13-Magnetic Circuits Analysis

ECEGR 3500

Electrical Energy Systems

Professor Henry Louie

Overview

- Introduction
- Magnetic Circuit Elements
- Magnetic Circuit Analysis

Questions

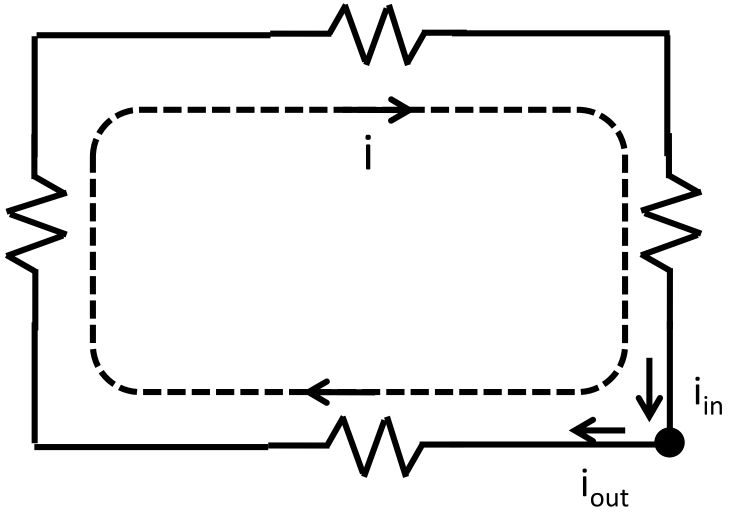
- What is the electric circuit analog of flux?
- What is the magnetic circuit analog of voltage?
- What is the magnetic circuit analog of resistance?

Introduction

- Electrical engineers are trained at solving circuits
- Fortunately, we can derive a loose analogy between magnetic flux and current
- We will be solving "magnetic circuits"
- Assumptions were discussed in previous lecture

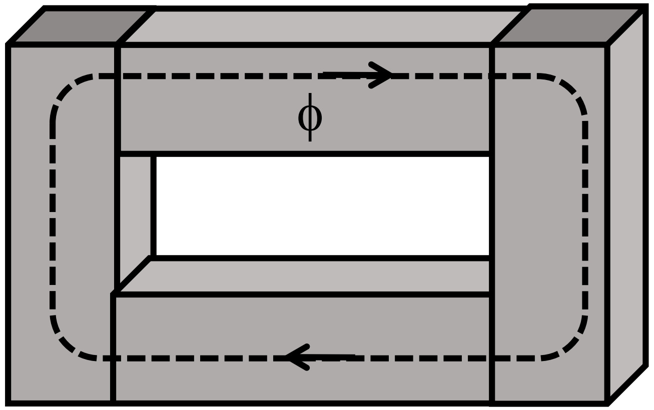
Magnetic Circuit Assumptions

Electric Circuit



current entering node =
current leaving node

Magnetic Circuit



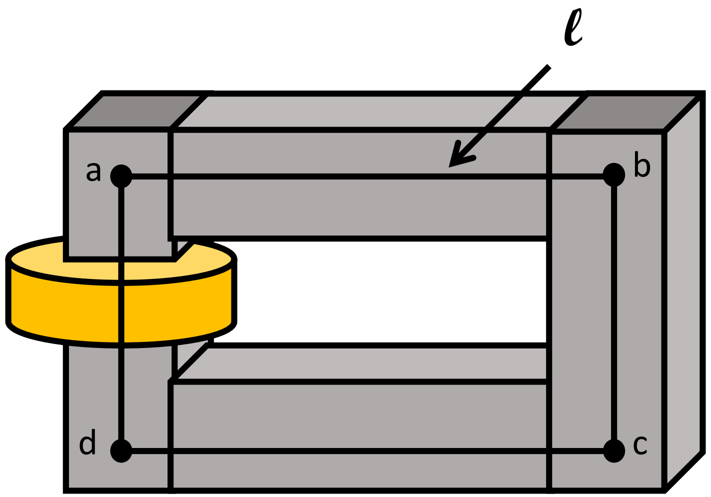
magnetic flux entering a boundary=
magnetic flux leaving a boundary

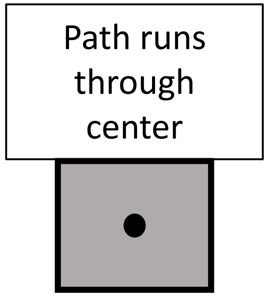
(from $\oint_s \mathbf{B}\cdot d\mathbf{s} = 0$)

Magnetic Circuit Analysis

Let ℓ be the mean length of the magnetic path (m)

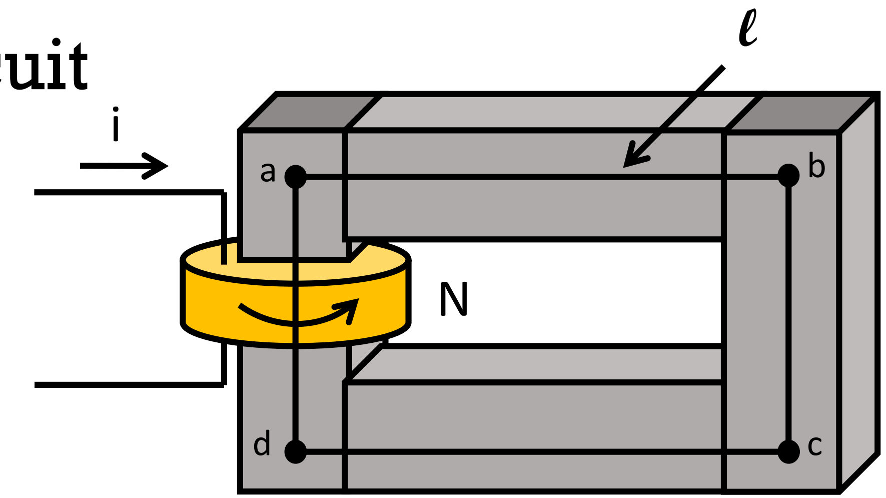
$$\ell = \ell_{ab} + \ell_{bc} + \ell_{cd} + \ell_{da}$$





Magnetomotive Force (mmf)

- The current enclosed by the closed path through the core is (Ampere's Law) $\oint_C \mathbf{H} \cdot d\ell = Ni = \mathcal{F}$
- where
  - $\mathcal{F}$: magnetomotive force (A-t)
- mmf is analogous to voltage in a circuit



