14-Ideal Transformers

Text 11.1

ECEGR 3500

Electrical Energy Systems

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Overview

- Self Inductance
- Transformer Theory of Operation
- Ideal Single Phase Transformers

Introduction

Introduction

“homemade” Zambian transformer

Introduction

- Transformers are important electrical-electrical energy conversion components
- One important reason we use AC is because we can easily change the voltage levels, which reduces losses
- Transformers enable this conversion of voltage level
  - High efficiency (up to 99%)
  - No or few moving parts (low maintenance)

Transformers

- Shift between voltage levels
  - generation 11 kV to 30 kV
  - transmission up to 765 kV
  - distribution around 69 kV
  - residential 240/120 V
- Controlling voltages, power flows
  - regulating transformers
- Isolation (dc current)
- Instrument
  - PTs, CTs

Inductance

- Transformers and other machines have coils of wire wrapped around permeable material
- Transformers are made of one or more inductors on a common core
- We will start with a qualitative description of inductance

Inductance

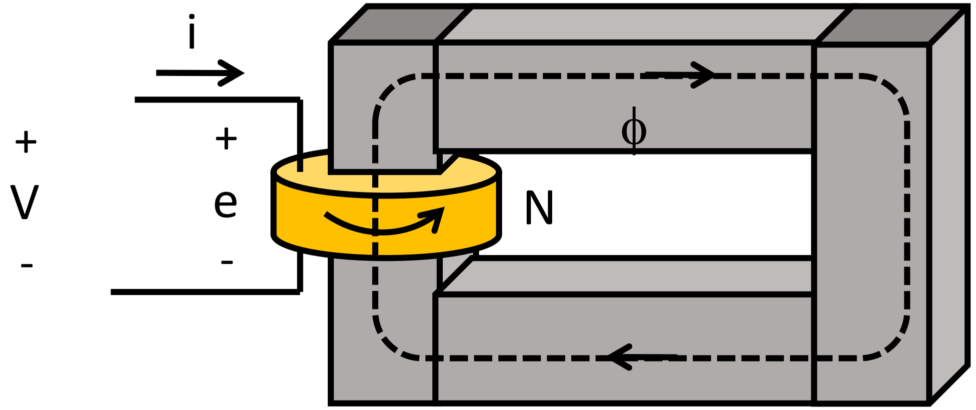
- Inductive reactance X_L exists due to Faraday's Law
  - $jX_L = j\omega L$
- The j operator accounts for the 90 degree phase shift between current and induced voltage
- ω accounts for the dependency on frequency
- L is a description of how strong the current links the flux through the coil
- Next we examine inductance

Inductance

- Recall that

$$e = N\frac{d\phi}{dt}$$ (note the polarity in the figure)

  - $N\phi$ is also known as the flux linkages (λ)

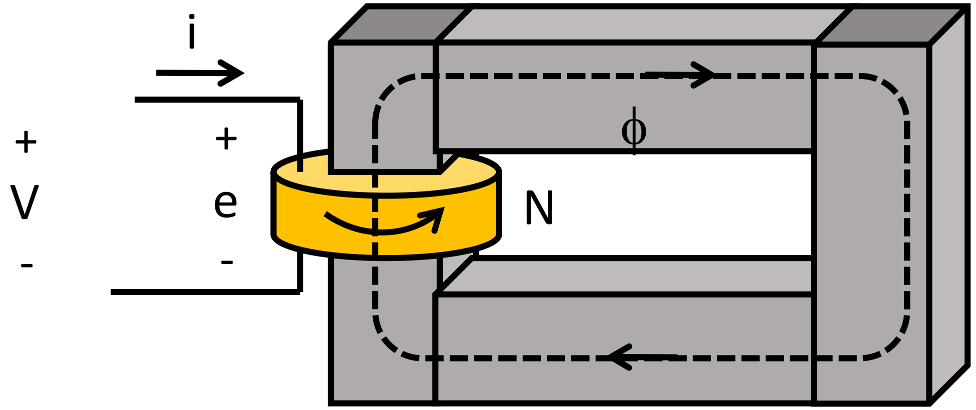


Inductance

- Self-inductance (inductance) is defined as:

$$L \triangleq N\frac{d\phi}{di}$$

- Large inductance: great sensitivity of flux wrt current



Inductance describes how the flux linking a coil changes with the applied current

Self Inductance

- Inductance depends on the physical characteristics of the magnetic circuit
- Recall that

$$\phi = \frac{Ni}{\Re}$$

$$L \triangleq N\frac{d\phi}{di} \qquad \text{therefore}$$

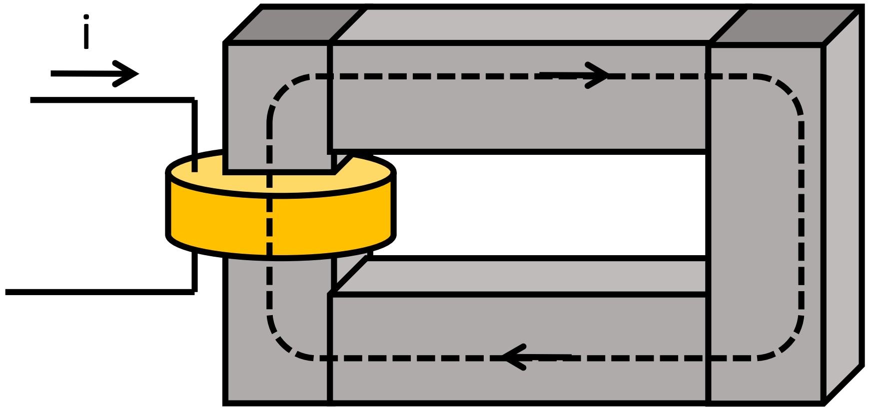
$$L = \frac{N^2}{\Re}$$

- Inductance is constant if the permeability of the magnetic circuit is itself constant (not the case in ferromagnetic materials)
- We will assume that we are operating in the linear region of B-H curve

