

21-Rotating Magnetic Field

ECEGR 3500

Text: 12.1

Electrical Energy Systems

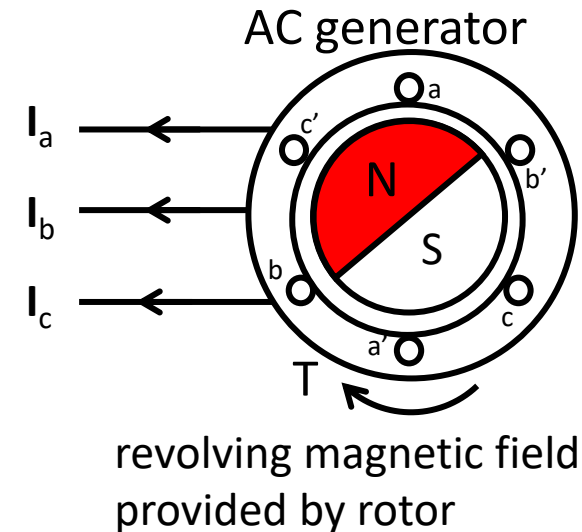
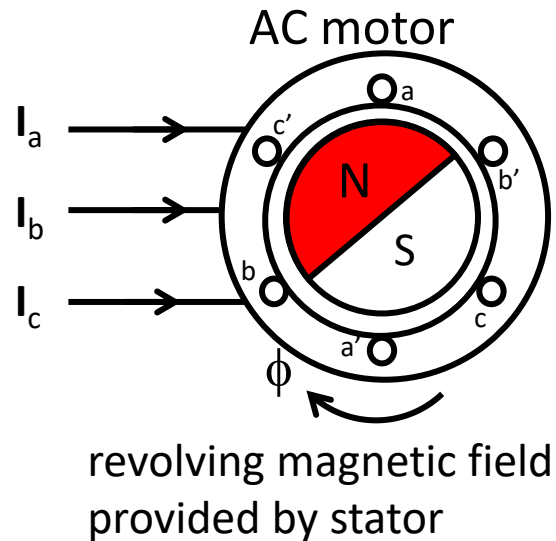
Professor Henry Louie

➤ Overview

- Introduction
- Rotating Magnetic Field
- Magnetic Field Rotational Speed
- Synchronous Speed

Introduction

- AC machines rely on a rotating magnetic field
- Stator houses current-carrying conductors
 - Stator is the armature



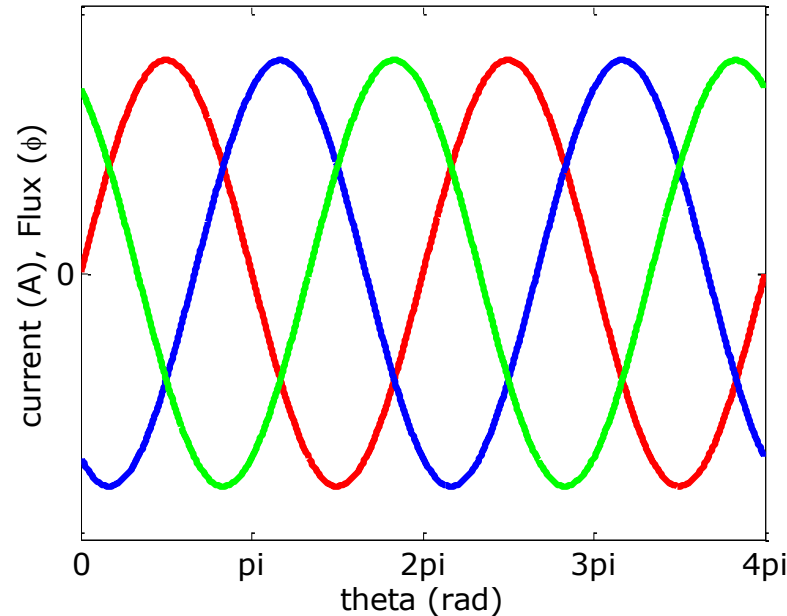
Rotating Magnetic Field

- Three-phase motors are very common
- Requires a three-phase source
- Under linear conditions, flux and current will have similar waveforms