Flux in Magnetic Circuits

- Assuming that **H** is uniform in the material, then

$$H\ell = Ni$$

- The magnetic flux density in the material is also uniform and

$$B = \mu H = \frac{\mu Ni}{\ell}$$

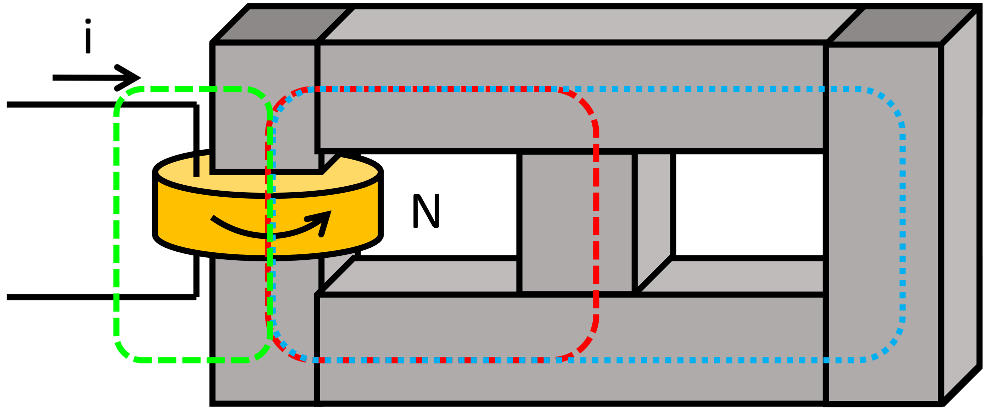
- The flux is:

$$\phi = BA = \frac{\mu NiA}{\ell}$$

  - A: cross sectional area of the material (m^2)

Exercise

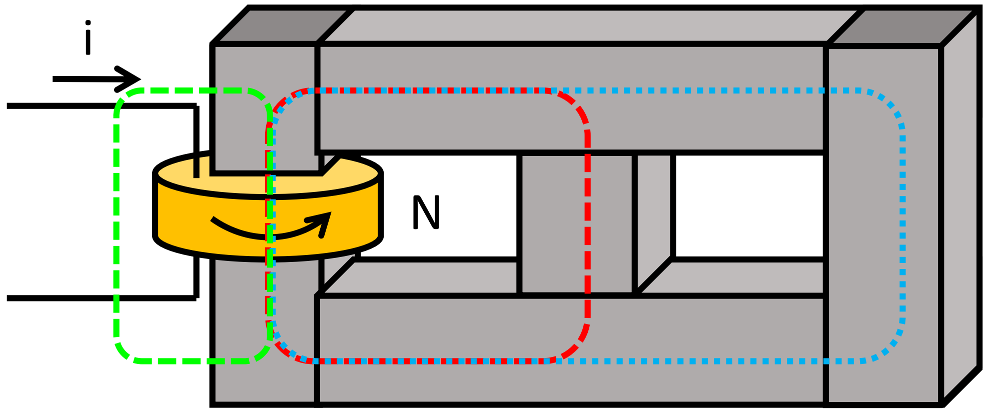
Which path (red, blue or green) results in the greatest mmf?



Cross section is uniform

Exercise

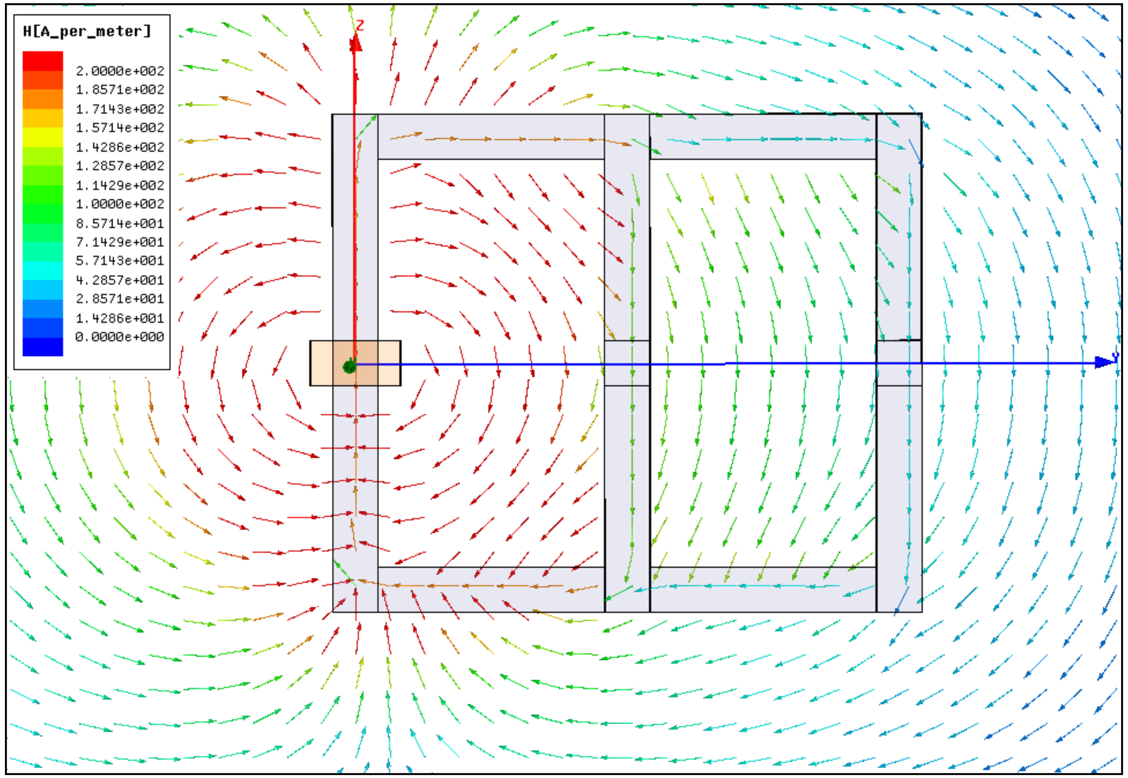
Which path (red, blue or green) results in the greatest mmf?