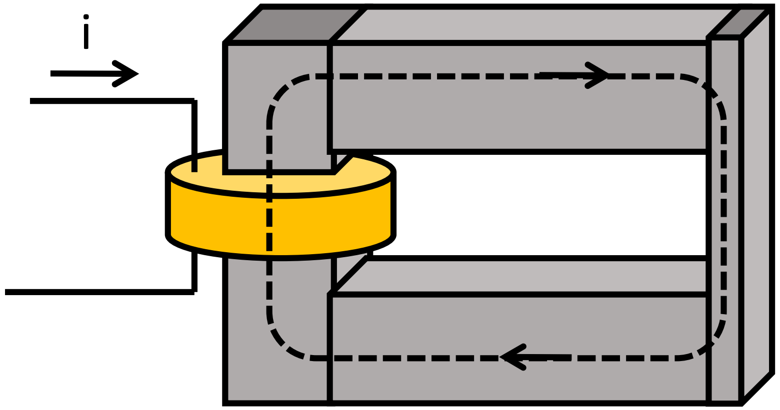
Question

Which circuit has greater inductance?

A.



B.



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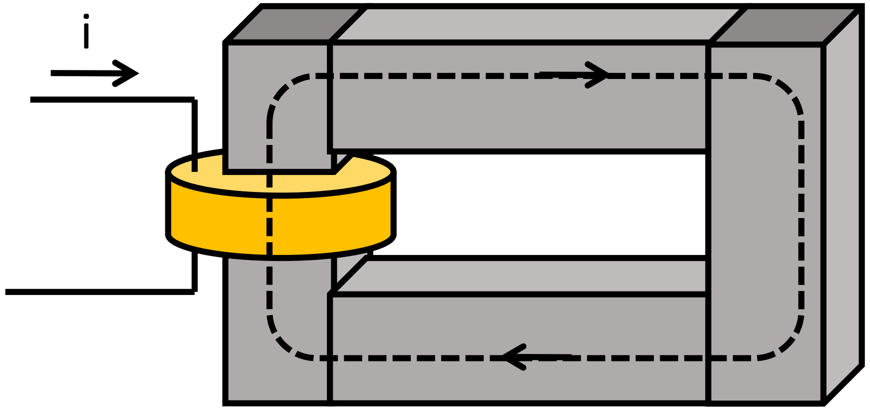
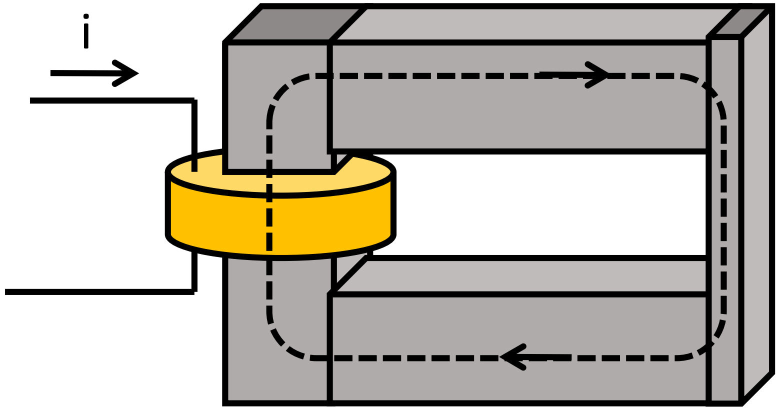


B.



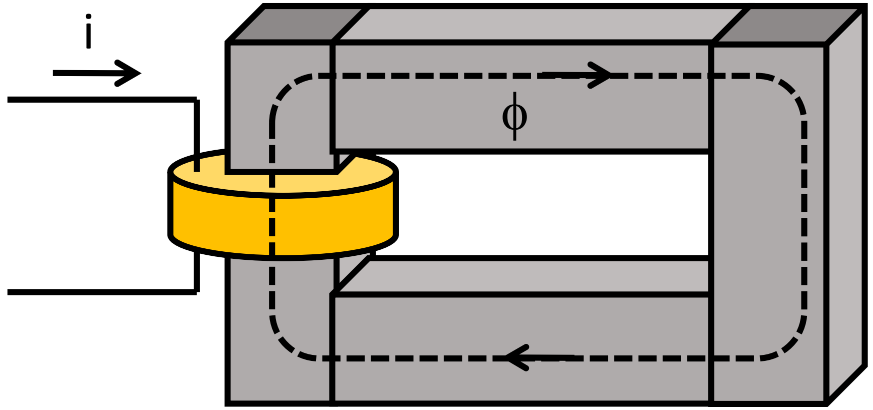
A. Has smaller reluctance. Current gives rise to greater flux so the inductance is larger.

Self Inductance

- Inductance is related to emf by:

$$\mathbf{e} = N\frac{d\phi}{dt} = N\frac{d\phi}{di}\frac{di}{dt} = L\frac{di}{dt}$$

- A coil with 1 H of inductance will have 1 volt induced in it if the current changes at a rate of 1 A/s
- <u>If we know the inductance, we do not need to compute the flux</u>

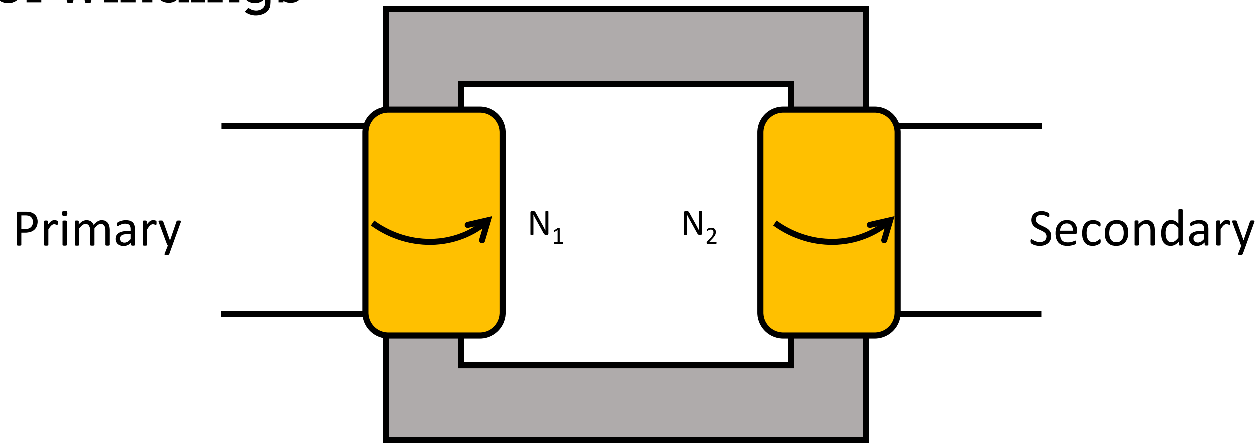


Questions

- Why are transformers used in power systems?
- Is it possible to use a “dc” transformer?
- Are transformers efficient?
- How is the power into a transformer related to the power out of a transformer?