$$\mathbf{I}_a = i_{\max} \sin(\omega t)$$

$$\mathbf{I}_b = i_{\max} \sin(\omega t - 120^\circ)$$

$$\mathbf{I}_c = i_{\max} \sin(\omega t + 120^\circ)$$

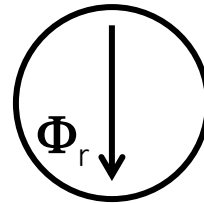
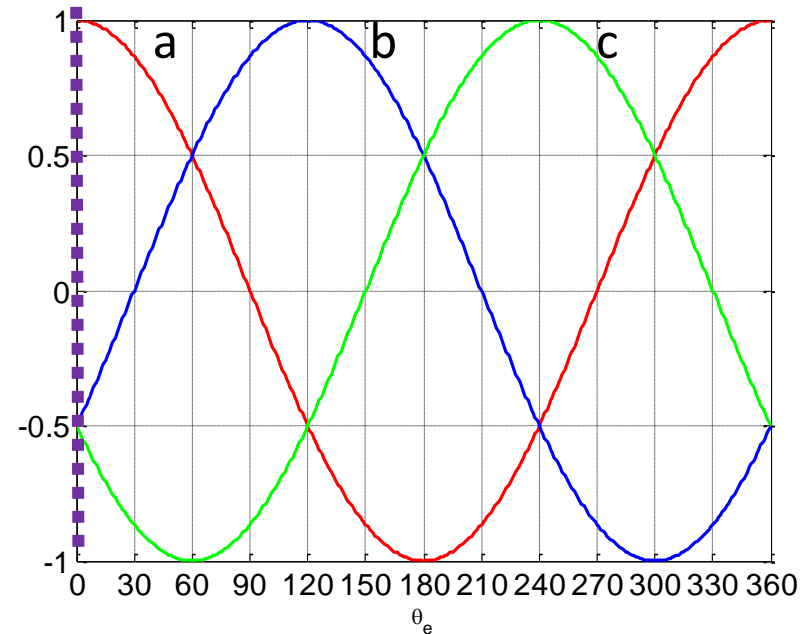
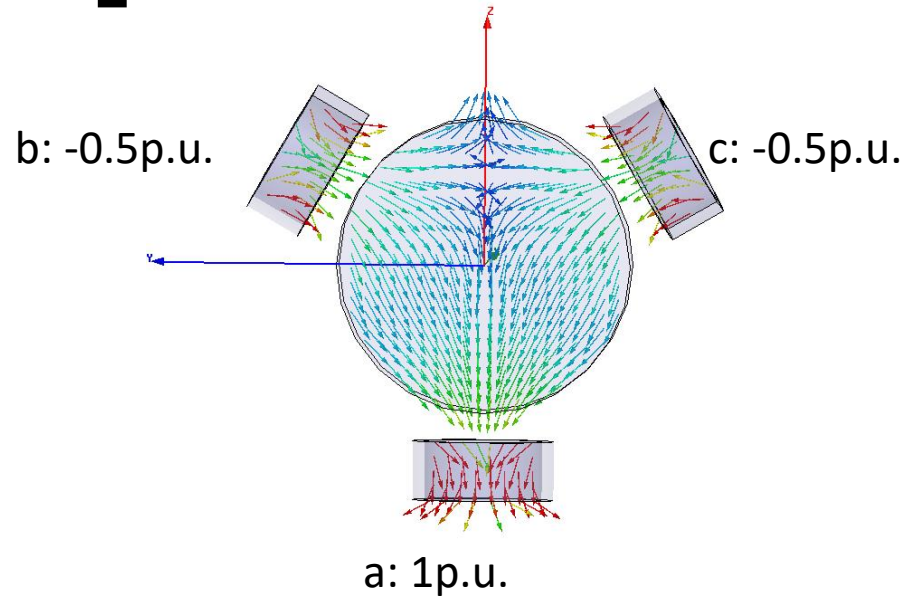


