07-Three Phase Analysis

Text: Chapter 8.3

ECEGR 3500

Electrical Energy Systems

Professor Henry Louie

Overview

- Three Phase Power
- Three Phase Analysis
- Per Phase Analysis
- Practical Considerations

Questions

- How does three phase power compare to single phase power?
- What is the "neutral" connection in a three phase system?

Conventions & Assumptions

- Voltage: line-to-line in rms
- Current: line in rms
- Current direction: source to load
- Balanced three phase

Three-Phase Power

Benefits of three-phase power:
- efficient use of conductors over three, single phases
- rotating field is needed for some loads (e.g. three phase motors)
- effective for power transfer
- per-phase analysis can be used in many cases
- constant power delivery to three phase loads

Three-Phase Power

- Total power delivered is the sum of power delivered by each phase

$$\mathbf{S}_{3\phi} = \mathbf{V}_{an}\mathbf{I}^*_{na} + \mathbf{V}_{bn}\mathbf{I}^*_{nb} + \mathbf{V}_{cn}\mathbf{I}^*_{nc}$$
$$= 3\mathbf{V}_{an}\mathbf{I}^*_{na} \quad \text{(due to symmetry)}$$
$$= 3\mathbf{S}$$

where
- $\mathbf{S}_{3\phi}$: total three phase complex power (VA)
- $\mathbf{S}$: single phase complex power (VA)

Three-Phase Power

Similarly

$$P_{3\phi} = \text{Re}\{\mathbf{V}_{an}\mathbf{I}^*_{na}\} + \text{Re}\{\mathbf{V}_{bn}\mathbf{I}^*_{nb}\} + \text{Re}\{\mathbf{V}_{cn}\mathbf{I}^*_{nc}\}$$
$$= 3\text{Re}\{\mathbf{V}_{an}\mathbf{I}^*_{na}\}$$
$$= 3P$$

and

$$Q_{3\phi} = \text{Im}\,\{\mathbf{V}_{an}\mathbf{I}^*_{na}\} + \text{Im}\,\{\mathbf{V}_{bn}\mathbf{I}^*_{nb}\} + \text{Im}\,\{\mathbf{V}_{cn}\mathbf{I}^*_{nc}\}$$
$$= 3\text{Im}\{\mathbf{V}_{an}\mathbf{I}^*_{na}\}$$
$$= 3Q$$

Assume that power (real, reactive, complex) is stated as the total power, not the per-phase power.

Exercise

A motor consumes 3000 W and 150 Var. What is the complex power provided by B-phase?

Exercise

A motor consumes 3000 W and 150 Var. What is the complex power provided by B-phase?

Answer: **S** = 1000 + 50 VA. The power is evenly divided by each of the phases.

Three-Phase Power

What about instantaneous power?

$$p_{3\phi}(t) = i_{max} v_{max} \cos(\omega t + 0^\circ)\cos(\omega t + \theta_{ia}^\circ) + i_{max} v_{max} \cos(\omega t - 120^\circ)\cos(\omega t + \theta_{ia}^\circ - 120^\circ) + i_{max} v_{max} \cos(\omega t + 120^\circ)\cos(\omega t + \theta_{ia}^\circ + 120^\circ)$$

$$p_{3\phi}(t) = \frac{1}{2} v_{max} i_{max} [\cos(-\theta_{ia}) + \cos(2\omega t + \theta_i) + \cos(-\theta_{ia}) + \cos(2\omega t + \theta_i - 120^\circ) + \cos(-\theta_{ia}) + \cos(2\omega t + \theta_i + 120^\circ)]$$