Ideal Single-Phase Transformer

- Two magnetically coupled coils
  - Primary: N_1 turns
  - Secondary: N_2 turns
- Primary and secondary can be arbitrarily assigned
- Note direction of windings



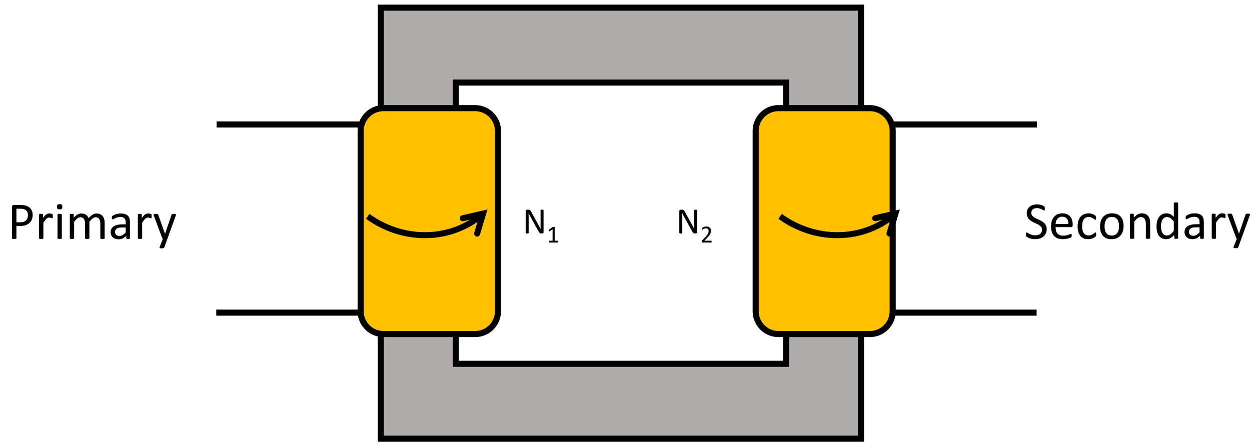
Ideal Single-Phase Transformer

Ideal assumptions

- No flux leakage
- No eddy currents

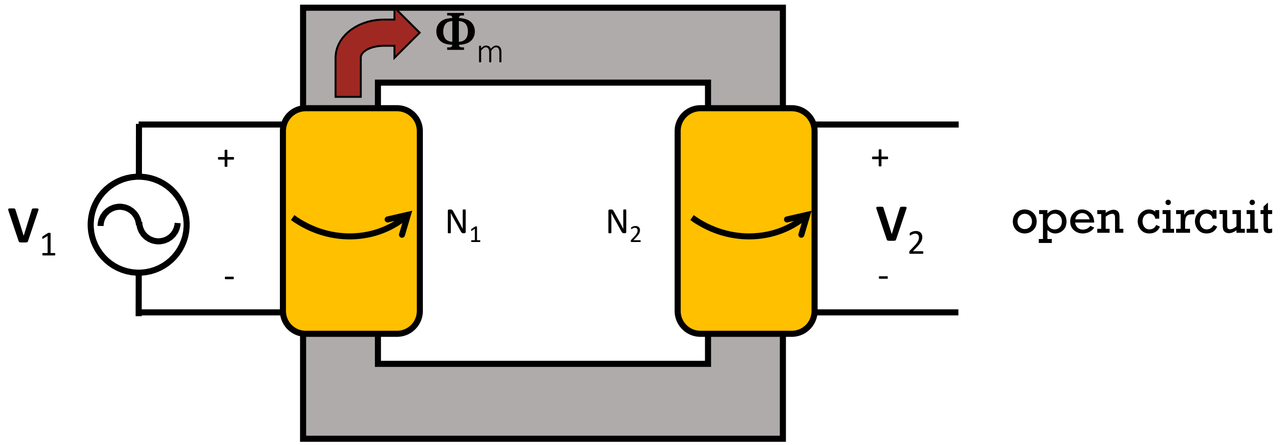
Recall from magnetic circuits lecture

- No winding resistance
- <u>Near infinite core permeability</u>



Single-Phase Transformer

- Primary directly connected to AC voltage source
- Voltage across coil has sinusoidal flux associated with it $e = -\frac{d\phi}{dt}$ (Faraday's Law)



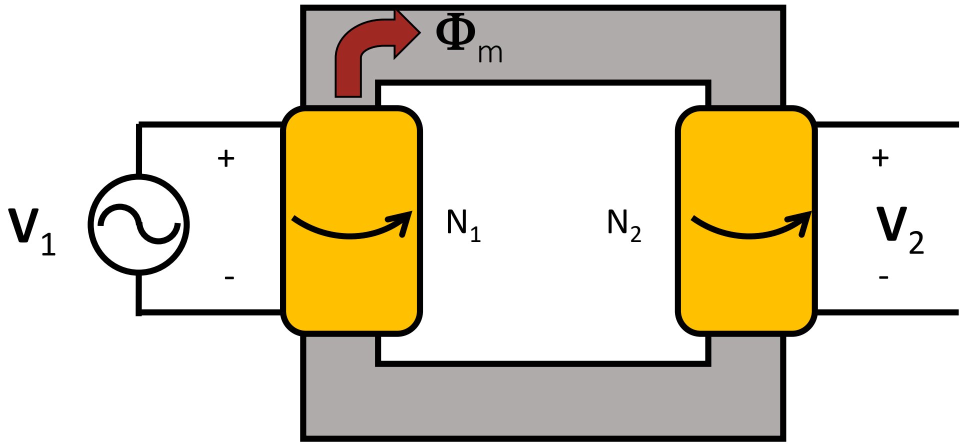
Single-Phase Transformer

- Same flux passes through each coil
  - Φ_m: mutual flux (phasor)
- Therefore:

