11-Transformer EMF

Text: Chapter 11.1

ECEGR 3500

Electrical Energy Systems

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Overview

- Introduction
- Notation, Variables and Units
- Maxwell's Equations
- Faraday's Law

Introduction

- We know that transformers convert electrical power from one voltage level to another, but how do they work?

- Fundamental operating principle is derived from one of Maxwell's equations (Faraday's Law)

Notation, Variables and Units

- **Bold**: vector quantity
- **E**: electric field intensity (V/m)
- **B**: magnetic flux density (Wb/m^2) or Tesla (T)
- Φ: magnetic flux, (Wb)
- **H**: magnetic field intensity, (A/m)
- **J**: volume current density (C/m^2)
- ρ: volume charge density (C/m^3)

Maxwell's Equations

- Named for James Clerk Maxwell (1831-1879)
- Describe the nature of electromagnetic fields
- Set of four equations
  - Ampere's Law
  - Faraday's Law
  - Gauss's Law (flux)
  - Gauss's Law (charge)

Maxwell's Equations

Faraday's Law and Ampere's Law are especially important for electrical machines, so we focus on them

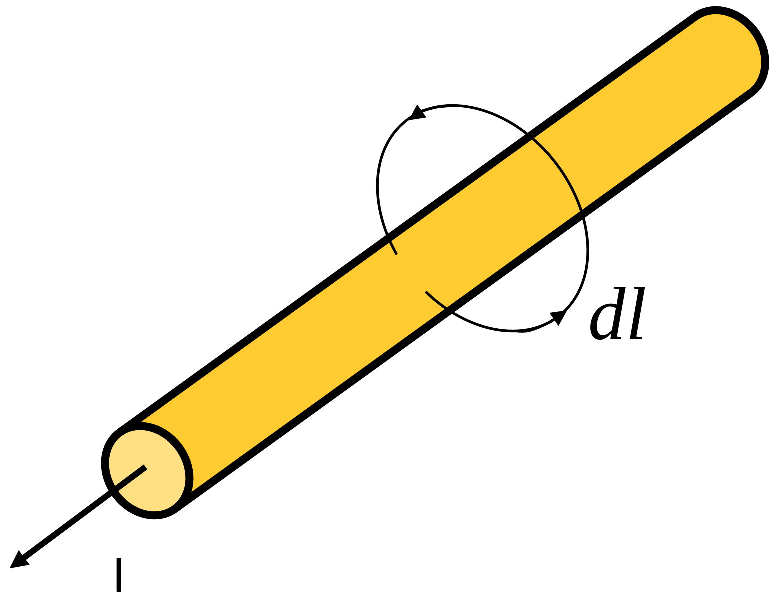
Ampere's Law

- Current through a conductor will produce a circular magnetic field around it
  - Line integral of the magnetic field around the conductor equals the current through it
- Mathematically

$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}$$

or

$$\oint_C \mathbf{H} \cdot d\ell = \oint_s \mathbf{J} \cdot d\mathbf{s} + \int_s \frac{\partial \mathbf{D}}{\partial t} \cdot d\mathbf{s}$$

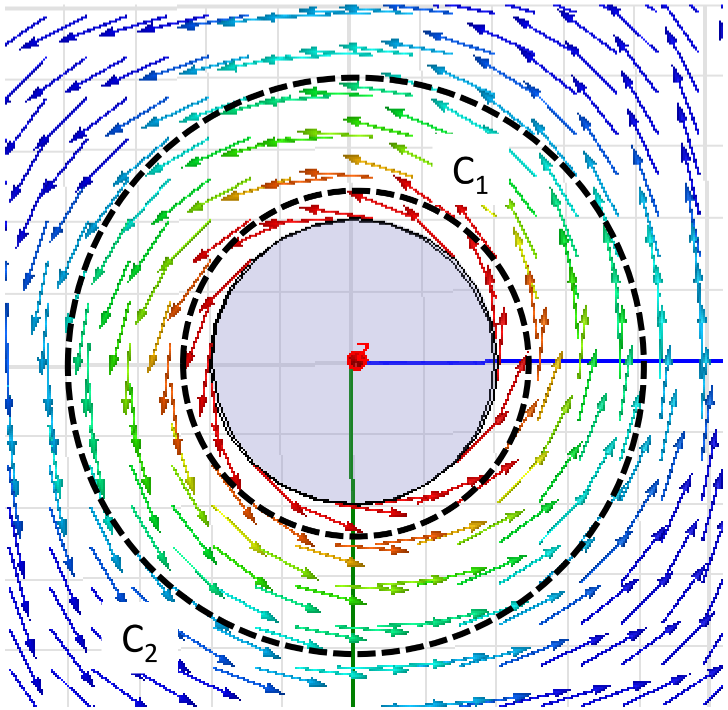


Ampere's Law

- Direction of **H** field is CCW
  - Right Hand Rule
- Field strength decreases as path length increases
  - Field becomes weaker as distance increases