Using Product-to-sum identity

$$\cos(u)\cos(v) = \frac{1}{2}[\cos(u-v) - \cos(u+v)]$$

Next use: $\cos(x+y) = \cos x \cos y - \sin x \sin y$

Three-Phase Power

$$p_{3\phi}(t) = \frac{1}{2} v_{max} i_{max} [3\cos(-\theta_{ia}) + \cos(2\omega t + \theta_i) +$$
$$\cos(2\omega t + \theta_i)\cos(-120^\circ) - \sin(2\omega t + \theta_i)\sin(-120^\circ)$$
$$\cos(2\omega t + \theta_i)\cos(120^\circ) - \sin(2\omega t + \theta_i)\sin(120^\circ)]$$

$$p_{3\phi}(t) = \frac{1}{2} v_{max} i_{max} [3\cos(-\theta_{ia}) + \cos(2\omega t + \theta_i) +$$
$$\frac{-1}{2}\cos(2\omega t + \theta_i) - \frac{-\sqrt{3}}{2}\sin(2\omega t + \theta_i)$$
$$\frac{-1}{2}\cos(2\omega t + \theta_i) - \frac{\sqrt{3}}{2}\sin(2\omega t + \theta_i)]$$

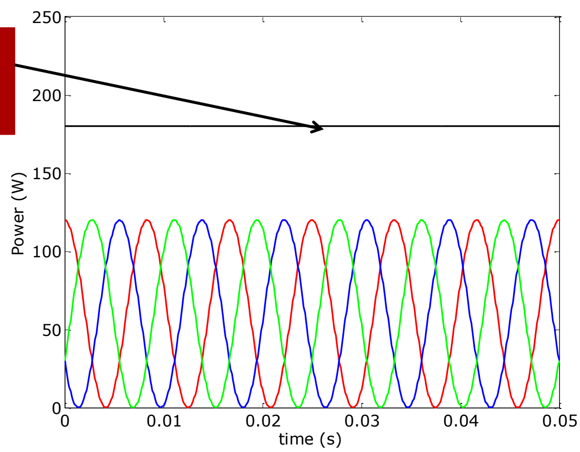
$$p_{3\phi}(t) = \frac{3}{2} v_{max} i_{max} \cos(\phi)$$

What does this say about the real power?

Three-Phase Power

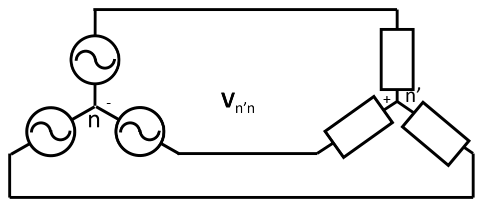
Total instantaneous power is constant!

This is an important advantage of using 3 phase (no pulsating torque), makes for smooth rotation of generators and motors.



Three-Phase Analysis

- Consider the Y-connected source and load
- Determine $\mathbf{V}_{n'n}$



Three-Phase Analysis

- Analysis is easier using admittance, **Y**

$$\mathbf{Y} = \frac{1}{\mathbf{Z}}$$

- Line current is equal to phase current

$$\left.\begin{aligned} \mathbf{I}_{na} &= \mathbf{Y}(\mathbf{V}_{an} - \mathbf{V}_{n'n}) \\ \mathbf{I}_{nb} &= \mathbf{Y}(\mathbf{V}_{bn} - \mathbf{V}_{n'n}) \\ \mathbf{I}_{nc} &= \mathbf{Y}(\mathbf{V}_{cn} - \mathbf{V}_{n'n}) \end{aligned}\right\} \text{due to wye connection}$$

Three-Phase Analysis

- Summing the line current

$$\mathbf{I}_{na} + \mathbf{I}_{nb} + \mathbf{I}_{nc} = \mathbf{Y}(\mathbf{V}_{an} + \mathbf{V}_{bn} + \mathbf{V}_{cn}) - 3\mathbf{Y}\mathbf{V}_{nn'}$$

- Previously we showed that

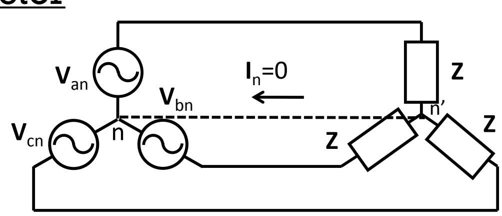
$$\mathbf{I}_{na} + \mathbf{I}_{nb} + \mathbf{I}_{nc} = 0$$
$$\mathbf{V}_{an} + \mathbf{V}_{bn} + \mathbf{V}_{cn} = 0$$