$$\mathbf{V}_1 = \frac{d\lambda_1}{dt} = N_1 \frac{d\Phi_m}{dt}$$

$$\mathbf{V}_2 = \frac{d\lambda_2}{dt} = N_2 \frac{d\Phi_m}{dt}$$

- Rewritten: $\frac{V_1}{V_2} = \frac{N_1}{N_2} \triangleq a \triangleq \frac{1}{n}$



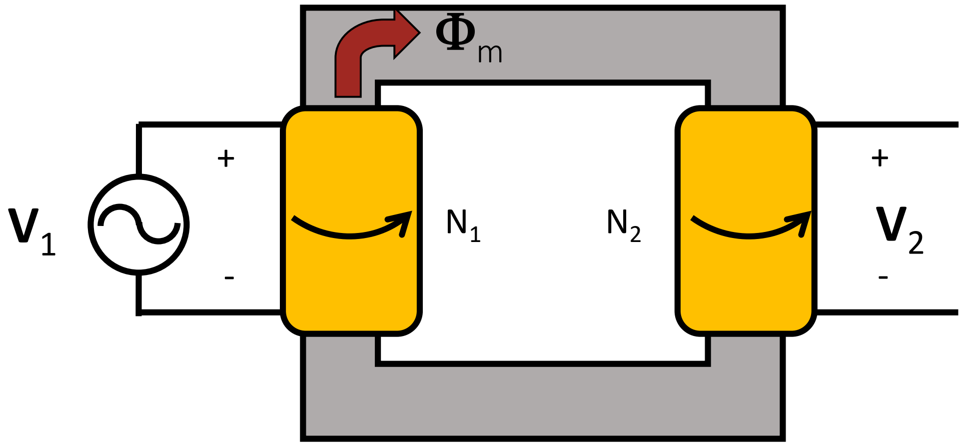
Single-Phase Transformer

- Ratio of voltages is the same as the ratio of turns

$$\frac{V_1}{V_2} = \frac{N_1}{N_2}$$

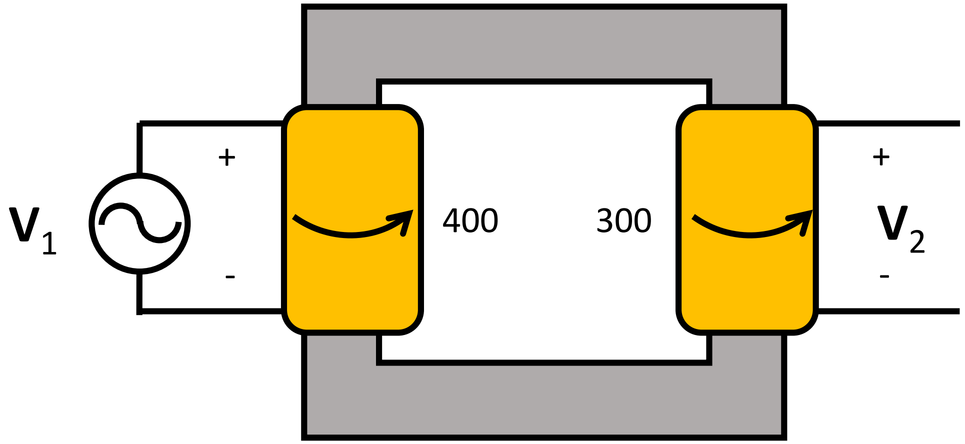
- Possible to transform voltage level from primary to secondary (and vice versa)
- Note: no current flows
  - (near infinite permeability)

$$\phi = BA = \frac{\mu N i A}{\ell}$$



Question

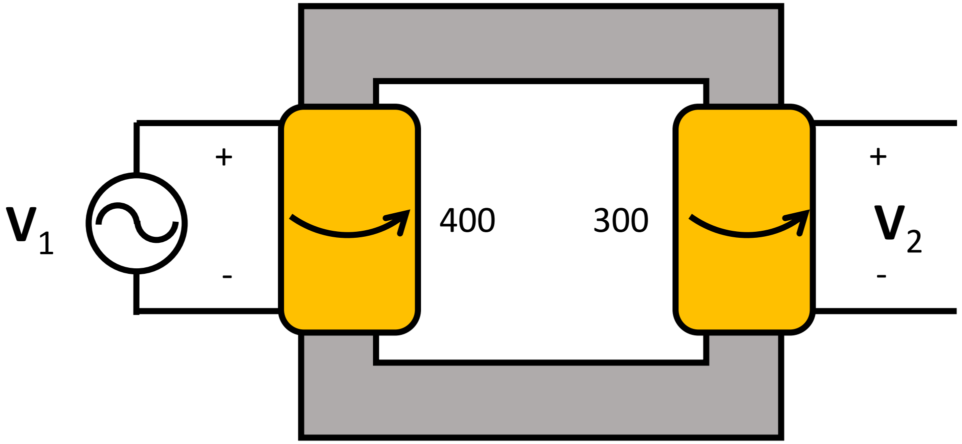
If $|\mathbf{V}_1| = 120V$, is $|\mathbf{V}_2|$ greater than 120V?



Question

If $|\mathbf{V}_1| = 120V$, is $|\mathbf{V}_2|$ greater than 120V?

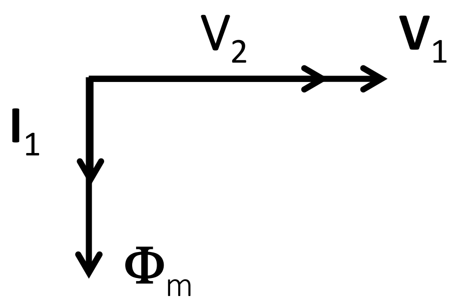
Less than 120V.