$$\Phi_a = \phi_{\max} \sin(\omega t)$$

$$\Phi_b = \phi_{\max} \sin(\omega t - 120^\circ)$$

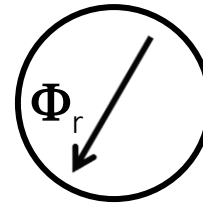
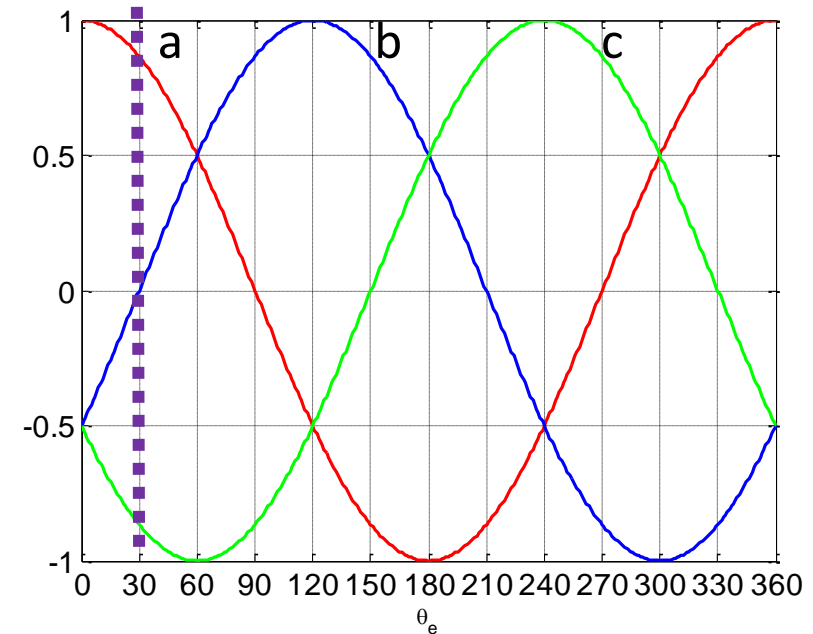
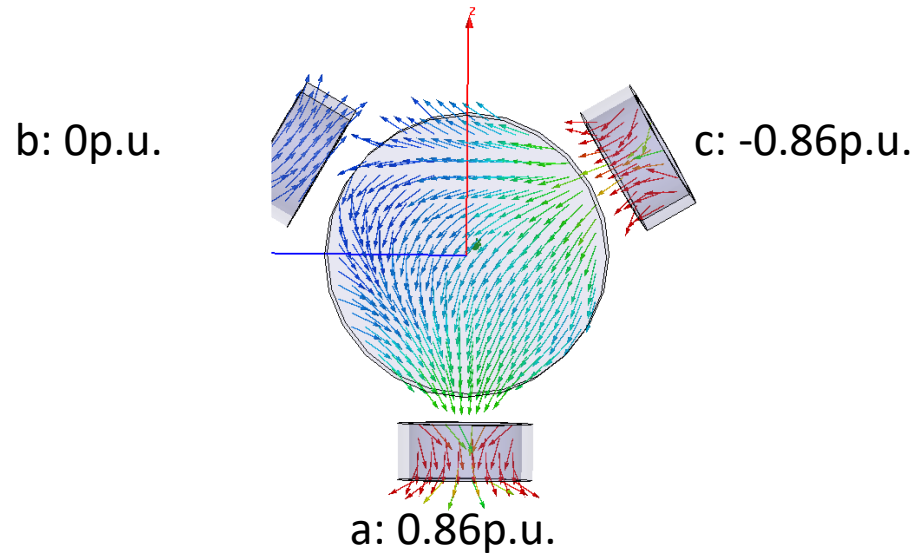
$$\Phi_c = \phi_{\max} \sin(\omega t + 120^\circ)$$