- Therefore

$$0 = \mathbf{Y}(0) - 3\mathbf{Y}\mathbf{V}_{nn'}$$
$$=> \mathbf{V}_{nn'} = 0$$

Three-Phase Analysis

- Since $\mathbf{V}_{nn'} = 0$, we can make a hypothetical connection without affecting the circuit
- Called the Neutral Conductor
- For <u>balanced sources and loads, no current flows on the neutral conductor</u>

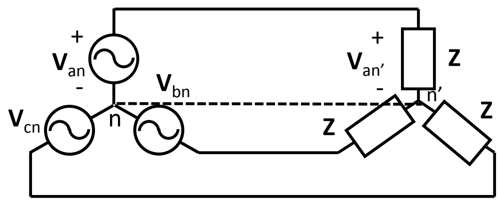


Three-Phase Analysis

Let's now analyze the current through the a-phase load

$$\mathbf{V}_{an} = \mathbf{V}_{an'}$$

$$\mathbf{I}_{na} = \frac{\mathbf{V}_{an'}}{\mathbf{Z}}$$

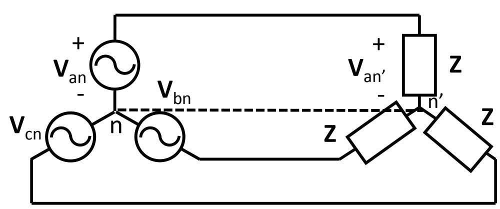


Three-Phase Analysis

Let's now analyze the current through the b-phase load

$$\mathbf{V}_{bn} = \mathbf{V}_{bn'}$$

$$\mathbf{I}_{nb} = \frac{\mathbf{V}_{bn'}}{\mathbf{Z}}$$



(b-phase current in terms of a-phase current)

But

$$\mathbf{V}_{bn} = \mathbf{V}_{an}(1\angle -120^\circ)$$

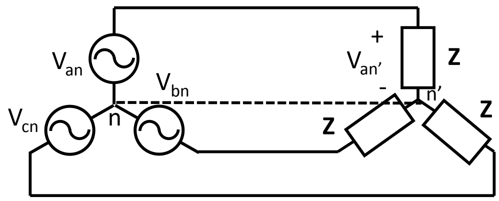
$$\mathbf{I}_{nb} = \frac{\mathbf{V}_{bn'}}{\mathbf{Z}} = \frac{\mathbf{V}_{an}(1\angle -120^\circ)}{\mathbf{Z}} = \mathbf{I}_{na}(1\angle -120^\circ)$$

Three-Phase Analysis

- Let's now analyze the current through the c-phase load

$$\mathbf{V}_{cn} = \mathbf{V}_{cn'}$$

$$\mathbf{I}_{nc} = \frac{\mathbf{V}_{cn'}}{\mathbf{Z}}$$



(c-phase current in terms of a-phase current)

- But

$$\mathbf{V}_{cn} = \mathbf{V}_{an}(1\angle 120^\circ)$$

$$\mathbf{I}_{nc} = \frac{\mathbf{V}_{cn'}}{\mathbf{Z}} = \frac{\mathbf{V}_{an}(1\angle 120^\circ)}{\mathbf{Z}} = \mathbf{I}_{na}(1\angle 120^\circ)$$

Three-Phase Analysis

- Once $\mathbf{I}_{na}$ is solved for, simply shift it by -/+120 degrees to find $\mathbf{I}_{nb}$ and $\mathbf{I}_{nc}$
- Phases can be conceptually decoupled
- No need to solve all three phases
  - Solve for a-phase (current or voltage)
  - Shift +120° for c-phase, and -120° for b-phase
- We can therefore do a per-phase analysis