The winding with more turns has greater voltage

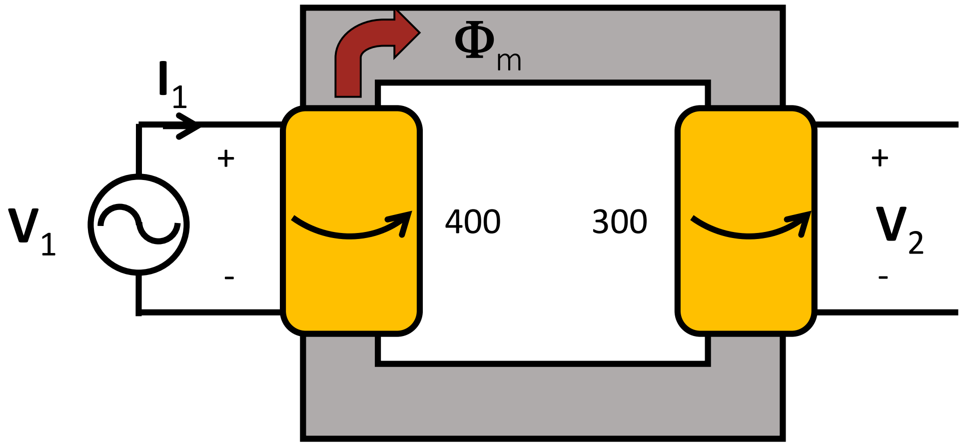
Single-Phase Transformer

Phasor Diagram (finite permeability, no load)



V_1, V_2 in phase.
Φ_m lags voltage by 90°
Current in phase with Φ_m

Note: $|\Phi|$ arbitrarily drawn



Single-Phase Transformer

- Now a resistive load is connected to the secondary
- $\mathbf{V}_2$ causes $\mathbf{I}_2$ to flow
- Examining mmf

$$\Im = N_1\mathbf{I}_1 - N_2\mathbf{I}_2 = \Re\mathbf{\Phi}_m$$

$$\Re = \frac{\ell}{\mu A}$$

- Infinite permeability

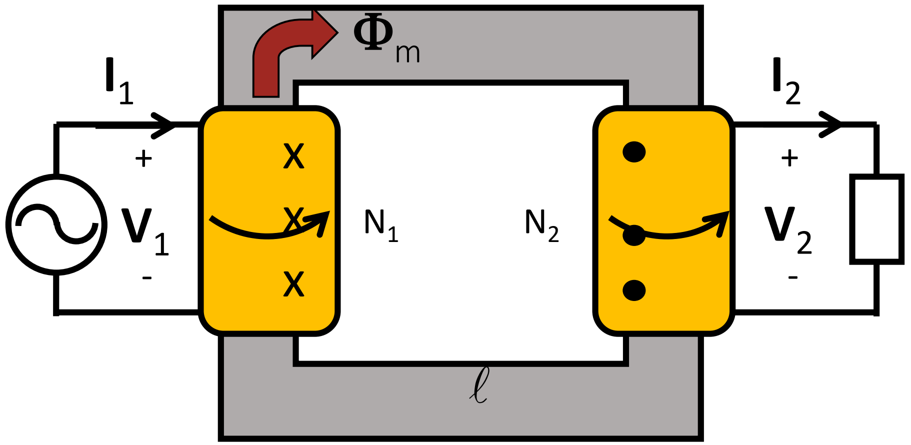
$$\Im = N_1\mathbf{I}_1 - N_2\mathbf{I}_2 = 0$$

$$N_1\mathbf{I}_1 = N_2\mathbf{I}_2$$

- Current gain

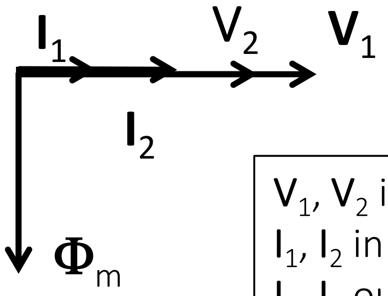
$$\frac{\mathbf{I}_2}{\mathbf{I}_1} = \frac{N_1}{N_2} = a$$

Compare to: $\frac{V_1}{V_2} = \frac{N_1}{N_2}$

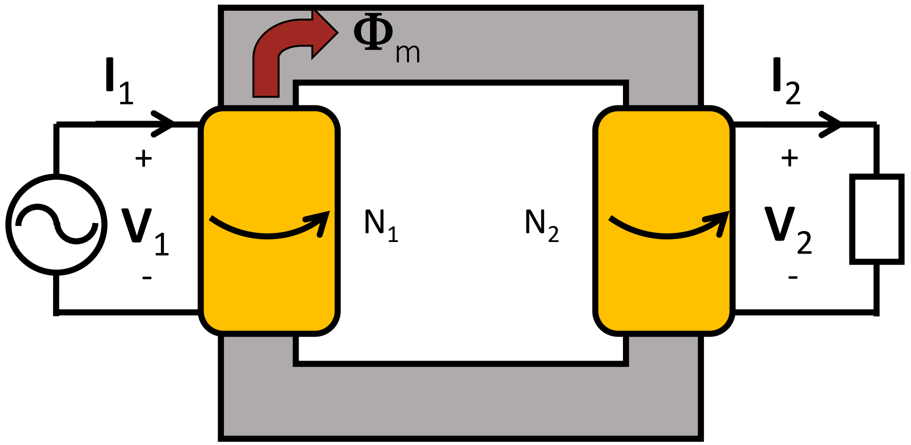


Single-Phase Transformer

Phasor Diagram
(ideal xfmr, resistive load)



V_1, V_2 in phase
I_1, I_2 in phase
I_1, I_2 out of phase with $\mathbf{\Phi}_m$
$\mathbf{\Phi}_m$ lags voltage by 90°



Note: $|\mathbf{\Phi}|$ arbitrarily drawn

Question

In an ideal transformer serving a load, if $|\mathbf{V}_1| > |\mathbf{V}_2|$, is $|\mathbf{I}_1| > |\mathbf{I}_2|$?