$$\oint_{C_1} \mathbf{H} \cdot d\ell = \oint_{C_2} \mathbf{H} \cdot d\ell = \text{constant}$$

Magnetic Field Intensity **H**



View "down" Z axis (Current out of slide)

Magnetic Flux Density

- Related to **H** is the Magnetic Flux Density (**B**)

$$\mathbf{B} = \mu_0(\mathbf{H}+\mathbf{M})$$

We will assume that **M** = 0

- where:
  - **B**: magnetic flux density (Wb/m^2)
  - **M**: magnetization field (A/m)
  - μ_0: permeability of free space $4\pi E\text{-}7$ (H/m)
  - μ_r: relative permeability

A material is magnetized if $\mathbf{M} \neq 0$
Note: **H** and **M** may have different signs (directions)

Magnetic Flux Density

- Inside <u>linear</u> isotropic homogeneous (LIH) medium

$$\mathbf{B} = \mu_0(\mathbf{H}+\mathbf{M})$$
$$\mathbf{M} = \chi\mathbf{H}$$
$$\mathbf{B} = \mu_0(1+\chi)\mathbf{H}$$
$$\mathbf{B} = \mu_0\mu_r\mathbf{H} = \mu\mathbf{H}$$

Note: permanent magnets are not LIH

- where:
  - χ: magnetic susceptibility of the medium
  - μ_r: relative permeability

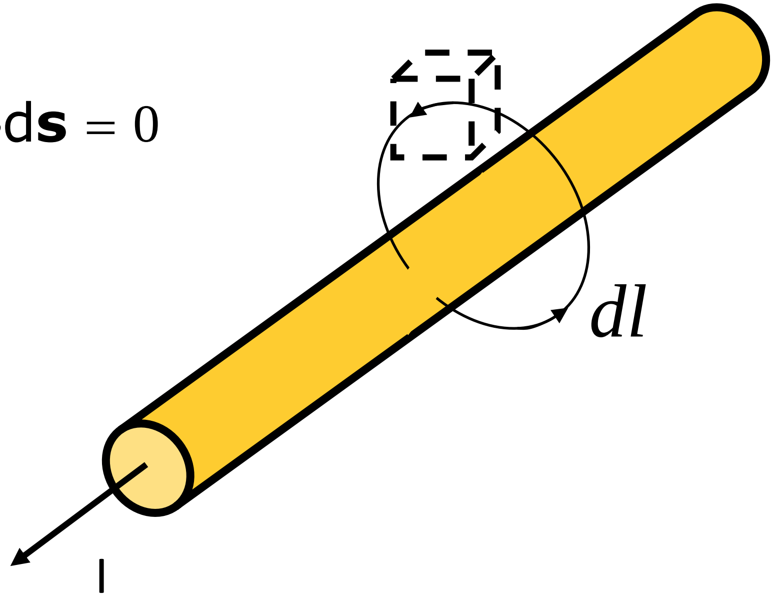
Note: magnetic susceptibility can be positive or negative

Gauss's Law for Magnetic Fields

- The magnetic flux entering a volume is equal to the magnetic flux leaving it (it is continuous)
- Mathematically