The mmf is the same. The same number of Ampere-turns is enclosed by each loop (Ni).

$$\oint_C \mathbf{H} \cdot d\ell = Ni = \mathscr{F}$$

Exercise



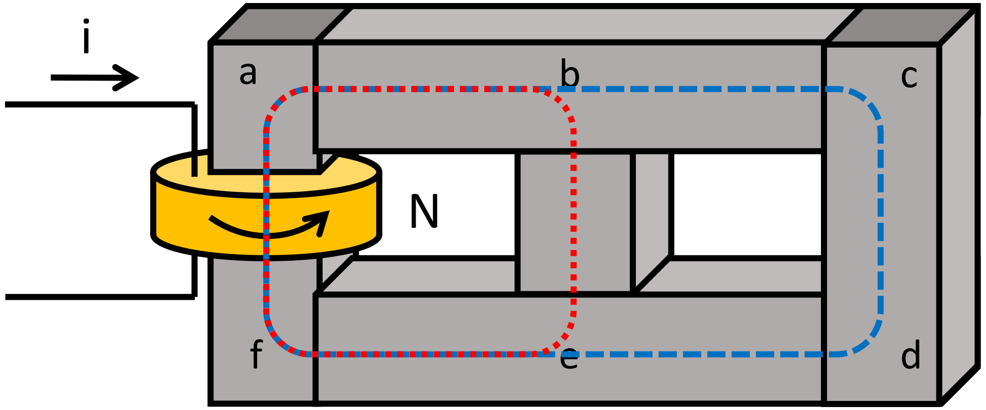
H[A_per_meter]
2.0000e+002
1.8571e+002
1.7143e+002
1.5714e+002
1.4286e+002
1.2857e+002
1.1429e+002
1.0000e+002
8.5714e+001
7.1429e+001
5.7143e+001
4.2857e+001
2.8571e+001
1.4286e+001
0.0000e+000
Z
Y

Magnetic Circuit Analysis

mmf is the same, no matter which path is used

$$Ni = H_{fa}\ell_{fa} + H_{ab}\ell_{ab} + H_{be}\ell_{be} + H_{ef}\ell_{ef}$$

$$= H_{fa}\ell_{fa} + H_{ab}\ell_{ab} + H_{bc}\ell_{bc} + H_{cd}\ell_{cd} + H_{de}\ell_{de} + H_{ef}\ell_{ef}$$



Cross section is uniform

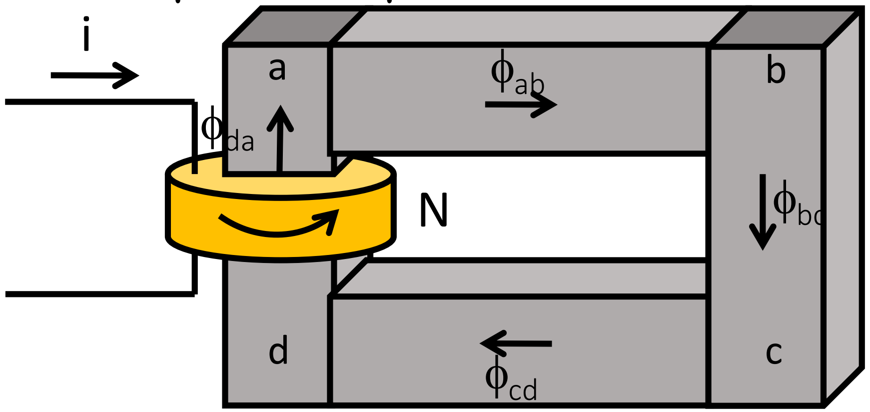
Magnetic Circuit Analysis

$$Ni = H_{da}\ell_{da} + H_{ab}\ell_{ab} + H_{bc}\ell_{bc} + H_{cd}\ell_{cd}$$

$$\mathcal{F} = \frac{B_{da}}{\mu}\ell_{fa} + \frac{B_{ab}}{\mu}\ell_{ab} + \frac{B_{bc}}{\mu}\ell_{bc} + \frac{B_{cd}}{\mu}\ell_{cd} \quad \text{using: } H = B/\mu$$

$$\mathcal{F} = \left(\frac{B_{da}}{\mu}\ell_{fa} + \frac{B_{ab}}{\mu}\ell_{ab} + \frac{B_{bc}}{\mu}\ell_{bc} + \frac{B_{cd}}{\mu}\ell_{cd}\right)\frac{A}{A}$$

$$\mathcal{F} = \phi_{da}\frac{\ell_{da}}{A\mu} + \phi_{ab}\frac{\ell_{ab}}{A\mu} + \phi_{bc}\frac{\ell_{bc}}{A\mu} + \phi_{cd}\frac{\ell_{cd}}{A\mu} \quad \text{using: } \phi = BA$$



Cross section is uniform

Magnetic Circuit Analysis

$$\mathcal{F} = \phi_{da}\frac{\ell_{da}}{A\mu} + \phi_{ab}\frac{\ell_{ab}}{A\mu} + \phi_{bc}\frac{\ell_{bc}}{A\mu} + \phi_{cd}\frac{\ell_{cd}}{A\mu}$$

$$\phi = \phi_{da} = \phi_{ab} = \phi_{bc} = \phi_{cd}$$

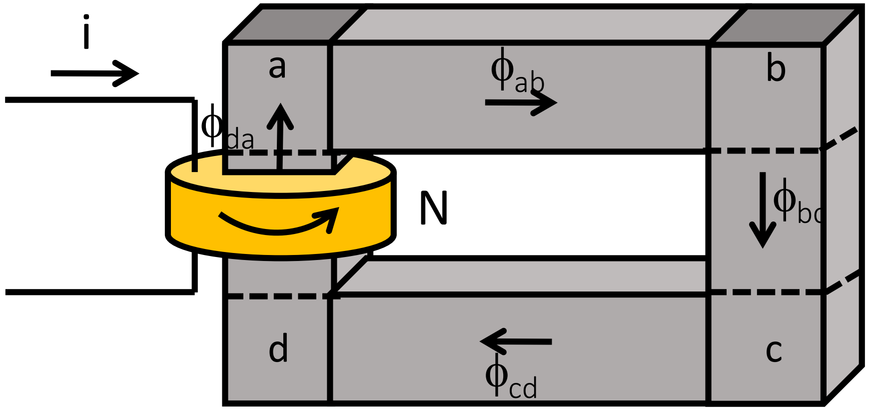
Since flux entering a boundary = flux leaving the boundary

$$\oint_s \mathbf{B \cdot ds} = 0$$

$$\mathcal{F} = \phi\left(\frac{\ell_{da}}{A\mu} + \frac{\ell_{ab}}{A\mu} + \frac{\ell_{bc}}{A\mu} + \frac{\ell_{cd}}{A\mu}\right)$$

$$\mathcal{F} = \phi(\Re_{da} + \Re_{ab} + \Re_{bc} + \Re_{cd}) \quad \left\{ \Re \triangleq \frac{\ell}{\mu A} \right.$$

