15-Intro. to Three Phase Transformers

ECEGR 3500

Text: 11.4

Electrical Energy Systems

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Overview

- Y-Y Transformer Connection
- Delta-Delta Transformer Connection
- Y-Delta Transformer Connection
- Delta-Y Transformer Connection

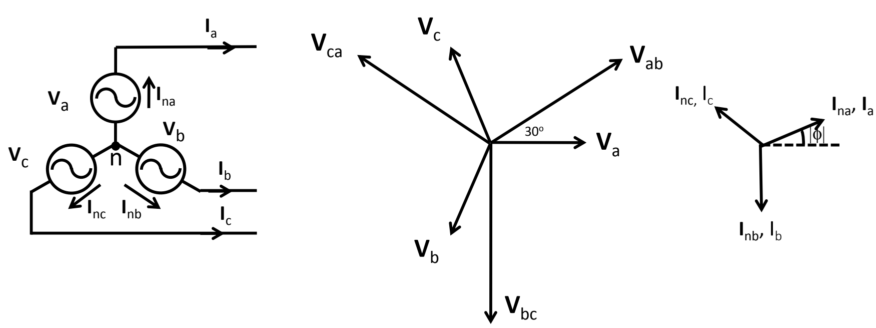
Three-Phase Transformer Connections

- We are generally more concerned with three phase transformers
- Three-phase transformers can be composed of three single-phase transformers or be wound on the same core
- Many combinations are possible:
  - Y-Y
  - Delta-Delta
  - Y-Delta
  - Delta-Y

Y-Connected Circuits

- Recall from previous lectures:
  - Line current = Phase current
  - Line voltage = Phase voltage x $\sqrt{3}\angle 30^{\circ}$

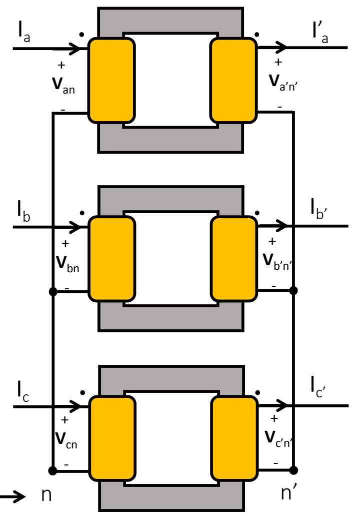
Vector Diagrams



Y-Y Transformer

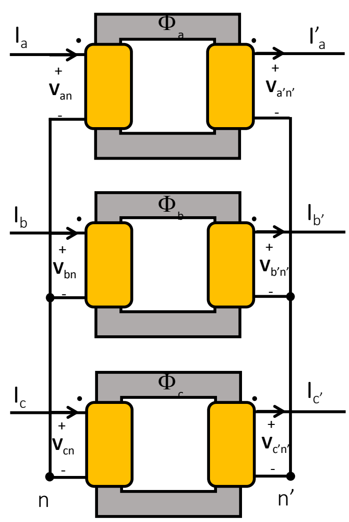
- Each side has a common point, n or n'
- Neutral points usually grounded
- Tertiary winding (Delta) sometimes connected to avoid distortion if harmonics are present
- <u>Line-neutral voltages appear across the coils on each side</u>
- Insulation is stressed to only 58% of line voltage