Conceptual Illustration $\theta_e = 0^\circ$



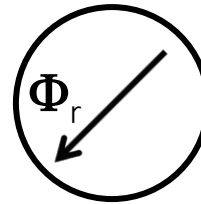
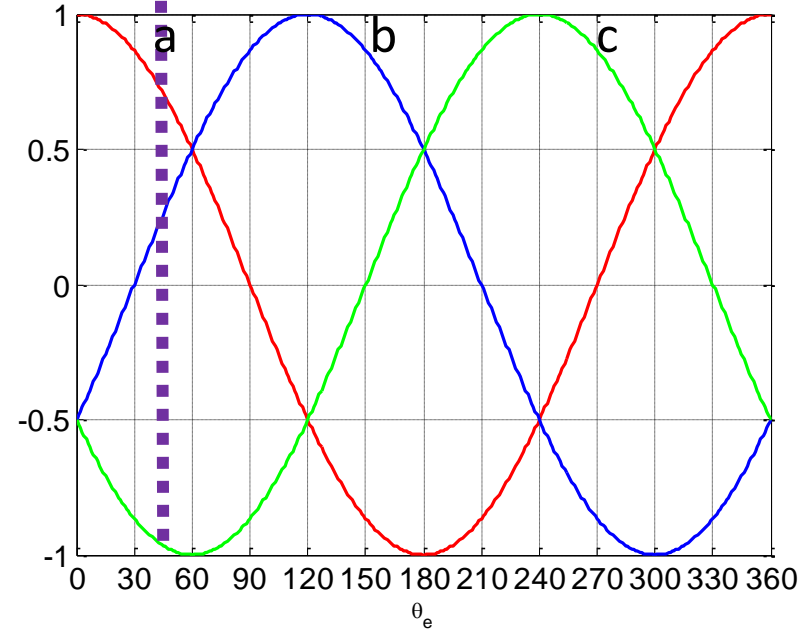
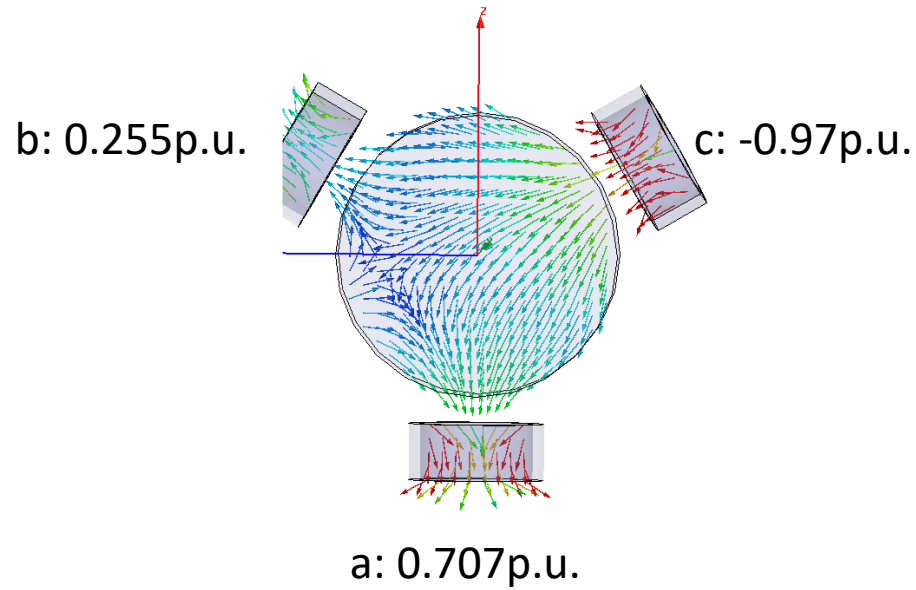
direction of flux through rotor