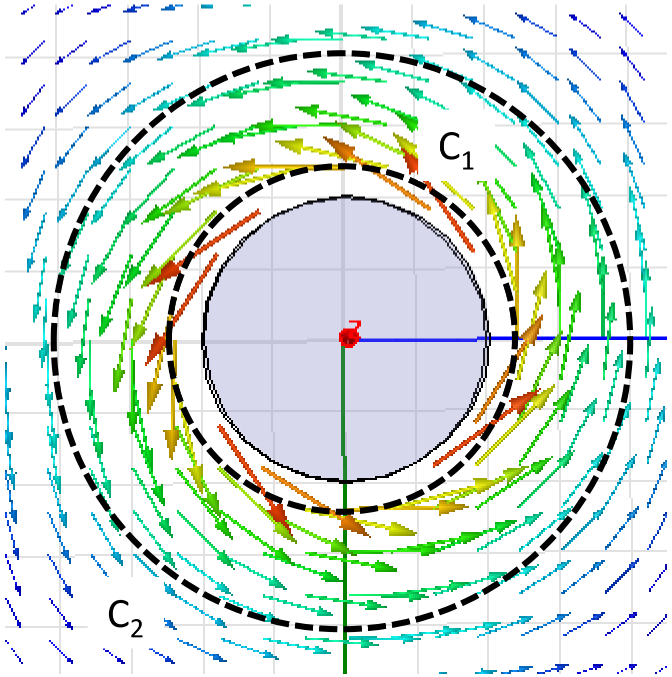
$\nabla \bullet \mathbf{B} = 0$ or $\oint_s \mathbf{B} \bullet d\mathbf{s} = 0$

(divergence is 0)



Magnetic Flux Density

Magnetic Flux Density **B**

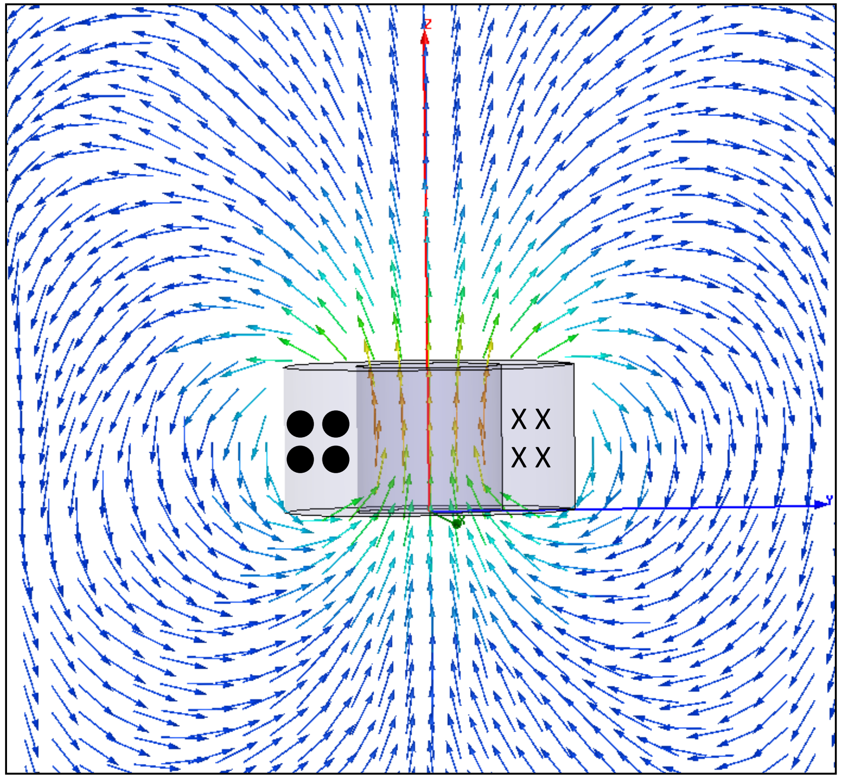


View "down" Z axis (Current out of slide)

- Direction of **B** field is inline with **H** field (**M** = 0 in a vacuum)
- Field strength decreases as path length increases
  - Field becomes weaker as distance increases

Magnetic Fields Around a Solenoid

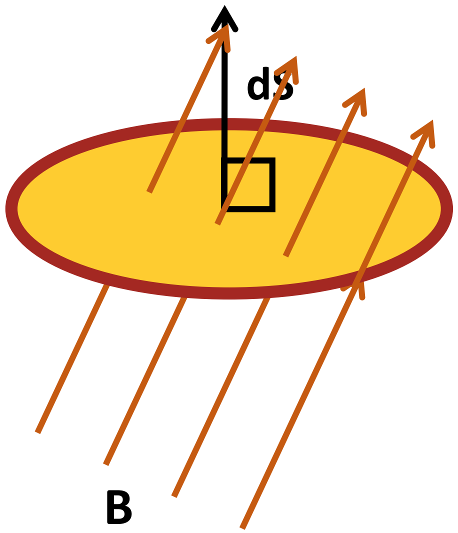
Magnetic Flux Density **B**



Magnetic Flux

- The total magnetic flux, Φ, passing through a surface is:

$$\Phi = \int_s \mathbf{B} \cdot d\boldsymbol{s}$$



- where:

