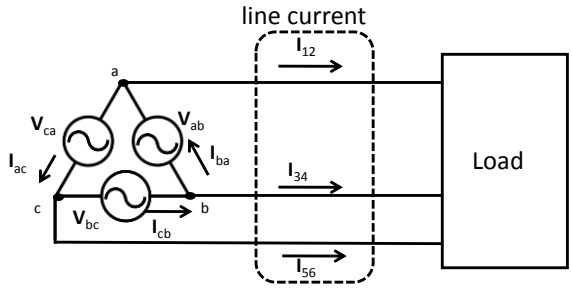
V_{ab} is used as reference

Three Phase Current

Line currents in Delta-connected sources:

$$\mathbf{I}_{12} = \mathbf{I}_{ba} - \mathbf{I}_{ac} = \mathbf{I}_{ba}(\sqrt{3}\angle -30^\circ)$$
$$\mathbf{I}_{34} = \mathbf{I}_{cb} - \mathbf{I}_{ba} = \mathbf{I}_{cb}(\sqrt{3}\angle -30^\circ)$$
$$\mathbf{I}_{56} = \mathbf{I}_{ac} - \mathbf{I}_{cb} = \mathbf{I}_{ac}(\sqrt{3}\angle -30^\circ)$$

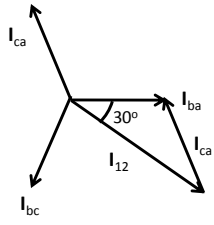
KCL at nodes a, b, c



Three Phase Current

Using $\mathbf{I}_{ba}$ as reference

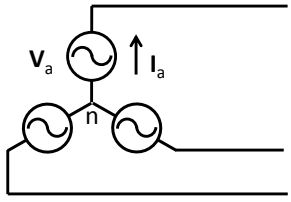
$$\mathbf{I}_{12} = \mathbf{I}_{ba} - \mathbf{I}_{ac} = \mathbf{I}_{ba}\sqrt{3}\angle -30^\circ$$



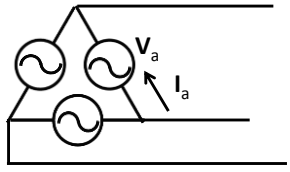
Summary

Line Current = $\mathbf{I}_a$
Phase Current = $\mathbf{I}_a$
Line Voltage = $\mathbf{V}_a \times \sqrt{3}\angle 30°$
Phase Voltage = $\mathbf{V}_a$

Line Current = $\mathbf{I}_a \times \sqrt{3}\angle -30°$
Phase Current = $\mathbf{I}_a$
Line Voltage = $\mathbf{V}_a$
Phase Voltage = $\mathbf{V}_a$



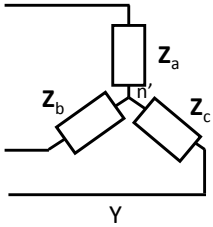
Y (Wye)



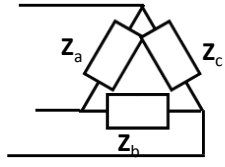
Delta

Three Phase Loads

- Three phase sources are connected to three phase loads in two common configurations
  - Y (wye)
  - Delta
- Y sources can be connected to delta and/or Y loads
- Delta sources can be connected to delta and/or Y loads



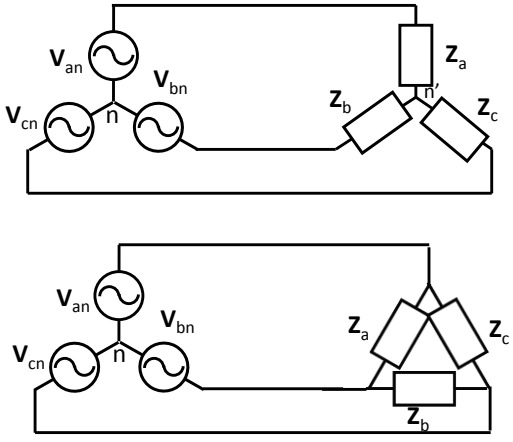
Y



Delta

Three Phase Loads

Examples with a Y source



Three Phase Loads

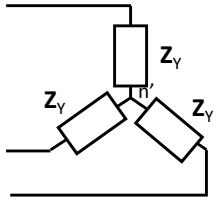
- Circuit analysis is easier if loads are connected as Y
- We can transform balanced Delta connected loads into balanced Y connected loads mathematically by

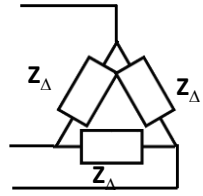
$$\mathbf{Z}_Y = \frac{\mathbf{Z}_\Delta}{3}$$

  - $\mathbf{Z}_Y$: complex impedance of Y-connected load (Ohms)
  - $\mathbf{Z}_\Delta$: complex impedance of a Delta-connected load (Ohms)
- Results only apply to terminal conditions

Three Phase Loads Example

Each phase of a Y-connected load has an impedance of 6 + j12. Find the impedance of the equivalent delta-connected load.

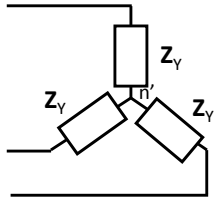


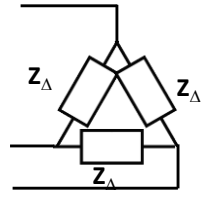


Three Phase Loads Example

- Each phase of a Y-connected load has an impedance of 6 + j12. Find the impedance of the equivalent delta-connected load.

Answer: $Z_\Delta = 18 + j36\Omega$





Summary

- Three phase systems: more efficient use of conductors; provides rotating magnetic fields; non-pulsating power delivery
- Balanced three phase: a,b,c (voltage, current) phases displaced by 120 degrees and have equal magnitude
- Line-line and line-neutral voltage relationship: $\mathbf{V}_{LL} = \mathbf{V}_{LN}(\sqrt{3}\angle 30^{\circ})$