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In an ideal transformer serving a load, if $|\mathbf{V}_1| > |\mathbf{V}_2|$, is $|\mathbf{I}_1| > |\mathbf{I}_2|$?

No. The transformer would be creating energy.

Question

How are the transformer input and output power related? Find α in $P_1 = \alpha P_2$

Single-Phase Transformer

- Power into the transformer

$$P_1 = \mathrm{Re}\{\mathbf{V}_1\mathbf{I}_1{*}\}$$

- Power out of the transformer

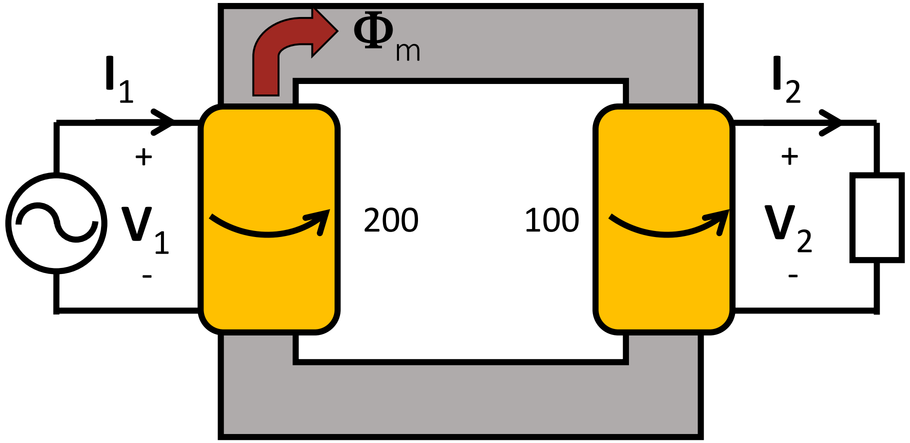
$$P_2 = \mathrm{Re}\{\mathbf{V}_2\mathbf{I}_2^*\} = \mathrm{Re}\{\frac{1}{a}\mathbf{V}_1\mathbf{I}_2^*\} = \mathrm{Re}\{\frac{1}{a}\mathbf{V}_1 a\mathbf{I}_1^*\} = P_1$$

- Power is conserved

Question

- Now a load with PF = 0.707 lagging is connected to the secondary
- Draw the phasor diagram of

  $\mathbf{V}_1, \mathbf{V}_2, \mathbf{I}_1, \mathbf{I}_2, \Phi_m$



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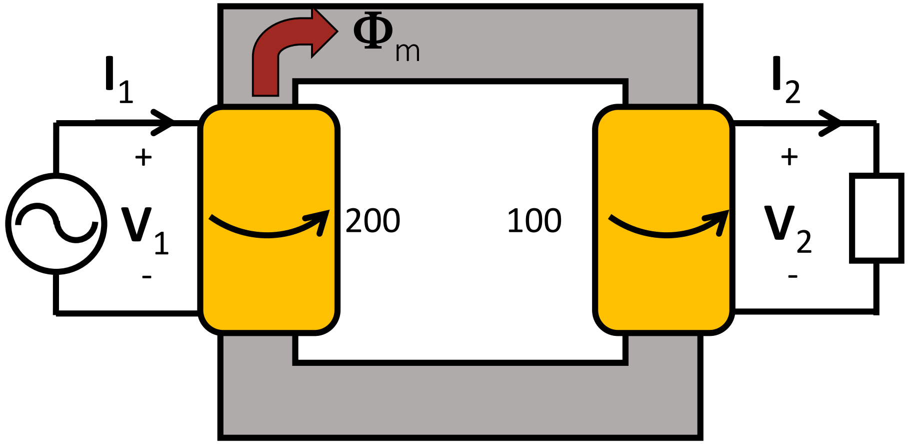
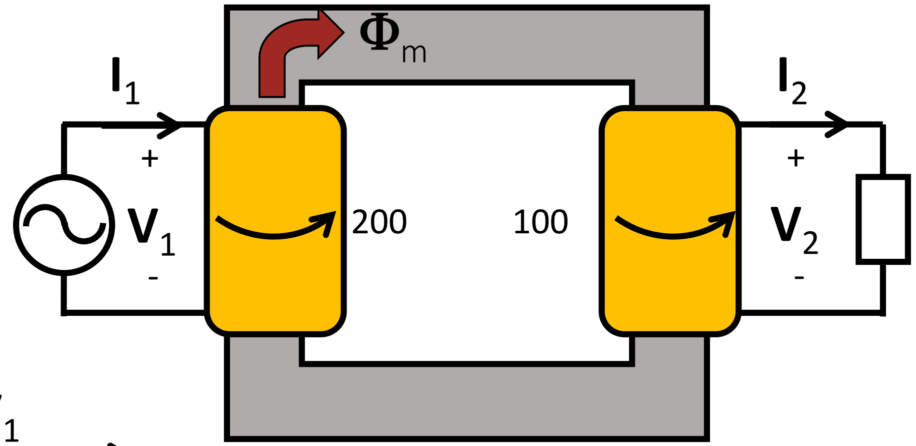
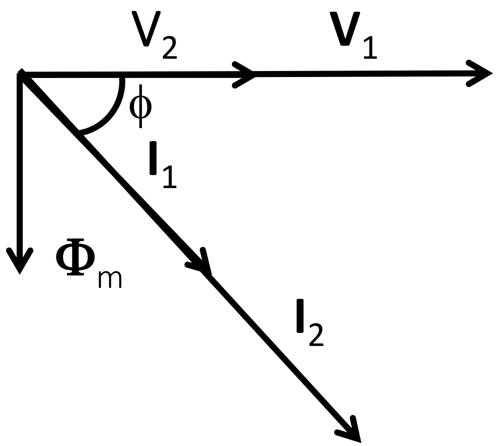


$|\mathbf{\Phi}_m|$, $|\mathbf{I}|$ can be arbitrarily drawn with respect to each other