$\mathcal{R}$: reluctance (A-t/Wb)



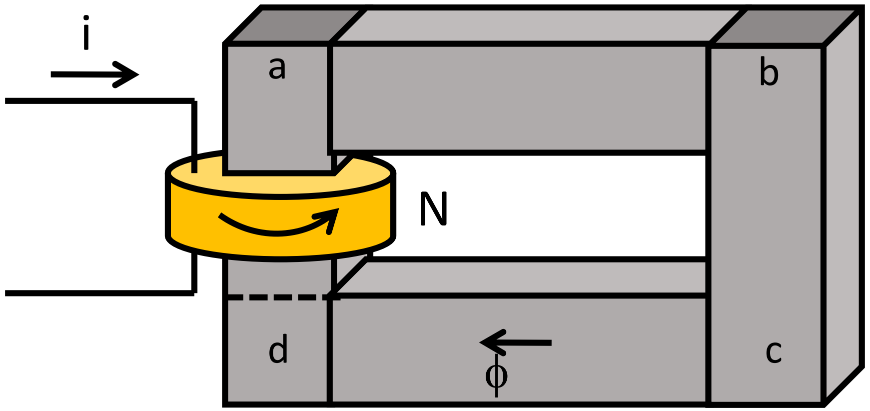
Cross section is uniform

Magnetic Circuit Analysis

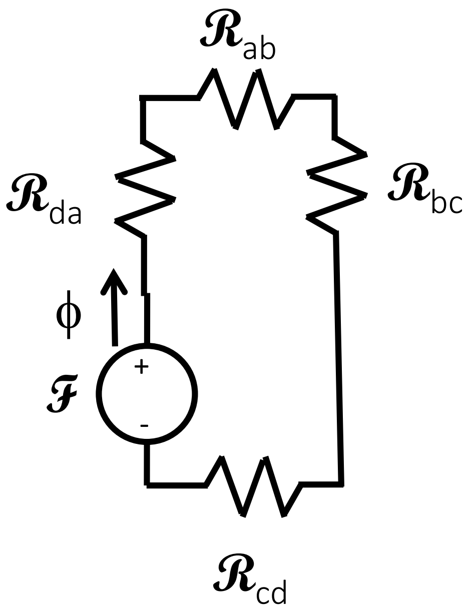
$$\mathcal{F} = \phi(\mathfrak{R}_{da} + \mathfrak{R}_{ab} + \mathfrak{R}_{bc} + \mathfrak{R}_{cd})$$

voltage ($\mathcal{F}$), current (ϕ), resistance ($\mathfrak{R}_{da} + \mathfrak{R}_{ab} + \mathfrak{R}_{bc} + \mathfrak{R}_{cd}$)

Equation can be modeled and solved like a circuit. Important!



Circuit equivalent



Magnetic Circuits

- Note that

$$\phi = \frac{\mathscr{F}}{\Re} = \frac{NiA\mu}{\ell}$$

- For electric circuits

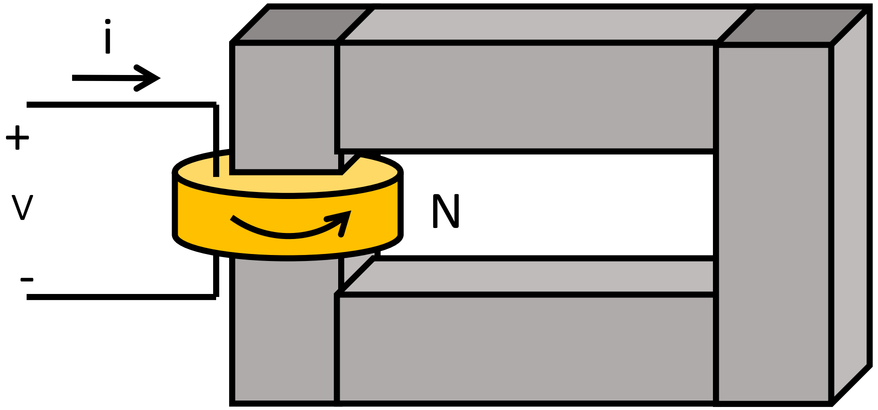
$$i = \frac{V}{R} = \frac{V}{\frac{\ell}{\sigma A}}$$

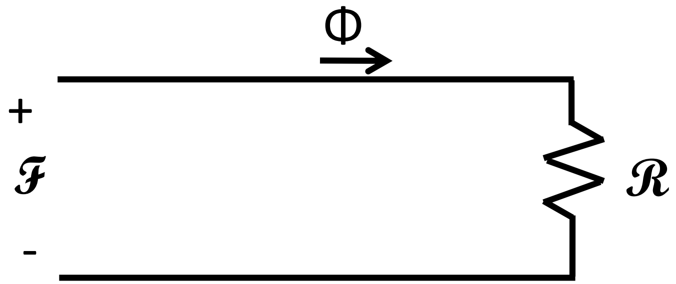
Analogous equations

  - σ: conductivity (S/m)
- Ohm's law for magnetic circuits $\mathscr{F} = \phi\Re$
  - mmf = flux x reluctance
- Checking the units
  - A-t = Wb x (A-t/Wb) = A-t

Magnetic Circuits

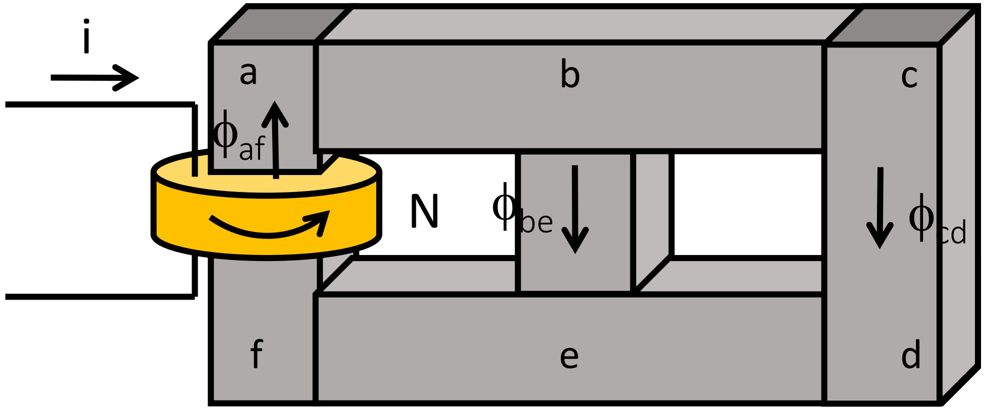
Circuit Quantity	Magnetic Quantity
Voltage, v (volt)	mmf, $\mathscr{F}$ (A-turns)
Current, i (Ampere)	magnetic flux, ϕ, (Wb)
Resistance, R (Ohm)	Reluctance, $\mathscr{R}$, (A-turns/Wb)
Conductivity, σ (S/m)	Permeability, μ (H/m)