Do not confuse this n with 1/a



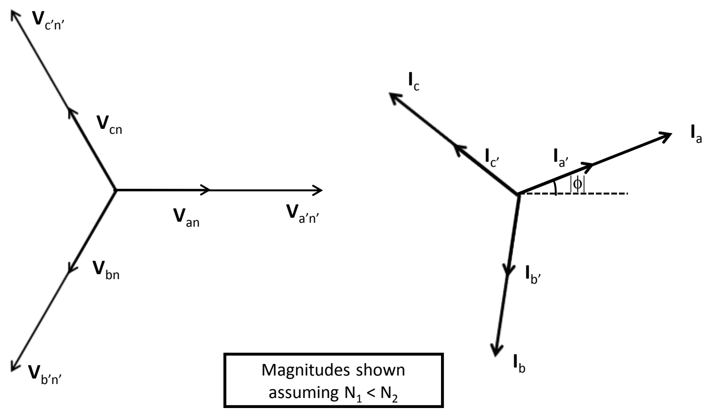
Y-Y Transformer Analysis

- Mutual flux in each core are out of phase by 120°
- Φ_a links primary and secondary
- Faraday's law: $\mathbf{V}_{an}$ and $\mathbf{V}_{a'n'}$ must be in phase and lead Φ_a by 90°
  - similar result for b, c phases
- Ampere's Law: $\mathbf{I}_a$ and $\mathbf{I}_{a'}$ are also in phase
  - similar result for b, c phases



Y-Y Transformer Analysis

Phasor Diagram for Ideal Y-Y Connected Transformer



Three-Phase Transformer Analysis: Y-Y

- Let k be the transformer voltage gain
- For Y-Y transformers:

$$\mathbf{V}_{a'n'} = k\mathbf{V}_{an} = \frac{N_2}{N_1}\mathbf{V}_{an}$$

therefore $k = n$ ← Do not confuse this n with the neutral point

$$\mathbf{I}_{a'} = \frac{\mathbf{I}_a}{k} = \frac{\mathbf{I}_a}{n}$$

$$\mathbf{V}_{a'b'} = \mathbf{V}_{a'n'}\sqrt{3}e^{\frac{j\pi}{6}} = (n\mathbf{V}_{an})\sqrt{3}e^{\frac{j\pi}{6}} \quad \text{(secondary line voltage)}$$

$$= \sqrt{3}\left(n\frac{\mathbf{V}_{ab}}{\sqrt{3}}e^{\frac{-j\pi}{6}}\right)e^{\frac{j\pi}{6}} = n\mathbf{V}_{ab}$$

Recall: $e^{\frac{j\pi}{6}} = 1\angle 30^\circ$

Three-Phase Transformer Analysis: Y-Y

- Line-neutral voltages

$\mathbf{V}_{a'n'} = n\mathbf{V}_{an}$, $\mathbf{V}_{a'b'} = n\mathbf{V}_{ab}$

$\mathbf{V}_{b'n'} = n\mathbf{V}_{bn}$, $\mathbf{V}_{b'c'} = n\mathbf{V}_{bc}$

$\mathbf{V}_{c'n'} = n\mathbf{V}_{cn}$, $\mathbf{V}_{c'a'} = n\mathbf{V}_{ca}$

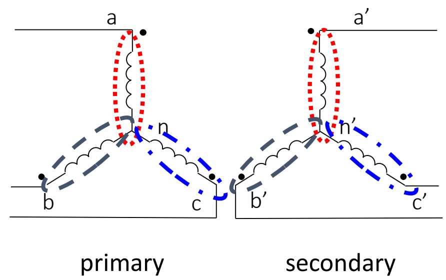
- Line currents

$$\mathbf{I}_{a'} = \frac{\mathbf{I}_a}{n}$$

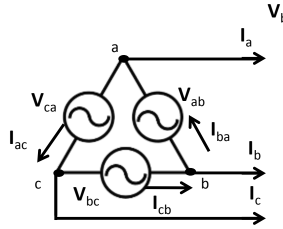
$$\mathbf{I}_{b'} = \frac{\mathbf{I}_b}{n}$$

$$\mathbf{I}_{c'} = \frac{\mathbf{I}_c}{n}$$

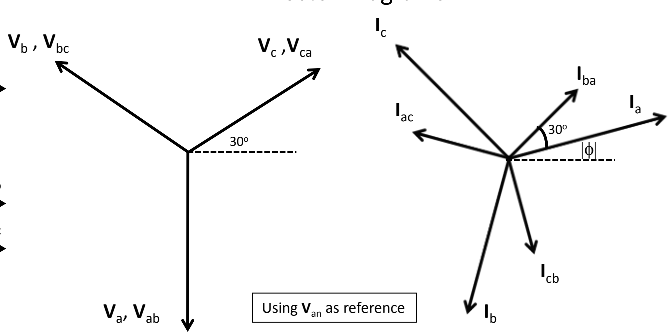
Recall: phase currents = line current for Y connections