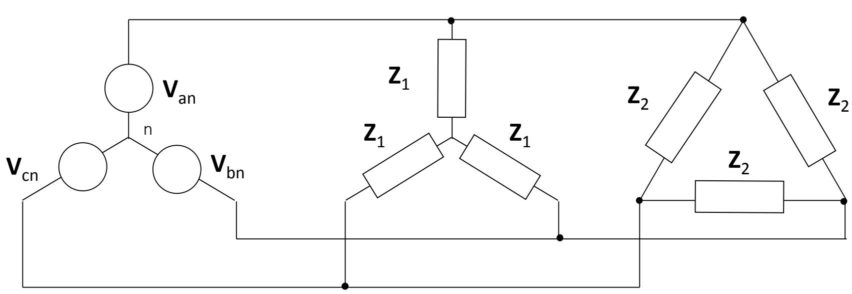
Three-Phase Analysis

- Balanced Three-Phase Theorem
- Assume:
  - balanced three-phase system
  - all loads and sources are Y-connected (or convert them to Y-connected loads/sources)
  - no mutual inductances between phases

then
  - all neutrals have the same voltage
  - the phases are completely decoupled
  - all corresponding network variables occur in balanced sets of the same sequence as the sources

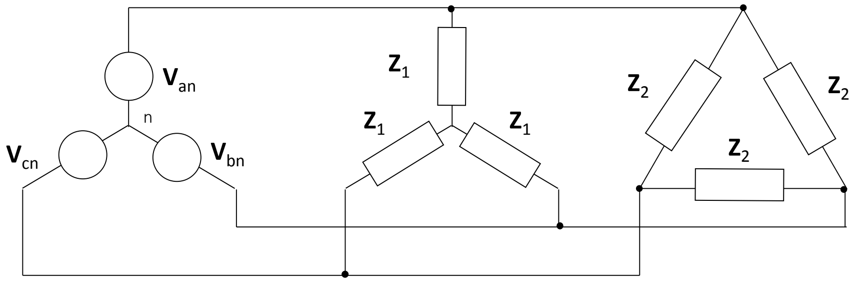
Per-Phase Analysis

- Consider the following circuit
- Is it balanced?



Per-Phase Analysis

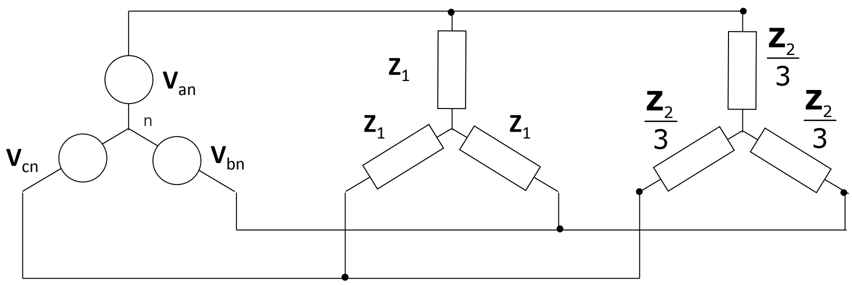
- To analyze this circuit, we first need to find a per-phase equivalent
- Need to transform the Delta load to Y load



SEATTLEU

Per-Phase Analysis

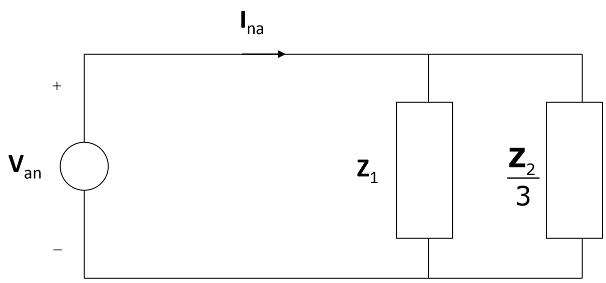
- To analyze this circuit, we first need to find a per-phase equivalent
- Need to transform the Delta load to Y load



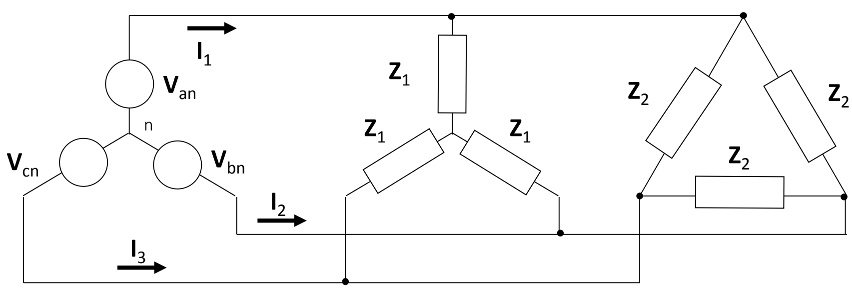
SEATTLEU

Per-Phase Analysis

- Analyze a single phase (arbitrarily a-phase)
- Analyze this phase to find the current, power, etc