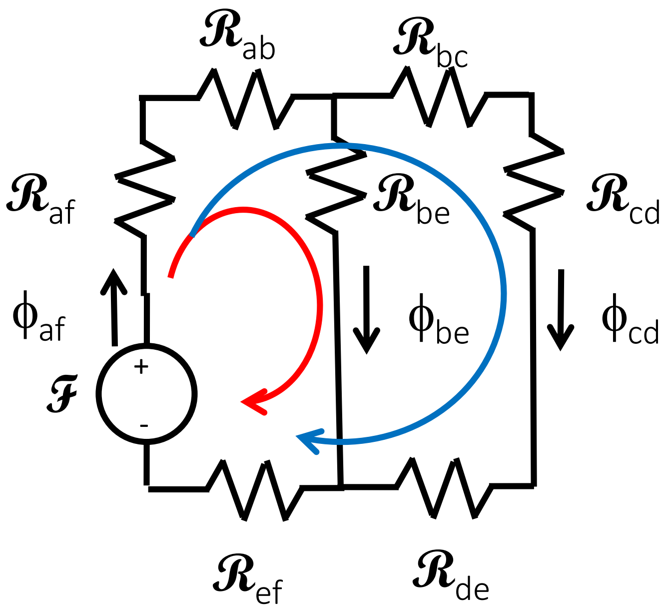




Magnetic Circuit Analysis

KVL and KCL and all other circuit theorems apply to equivalent electric circuit.



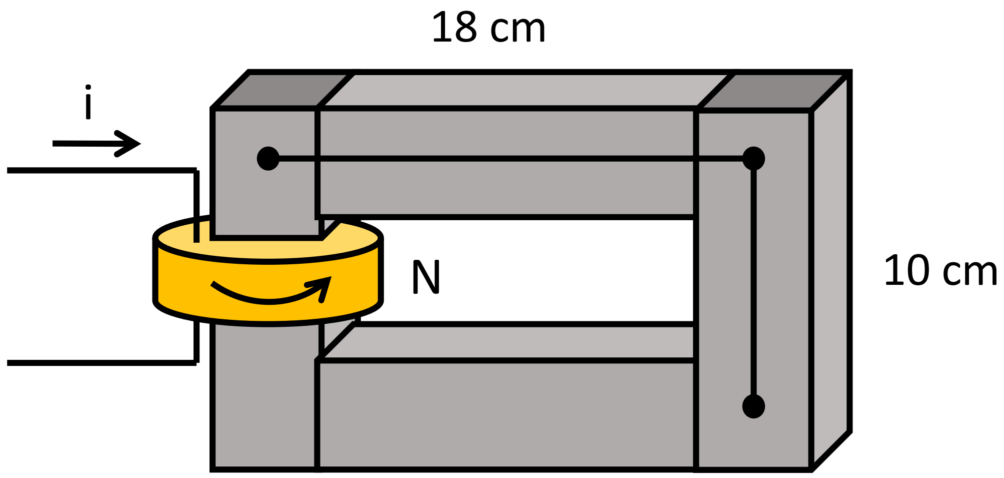


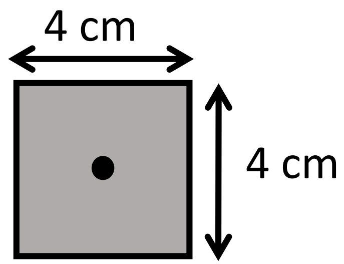
Magnetic Circuit Analysis

- Note: linear circuits assumed in analogy, therefore the magnetic circuit must be linear
  - Linear magnetic circuit = constant permeability
  - Ferromagnetic materials do not have constant permeability (see BH curve)
- Non-linear magnetic circuits can be solved iteratively

Example

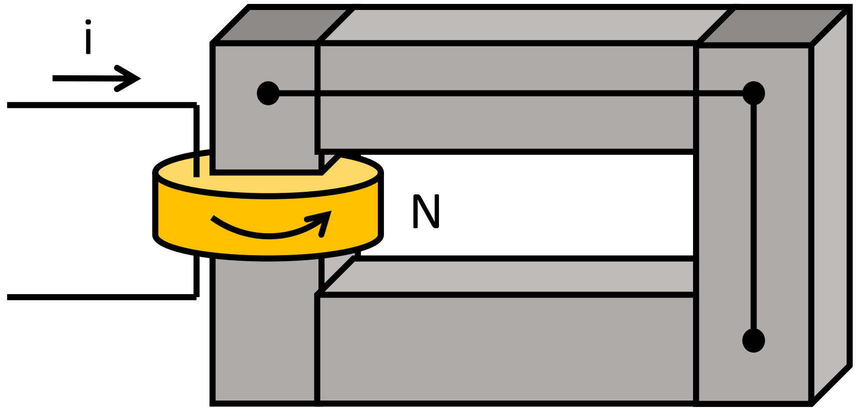
- Compute the flux flowing through the material given:
  - i = 1 A
  - N = 700
  - μ_r = 1000 (assumed to be constant)