Delta-Connected Circuits

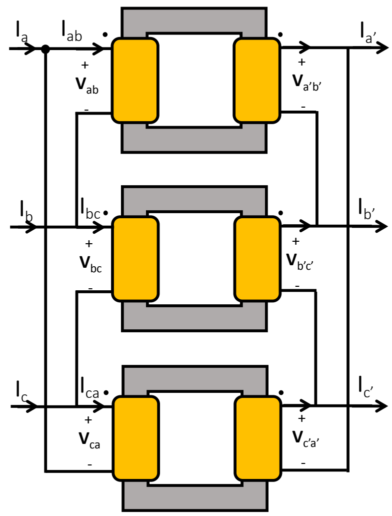


Vector Diagrams



Δ-Δ Transformer

- No neutral point
- $\mathbf{V}_{ab}$, $\mathbf{V}_{a'b'}$ in phase
- $\mathbf{I}_{ab}$, $\mathbf{I}_{a'b'}$ in phase
  - Similar results for b, c phase
- Line-line voltages appear across the coils on primary and secondary
- Best suited for lower voltage applications



Three-Phase Transformer Analysis: Δ-Δ

- Let k be the transformer voltage gain
- For Δ-Δ transformers: $\mathbf{V}_{a'b'} = k\mathbf{V}_{ab} = \frac{N_2}{N_1}\mathbf{V}_{ab}$

therefore $k = n$

$$\mathbf{I}_{a'b'} = \frac{\mathbf{I}_{ab}}{k} = \frac{\mathbf{I}_{ab}}{n}$$

$$\mathbf{V}_{a'n'} = \frac{\mathbf{V}_{a'b'}e^{\frac{-j\pi}{6}}}{\sqrt{3}} = \frac{n\mathbf{V}_{ab}e^{\frac{-j\pi}{6}}}{\sqrt{3}}$$

$$= \frac{n(\sqrt{3}\mathbf{V}_{an}e^{\frac{j\pi}{6}})e^{\frac{-j\pi}{6}}}{\sqrt{3}} = n\mathbf{V}_{an} \quad \text{(line-neutral voltage)}$$

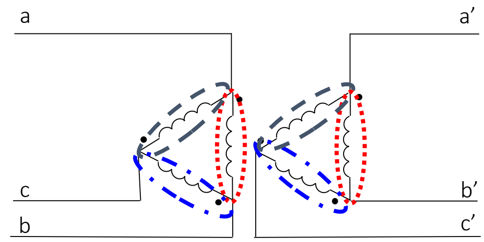
Three-Phase Transformer Analysis: Δ–Δ

- Voltage Relationships

$$\mathbf{V}_{a'b'} = n\mathbf{V}_{ab}$$
$$\mathbf{V}_{b'c'} = n\mathbf{V}_{bc}$$
$$\mathbf{V}_{c'a'} = n\mathbf{V}_{ca}$$

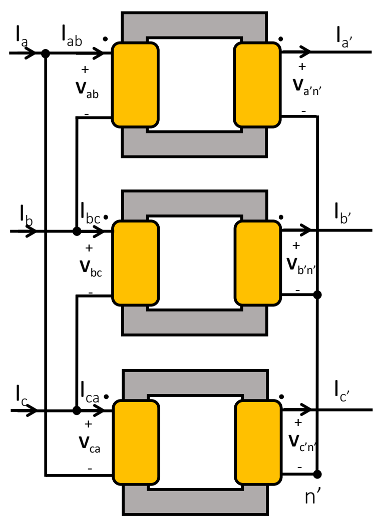
- Current Relationships

$$\mathbf{I}_{a'} = \frac{\mathbf{I}_a}{n}, \quad \mathbf{I}_{b'a'} = \frac{\mathbf{I}_{ab}}{n}$$
$$\mathbf{I}_{b'} = \frac{\mathbf{I}_b}{n}, \quad \mathbf{I}_{c'b'} = \frac{\mathbf{I}_{bc}}{n}$$
$$\mathbf{I}_{c'} = \frac{\mathbf{I}_c}{n}, \quad \mathbf{I}_{a'c'} = \frac{\mathbf{I}_{ca}}{n}$$