Question

- $\mathbf{V}_1, \mathbf{V}_2$ in phase: $\mathbf{V}_1 = \mathbf{V}_2 \dfrac{N_1}{N_2}$
- Φ_m lags voltage by 90°
- $\mathbf{I}_2$ lags $\mathbf{V}_2$ by 45°: $\phi = \cos^{-1}(0.707) = 45°$
- $\mathbf{I}_1, \mathbf{I}_2$ in phase: $\mathbf{I}_1 = \mathbf{I}_2 \dfrac{N_2}{N_1}$

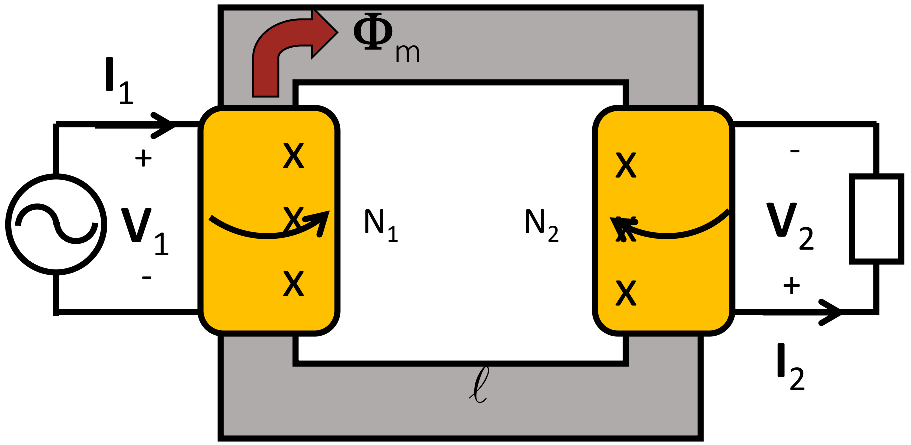




I_1, I_2 not in phase with Φ_m.

Transformer Polarity

What if the secondary coil was wound the opposite direction?



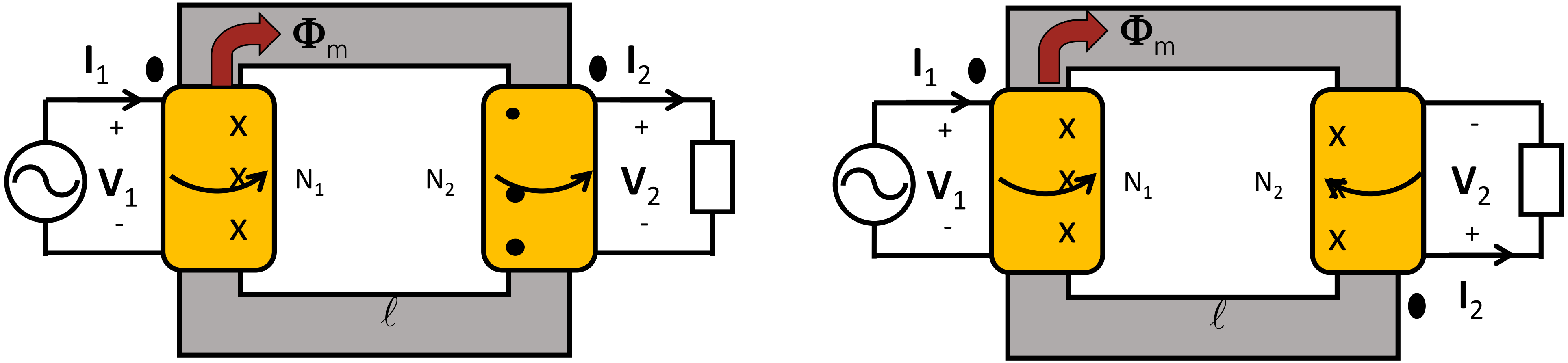
Examining the mmf:

$$\mathfrak{I} = N_1\mathbf{I}_1 + N_2\mathbf{I}_2 = 0$$
$$N_1\mathbf{I}_1 = -N_2\mathbf{I}_2$$

Current and voltage polarity reverses

Transformer Polarity

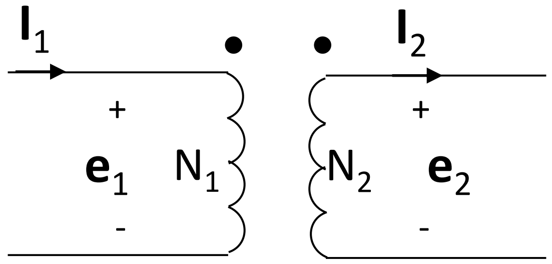
- Dot polarity:
  - Current entering polarity-marked terminals create flux in the same direction
  - When current enters one polarity-marked terminal, it leaves the other
  - Voltage of polarity-marked terminals are in phase (e.g. they are positive at the same time)



Transformer polarity is dictated by the direction of windings

Circuit Model

- New circuit element: Ideal Transformer
- Voltage relationship $\mathbf{e}_1 = a\mathbf{e}_2$
- Current relationship $\mathbf{I}_1 = \frac{1}{a}\mathbf{I}_2$



Ideal Transformer

Recall: $a = \frac{N_1}{N_2}$

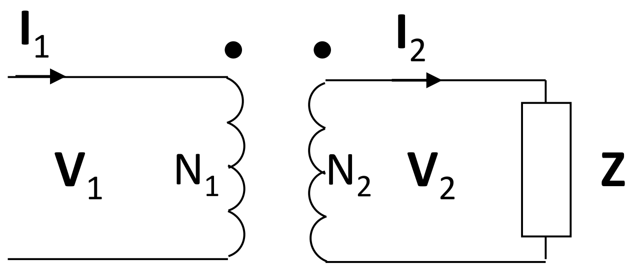
Circuit Model

- Now a load is connected to the secondary
- Solving for I_1

$$\mathbf{I}_1 = \frac{1}{a}\mathbf{I}_2$$

$$\mathbf{I}_1 = \frac{1}{a}\frac{\mathbf{V}_2}{\mathbf{Z}} \quad \text{using} \quad \mathbf{I}_2 = \frac{\mathbf{V}_2}{\mathbf{Z}}$$

$$\mathbf{I}_1 = \frac{\mathbf{V}_1}{a^2\mathbf{Z}} \quad \text{using} \quad \mathbf{V}_1 = a\mathbf{V}_2$$