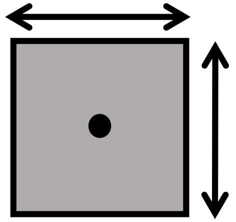




Example

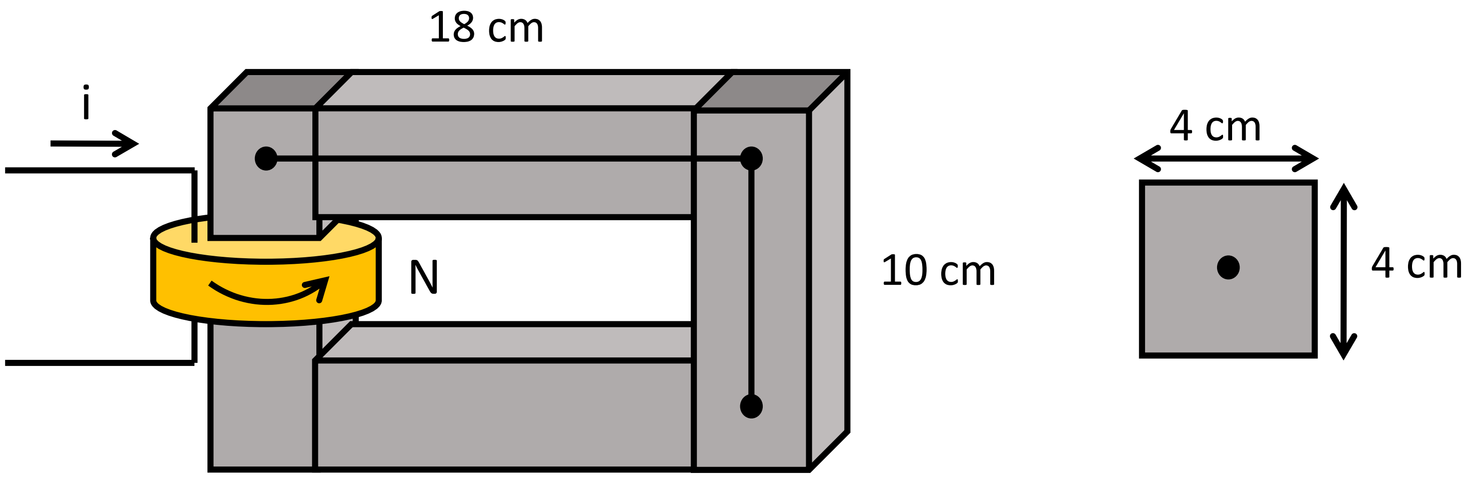
- Want to use: $\mathscr{F} = Ni = \Phi\Re$, $\Re = \dfrac{\ell}{\mu A}$
- First compute ℓ





Example

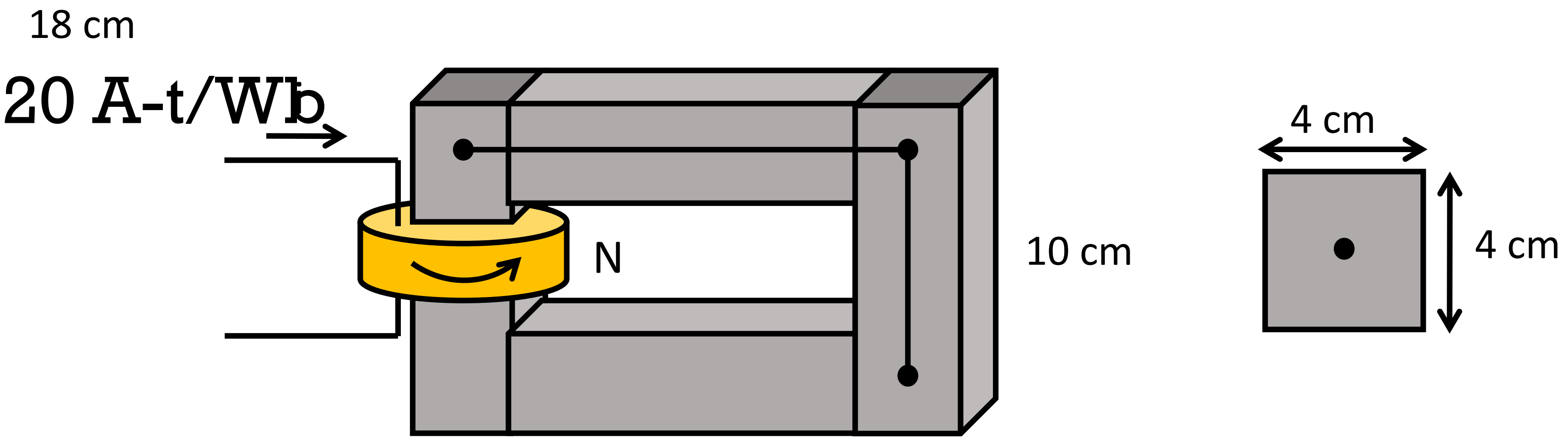
- First compute ℓ
  - ℓ = 18 + 10 + 18 + 10 = 0.56 m



Example

- Want to use: $\mathscr{F} = \mathrm{Ni} = \Phi\Re$, $\Re = \dfrac{0.56}{\mu A}$
- Computing A: $1000 \times 4\pi \times 10^{-7}$

  $A = 0.04 \times 0.04 = 0.0016\text{m}^2$
- Computing μ:
- Gives: $\mathscr{R}$ = 278,520 A-t/Wb