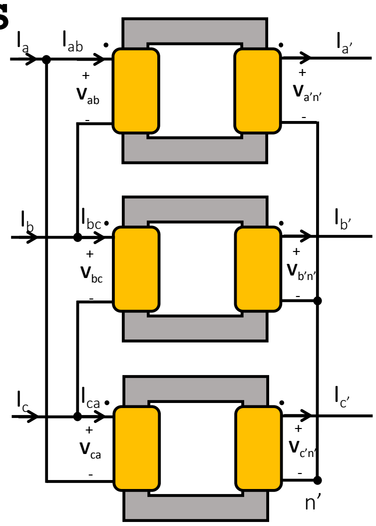
Δ-Y Transformer

- Secondary has a neutral connection
- Primary connected "top-to-bottom"
- Line-line voltages appear on the coils on primary
- Phase voltages appear on the coils on the secondary
- Less insulation needed on HV winding



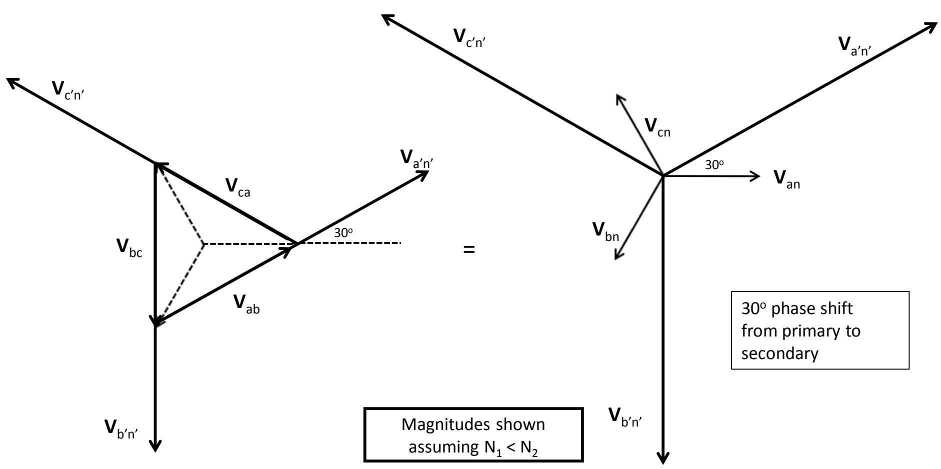
Δ-Y Transformer Analysis

- By Faraday's Law:
  - $\mathbf{V}_{ab}$ will be in-phase with $\mathbf{V}_{a'n'}$
  - Phase shift introduced
- By KCL:
  - $\mathbf{I}_a = \mathbf{I}_{ab} - \mathbf{I}_{ca}$
- By Ampere's Law:
  - $\mathbf{I}_{ab}$ will be in-phase with $\mathbf{I}_{a'}$
  - Phase shift introduced
- Similar results for b, c phases



Δ-Y Transformer Analysis

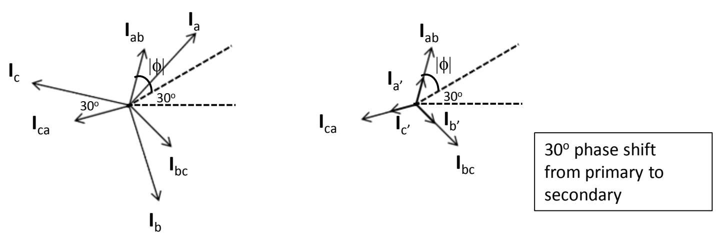
Phasor Diagram for Ideal Delta-Y Connected Transformer



Delta-Y Transformer Analysis

Phasor Diagram for Ideal Delta-Y Connected Transformer

$\mathbf{I}_a = \mathbf{I}_{ab} - \mathbf{I}_{ca}$
$\mathbf{I}_b = \mathbf{I}_{bc} - \mathbf{I}_{ab}$
$\mathbf{I}_c = \mathbf{I}_{ca} - \mathbf{I}_{bc}$



Three-Phase Transformer Analysis: Delta-Y

- Let k be the transformer voltage gain
- For Δ-Y transformers:

$$\mathbf{V}_{a'n'} = n\mathbf{V}_{ab} = n\mathbf{V}_{an}\sqrt{3}e^{\frac{j\pi}{6}} = k\mathbf{V}_{an}$$

therefore $k = n\sqrt{3}e^{\frac{j\pi}{6}}$ Important result!