Numerical Example



<u>given:</u>

balanced 3-phase source

$\mathbf{V}_{an} = 120\angle 0 \text{ V}$

$\mathbf{Z}_1 = 10 + j2\ \Omega$

$\mathbf{Z}_2 = 12\angle 45^\circ \Omega$

<u>find:</u>

I_{na}, I_{nb}, I_{nc}

total powered delivered to the loads

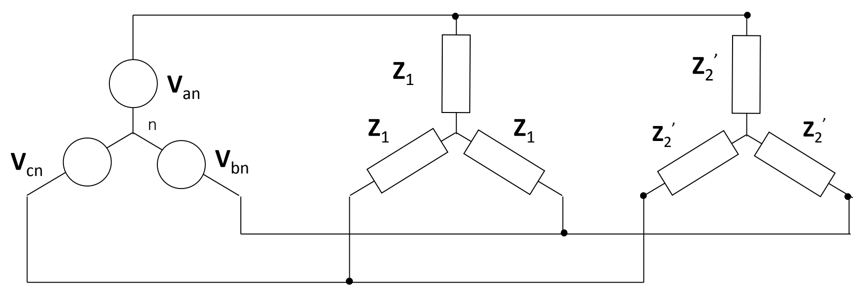
Exercise

Solution outline

- draw per-phase equivalent
- solve circuit for current
- compute single phase power
- translate per-phase values to 3-phase

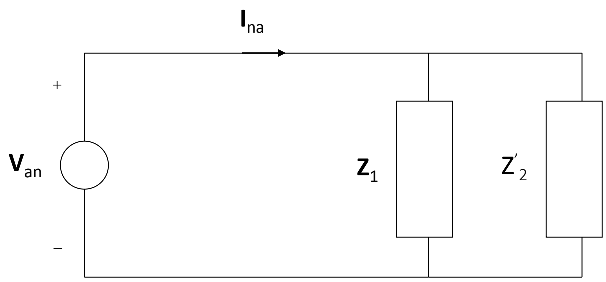
Exercise

- Convert Δ to Y $Z'_2 = 2.82 + j2.82\ \Omega$
- Redraw circuit



Exercise

- Redraw circuit
- Draw per-phase equivalent



$Z'_2 = 2.82 + j2.82\ \Omega$

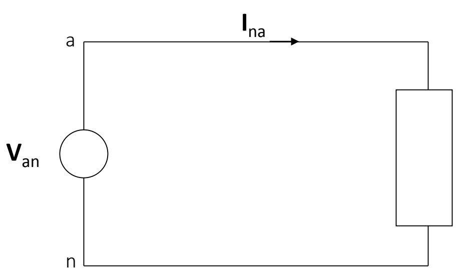
Exercise

- Draw per-phase equivalent
- Solve for I_{an} and S

$\mathbf{V}_{an} = 120\angle 0^\circ\ \text{V}$

$\mathbf{I}_{na} = 40.32\angle -35.68^\circ\ \text{A}$

$\mathbf{S} = 3930.2 + j2822.5\ \text{VA}$



$Z_{\parallel} = \dfrac{\mathbf{Z}_1\mathbf{Z}'_2}{\mathbf{Z}_1 + \mathbf{Z}'_2} = 2.417 + j1.736\Omega$

Exercise

- Shifting and solving

$$\mathbf{I}_{nb} = 40.32\angle\left(-35.68^\circ - 120^\circ\right) = 40.32\angle\left(-155.68^\circ\right)\text{A}$$