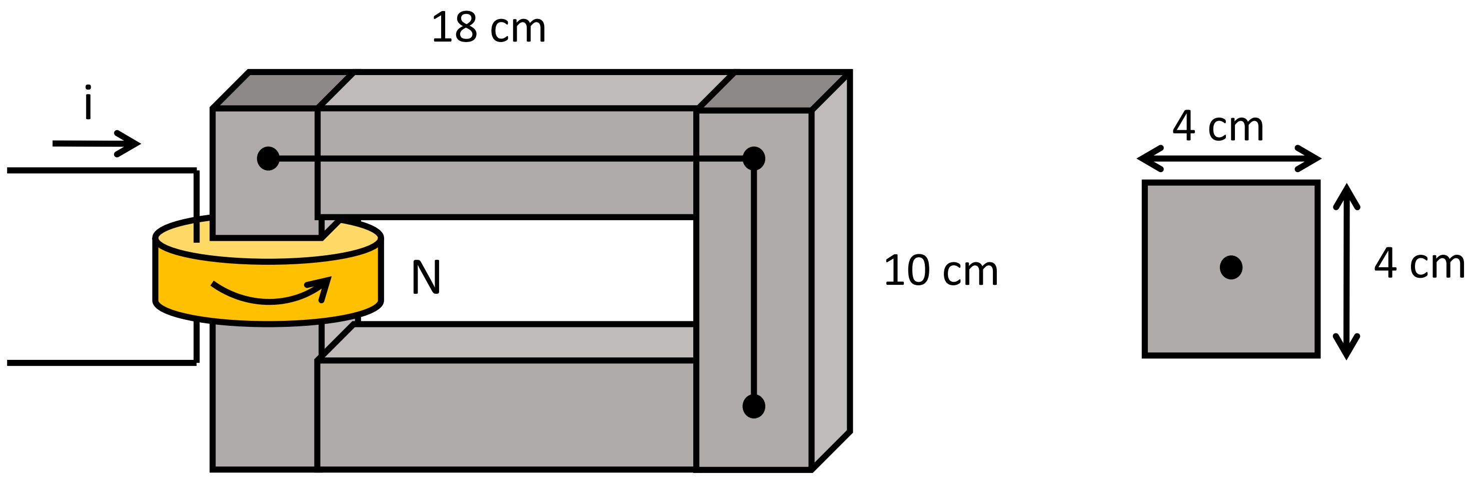


Example

- **Solving**

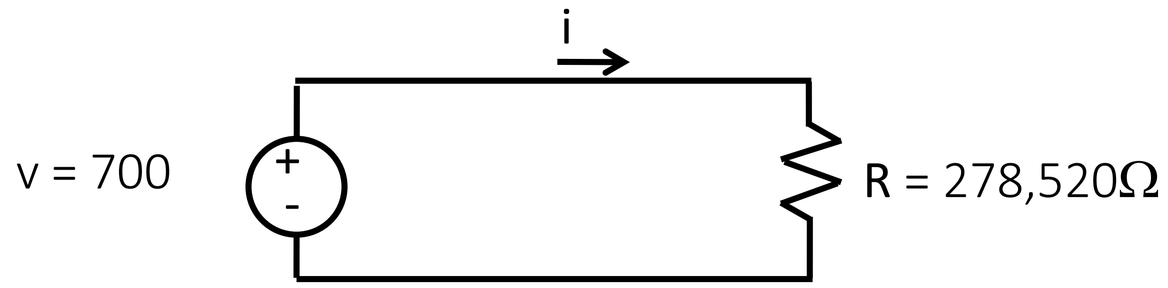
$$\mathscr{F} = Ni = \Phi \Re$$

$$\Phi = \frac{Ni}{\Re} = \frac{700 \times 1}{278520} = 0.0025 \text{ Wb}$$



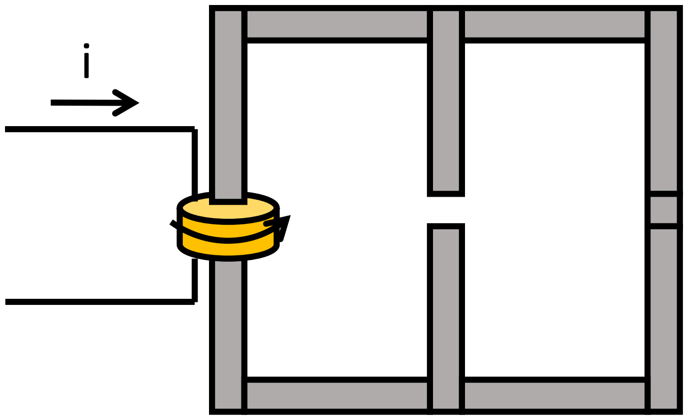
Example

Solution approach is the same as solving for the current in this circuit



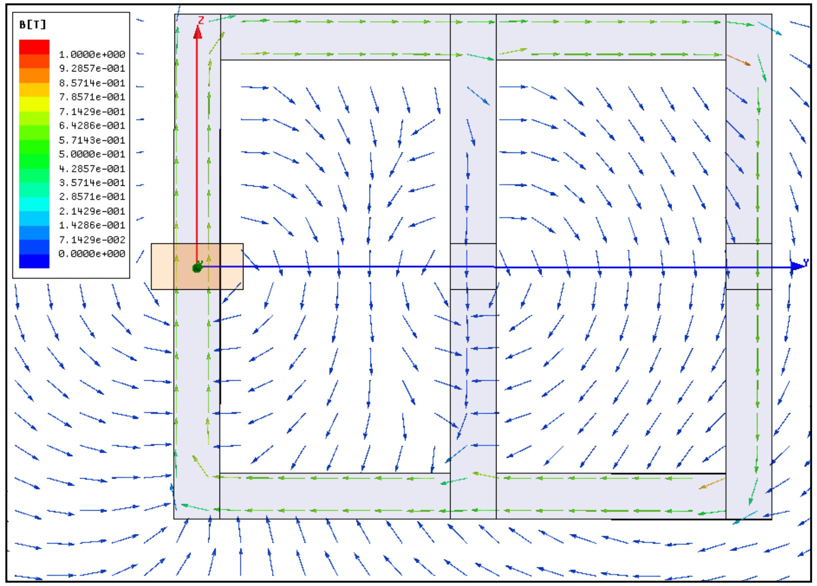
Example

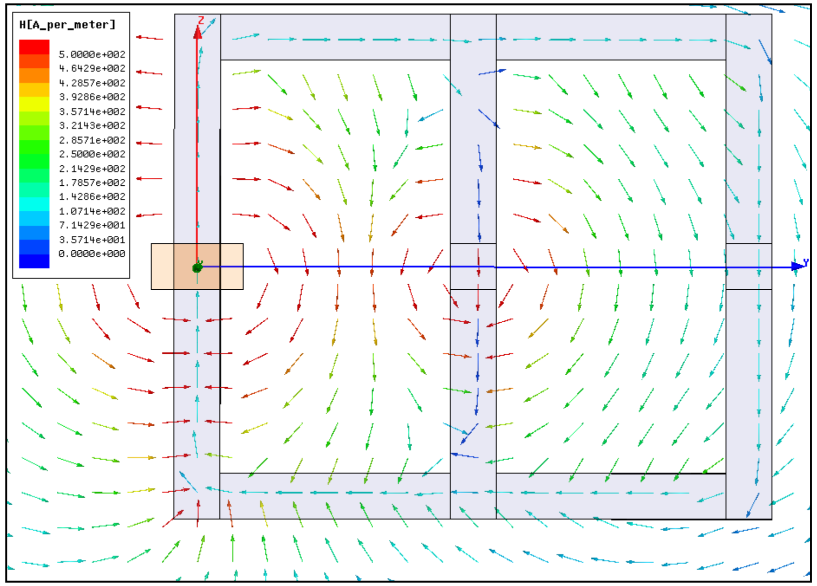
- Consider the shown magnetic circuit
  - Note the air gap in the center leg
- Determine:
  - direction of H, B within the circuit
  - which of the three legs has the greatest flux density, and which has the least
  - Which segment has the greatest field intensity
- Draw the equivalent electric circuit