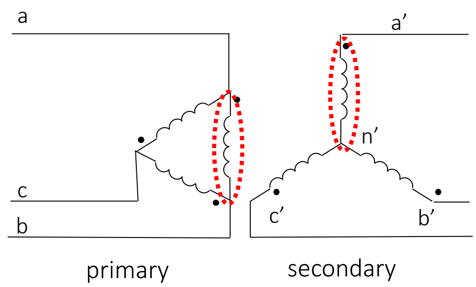
$$\mathbf{I}_{a'} = \frac{\mathbf{I}_{ab}}{n} = \frac{\mathbf{I}_a e^{\frac{j\pi}{6}}}{\sqrt{3}n}$$

$$= \frac{\mathbf{I}_a}{k^*}$$ } same phase shift as voltage

Delta-Y Transformer Analysis

- Voltage relationships

$$\mathbf{V}_{a'n'} = k\mathbf{V}_{an}, \quad \mathbf{V}_{a'b'} = k\mathbf{V}_{ab}$$
$$\mathbf{V}_{b'n'} = k\mathbf{V}_{bn}, \quad \mathbf{V}_{b'c'} = k\mathbf{V}_{bc}$$
$$\mathbf{V}_{c'n'} = k\mathbf{V}_{cn}, \quad \mathbf{V}_{c'a'} = k\mathbf{V}_{ca}$$



- Line current relationships

$$\mathbf{I}_{a'} = \frac{\mathbf{I}_a}{k^*}$$
$$\mathbf{I}_{b'} = \frac{\mathbf{I}_b}{k^*}$$
$$\mathbf{I}_{c'} = \frac{\mathbf{I}_c}{k^*}$$

Exercise

- Consider an ideal Delta-Y transformer, with 20 turns on each primary coil, and 80 turns on each secondary coil. If the primary side values are:

$$\mathbf{V}_{ab} = 208\angle 0° \text{ V}$$
$$\mathbf{I}_a = 10\angle 5° \text{ A}$$

- Compute: $\mathbf{V}_{a'b'}$, $\mathbf{V}_{a'n'}$ and $\mathbf{I}_a$'

Exercise

- Consider an ideal Delta-Y transformer, with 20 turns on each primary coil, and 80 turns on each secondary coil. If the primary side values are:

$\mathbf{V}_{ab} = 208\angle 0°\ \text{V}$

$\mathbf{I}_a = 10\angle 5°\ \text{A}$

- Compute: $\mathbf{V}_{a'b'}$, $\mathbf{V}_{a'n'}$ and $\mathbf{I}_a'$

$$k = n\sqrt{3}e^{\frac{j\pi}{6}} = 4\sqrt{3}e^{\frac{j\pi}{6}} = 6.92\angle 30°$$

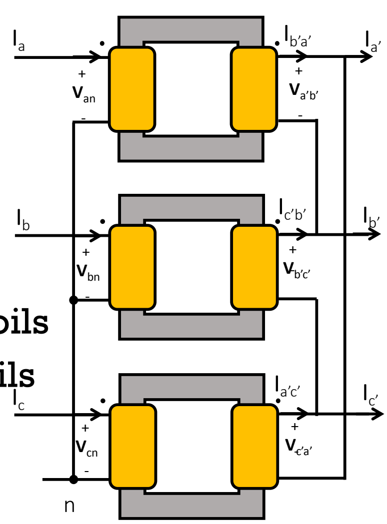
$$\mathbf{V}_{a'b'} = \mathbf{V}_{ab}k = 1436\angle 30°\ \text{V}$$

$$\mathbf{V}_{a'n'} = \mathbf{V}_{an}k = (120\angle -30°)(6.92\angle 30°) = 830.4\angle 0°\ \text{V}$$

$$\mathbf{I}_{a'} = \frac{\mathbf{I}_a}{k^*} = \frac{10\angle 5°}{6.92\angle -30°} = 1.45\angle 35°\ \text{A}$$