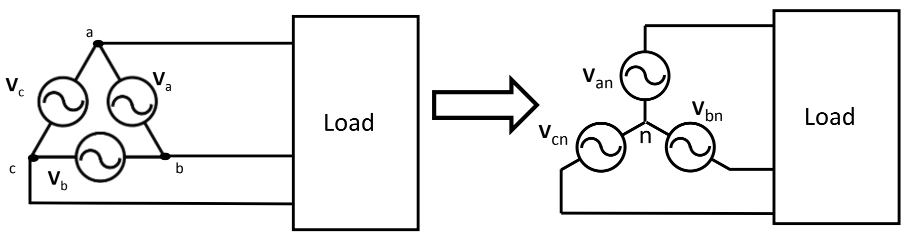
$$\mathbf{I}_{nc} = 40.32\angle\left(-35.68^\circ + 120^\circ\right) = 40.32\angle\left(84.32^\circ\right)\text{A}$$

$$\mathbf{I}_{n} = 0\text{A}$$

$$\mathbf{S}_{3\Phi} = 3\left(3930.19 + j2822.51\right) = 11790.57 + j8467.53\text{VA}$$

Per-Phase Analysis

- If Delta-connected voltage sources are present, convert them to their line-neutral equivalent and then do per-phase analysis



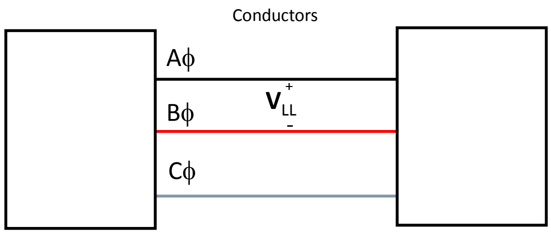
$$\mathbf{V}_{an} = \frac{\mathbf{V}_{ab}}{\sqrt{3}}\angle -30^\circ$$

$$\mathbf{V}_{bn} = \frac{\mathbf{V}_{bc}}{\sqrt{3}}\angle -30^\circ$$

$$\mathbf{V}_{cn} = \frac{\mathbf{V}_{ca}}{\sqrt{3}}\angle -30^\circ$$

For convenience, we usually re-define the reference angle as the a-phase line-neutral voltage

Practical Considerations



$$\mathbf{V}_{LL} = \mathbf{V}_{phase}(\sqrt{3}\angle 30^\circ)$$

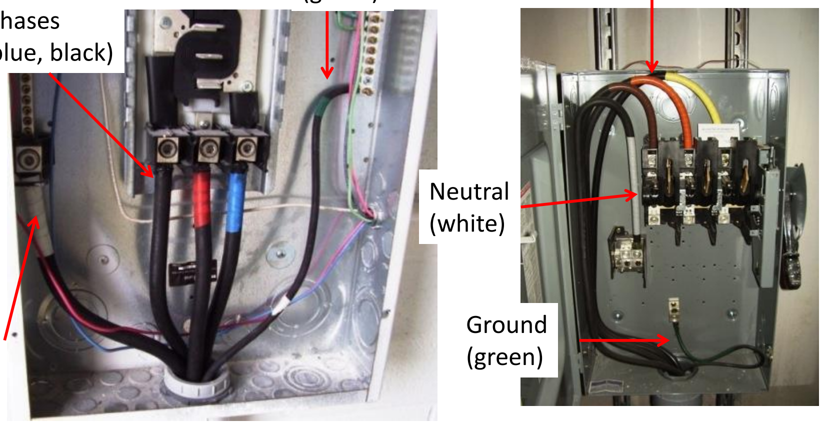
Three phase systems and components often referred to by RMS line-line voltages
e.g. 500kV, 13.6kV, 480V

Three phase systems and components often referred to by $|\mathbf{S}_{3f}|$
e.g. 100 MVA transformer

Practical Considerations

- Neutral usually bonded to ground at service panel (see NEC for details)
- Balanced load assumption can be questionable
  - Neutral current nonzero
  - Neutral not at ground potential

Practical Considerations



Source: www.electrical-design-tutor.com/

Summary

- Current (voltages) sum to zero and no current flows on neutral conductor
- Total **S**, P, Q = three times single phase **S**, P, Q
- Three phase systems analysis: convert into wye connections, solve per-phase