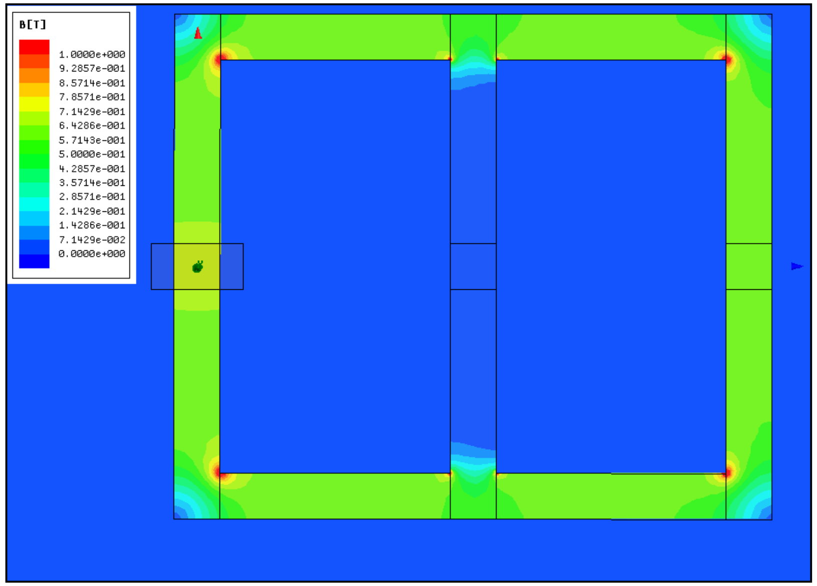
Assume uniform cross section

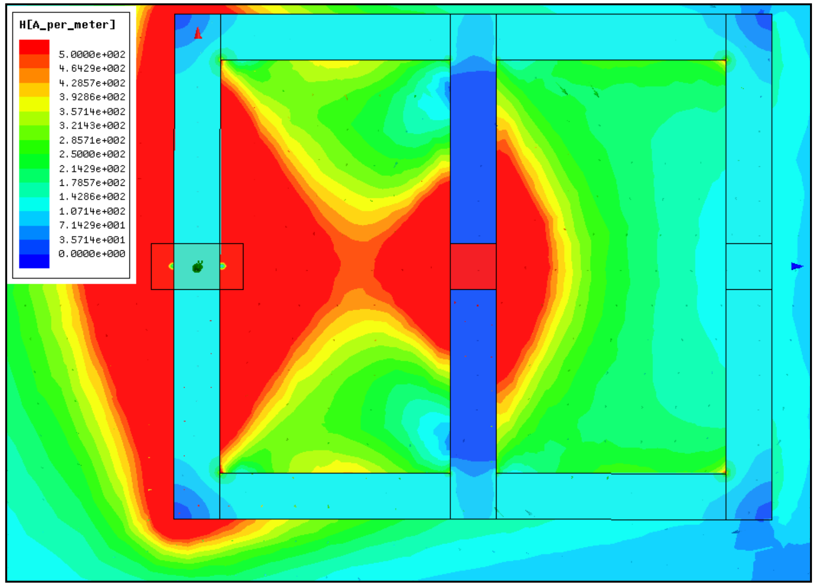
Example


B[T]
1.0000e+000
9.2857e-001
8.5714e-001
7.8571e-001
7.1429e-001
6.4286e-001
5.7143e-001
5.0000e-001
4.2857e-001
3.5714e-001
2.8571e-001
2.1429e-001
1.4286e-001
7.1429e-002
0.0000e+000


H[A_per_meter]
5.0000e+002
4.6429e+002
4.2857e+002
3.9286e+002
3.5714e+002
3.2143e+002
2.8571e+002
2.5000e+002
2.1429e+002
1.7857e+002
1.4286e+002
1.0714e+002
7.1429e+001
3.5714e+001
0.0000e+000

Example


B[T]
1.0000e+000
9.2857e-001
8.5714e-001
7.8571e-001
7.1429e-001
6.4286e-001
5.7143e-001
5.0000e-001
4.2857e-001
3.5714e-001
2.8571e-001
2.1429e-001
1.4286e-001
7.1429e-002
0.0000e+000

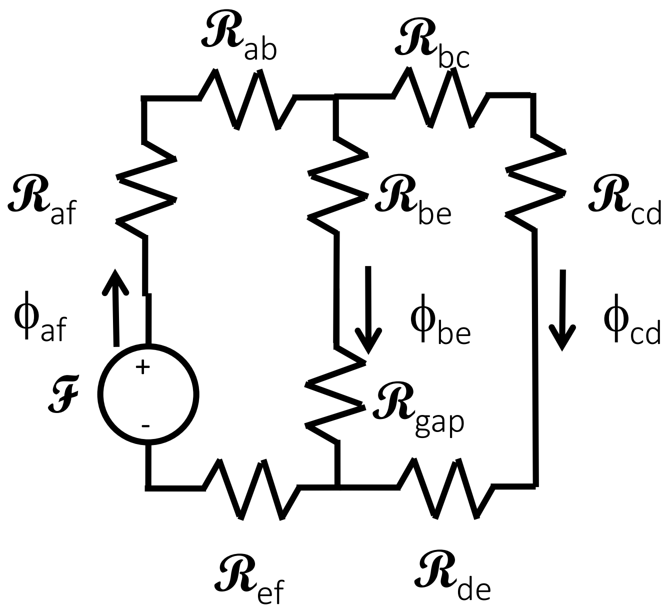

H[A_per_meter]
5.0000e+002
4.6429e+002
4.2857e+002
3.9286e+002
3.5714e+002
3.2143e+002
2.8571e+002
2.5000e+002
2.1429e+002
1.7857e+002
1.4286e+002
1.0714e+002
7.1429e+001
3.5714e+001
0.0000e+000

Example

Notes:

$\mathcal{R}_{gap} >> \mathcal{R}_{af}, \mathcal{R}_{ab}, \mathcal{R}_{be}, \mathcal{R}_{ef}, \mathcal{R}_{bc}, \mathcal{R}_{cd}, \mathcal{R}_{de}$

$\phi_{af} = \phi_{be} + \phi_{cd}$



Summary

- Magnetic circuits can be analyzed in an analogous fashion as electric circuits:

$$v = iR$$

$$\mathscr{F} = \phi \Re$$

- KVL, KCL, voltage divider, etc. all apply to magnetic circuit

- Reluctance, $\mathscr{R}$, increases with length, and decreases with permeability