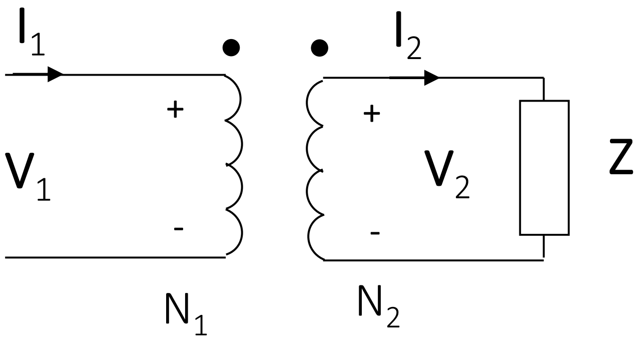
Ideal Transformer

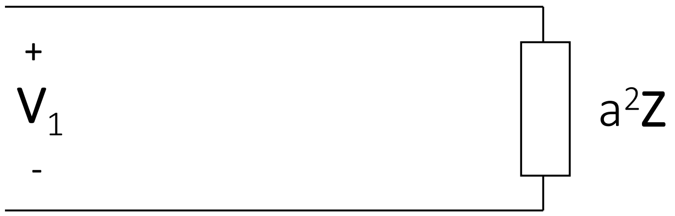
Circuit Model

From this result, <u>it is possible to analyze the circuit only using primary-side voltage and current</u> $(\mathbf{V}_1, \mathbf{I}_1)$

$$\mathbf{I}_1 = \frac{\mathbf{V}_1}{\boxed{a^2}\mathbf{Z}}$$

Secondary impedance referred to the primary side





equivalent circuit

Example

Consider an ideal transformer with $N_1 = 100$ and $N_2 = 500$. The primary is connected to a 100 V source. A load of 100 Ohms is connected to the secondary.

Find the power delivered to the load.

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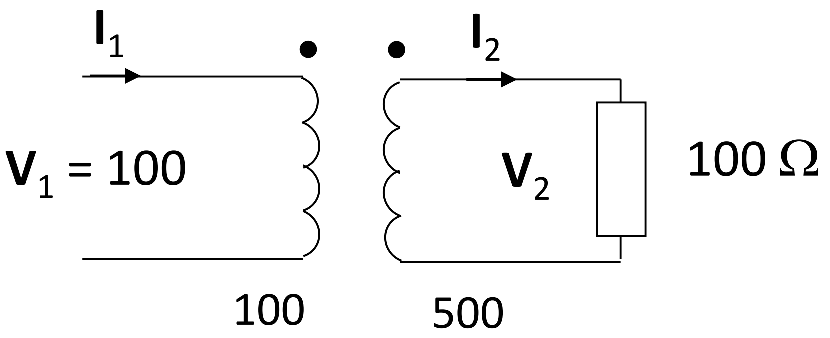
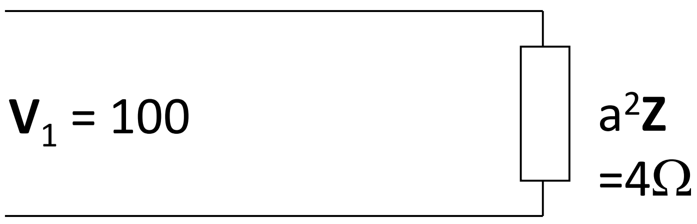
Consider an ideal transformer with N_1 = 100 and N_2 = 500. The primary is connected to a 100 V source. A load of 100 Ohms is connected to the secondary. Find the power delivered to the load.

$$a = \frac{N_1}{N_2} = \frac{100}{500} = 0.2$$





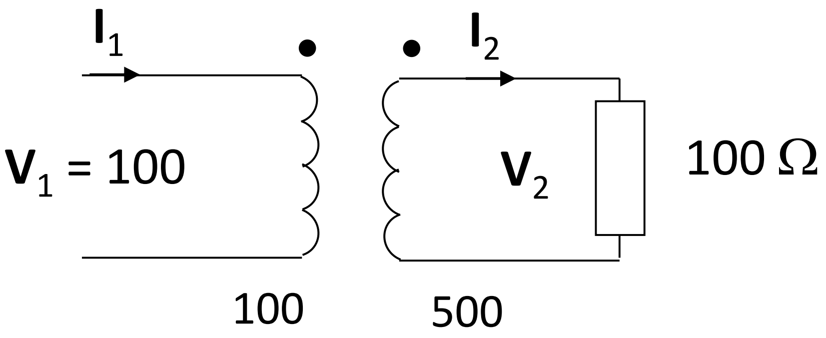
equivalent circuit

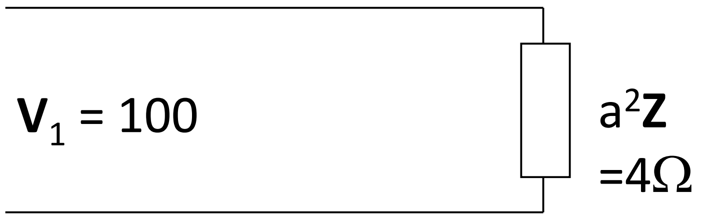
Example

Consider an ideal transformer with $N_1 = 100$ and $N_2 = 500$. The primary is connected to a 100 V source. A load of 100 Ohms is connected to the secondary.

Find the power delivered to the load.

$$P = \frac{|\mathbf{V}_1|^2}{R} = \frac{10,000}{4} = 2,500 \text{ W}$$





equivalent circuit

Summary

- Transformers are magnetically coupled coils
- Ratio of turns from primary to secondary is the "turns ratio". Side with greater number of turns has higher voltage, but lower current
- Ideal transformers: Power in = Power out
- Equivalent circuit is used to analyze transformers. Impedances can be transferred from secondary to primary by scaling by a^2