Y-Δ Transformer

- No neutral point on secondary
- $\mathbf{V}_{an}$, $\mathbf{V}_{a'b'}$ in phase
- $\mathbf{I}_{an}$, $\mathbf{I}_{b'a'}$ in phase
  - Similar results for b, c phase
- Line-neutral voltage across primary coils
- Line-line voltage across secondary coils



Three-Phase Transformer Analysis: Y-Δ

- Let k be the transformer voltage gain
- For Y-Δ transformers:

$$\mathbf{V}_{a'b'} = n\mathbf{V}_{an} = \frac{n\mathbf{V}_{ab}e^{\frac{-j\pi}{6}}}{\sqrt{3}} = k\mathbf{V}_{ab}$$

$$\text{therefore } k = \frac{ne^{\frac{-j\pi}{6}}}{\sqrt{3}}$$

Important result

Note: phase shift is -30°, whereas for Delta-Y transformer connection in previous slides it is +30°

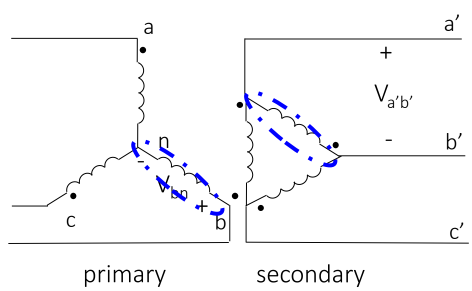
Three-Phase Transformer Analysis: Y-Δ

- Voltage relationship

$$\mathbf{V}_{a'b'} = -n\mathbf{V}_{bn} = \frac{n}{\sqrt{3}}\mathbf{V}_{ab}e^{\frac{j\pi}{6}} = k\mathbf{V}_{ab}$$

$$\text{therefore } k = \frac{n}{\sqrt{3}}e^{\frac{j\pi}{6}}$$

$$\mathbf{V}_{a'n'} = \frac{\mathbf{V}_{a'b'}}{\sqrt{3}}e^{\frac{-j\pi}{6}} = \frac{n\mathbf{V}_{an}}{\sqrt{3}}e^{\frac{j\pi}{6}} = k\mathbf{V}_{an}$$



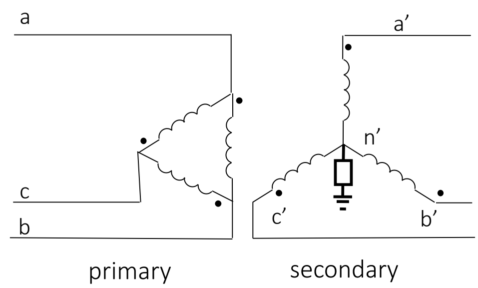
- For the other phases

$$\mathbf{V}_{a'n'} = k\mathbf{V}_{an}$$
$$\mathbf{V}_{b'n'} = k\mathbf{V}_{bn}$$
$$\mathbf{V}_{c'n'} = k\mathbf{V}_{cn}$$

Phase shift is now +30°

Practical Considerations

- Medium-voltage industrial facilities often use Δ-Y incoming transformers
- Y-side is grounded through a resistor
  - Reduces ground current during fault
  - Reduces voltage dip during fault



Summary

- Y-Y, Delta-Delta transformers result in magnitude changes of k = n from primary to secondary
  - No phase shifting occurs
- Y-Y transformers grant access to neutral point, which is usually grounded to prevent distortion
- Delta-Delta transformers have no neutral point, but are less prone to distortion
- Per phase analysis can be used on Y-Y, Delta-Delta transformers
  - Ensure impedances and voltages are properly converted
- There is a complex gain (magnitude and phase angle shift) associated with Y-Delta and Delta-Y transformers

Reading

- Also read:
  - 11.2 (Multi-Winding Transformer)
  - 11.3 (Autotransformer)