  Φ: magnetic flux (Wb)

Faraday's Law

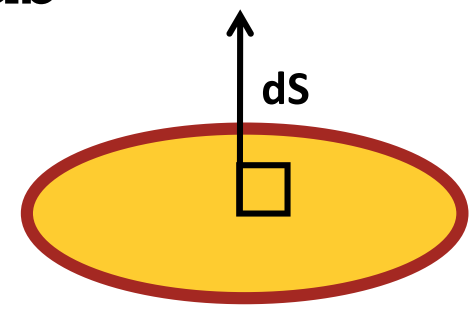
- The change in magnetic flux through a closed path induces a voltage
  - Known as induced electromotive force (emf)
- Mathematically

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \quad \text{or} \quad \oint_C \mathbf{E} \bullet d\ell = -\int_s \frac{\partial \mathbf{B}}{\partial t} \bullet d\mathbf{s}$$

- Understanding Faraday's Law is critical in understanding transformers and electric machines

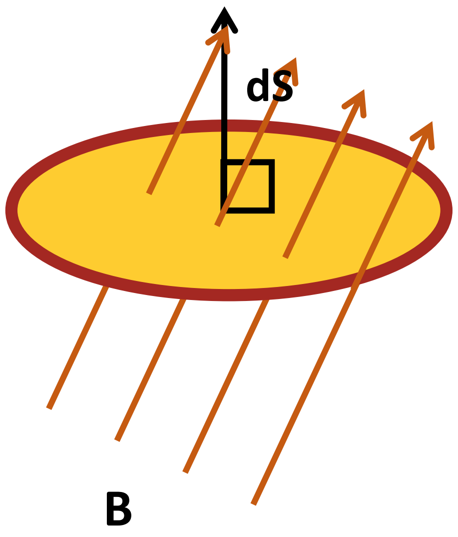
Faraday's Law

Consider a closed loop of wire in a time-varying magnetic field with differential surface d**S**



Faraday's Law

The total magnetic flux, Φ, passing through the loop is: $\Phi = \int_S \mathbf{B} \cdot d\mathbf{s}$



Faraday's Law

Voltage is the electric field multiplied by distance so that

$$\oint_C \mathbf{E} \cdot d\ell = -\int_s \frac{\partial \mathbf{B}}{\partial t} \cdot d\mathbf{s}$$

$$e = -\int_s \frac{\partial \mathbf{B}}{\partial t} \cdot d\mathbf{s} = -\frac{d\Phi}{dt}$$

e: induced electromotive force (emf), (V)

Exercise

Which of the following are ways to increase the voltage induced in a coil?

A) increase the area enclosed by the coil

B) reduce the resistance of the coil

C) increase the time rate of change of the flux density

Exercise

Which of the following are ways to increase the voltage induced in a coil?

A) increase the area enclosed by the coil

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C) increase the time rate of change of the flux density

$$\Phi = \int_s \mathbf{B} \cdot d\boldsymbol{s}$$

$$e = -\frac{d\Phi}{dt}$$

Faraday's Law of Induction

- Actually, the magnetic flux may be constant in time and we can still induce voltage
  - It's the change in flux that matters
- If the coil is stationary and the magnetic flux changes, then e is known as transformer emf
- If the coil is moving and the magnetic flux is constant, then e is known as motional emf

Important to understand this!

Faraday's Law

- A conductor moving in an area of constant flux will have voltage induced in it according to:

$$e = \oint_c (\mathbf{u} \times \mathbf{B}) \cdot d\ell$$

  - **u**: velocity of the conductor (m/s)
- Motional emf is due to a force acting on the free electrons in the conductor that moves them to one side or another
  - More on this force later
- Total induced emf is the sum of transformer and motional emf

Summary

- Current gives rise to **H** and **B** fields, which circulate around the conductor in accordance with the Right Hand Rule
- $|\mathbf{H}|$ decreases as distance from conductor increases
- $|\mathbf{B}|$ is related to $|\mathbf{H}|$ by characteristics of the medium
- Change in flux induces a voltage. The change can have mechanical origins