Conceptual Illustration $\theta_e = 30^\circ$



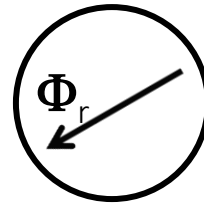
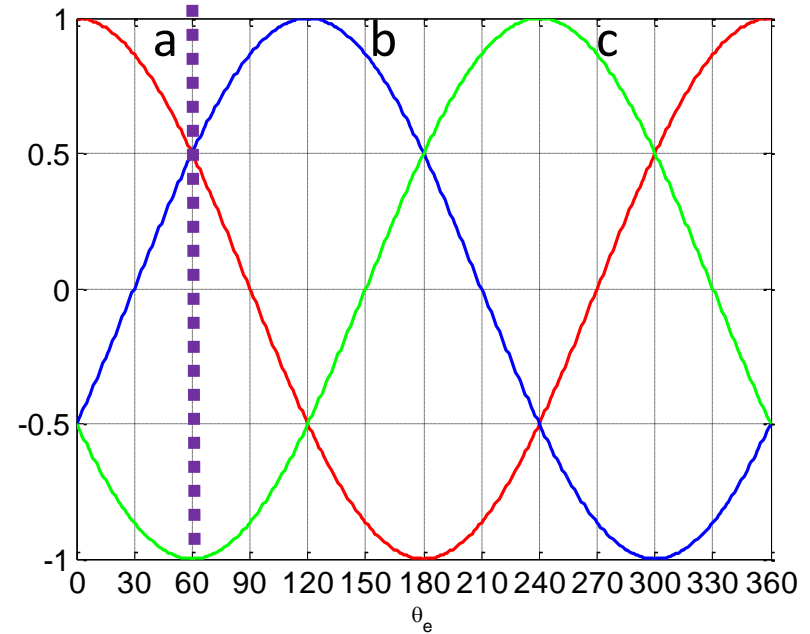
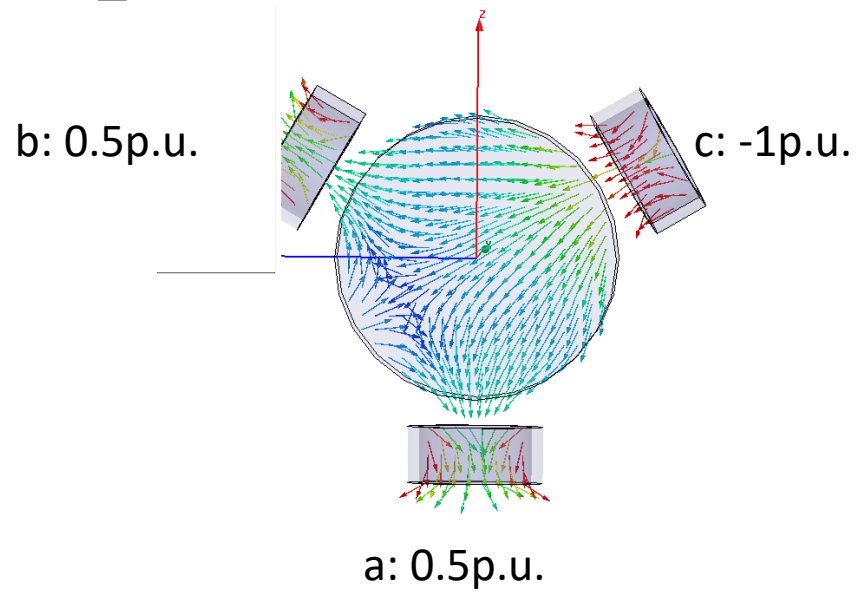
direction of flux through rotor

Conceptual Illustration $\theta_e = 45^\circ$



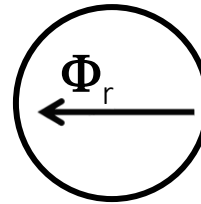
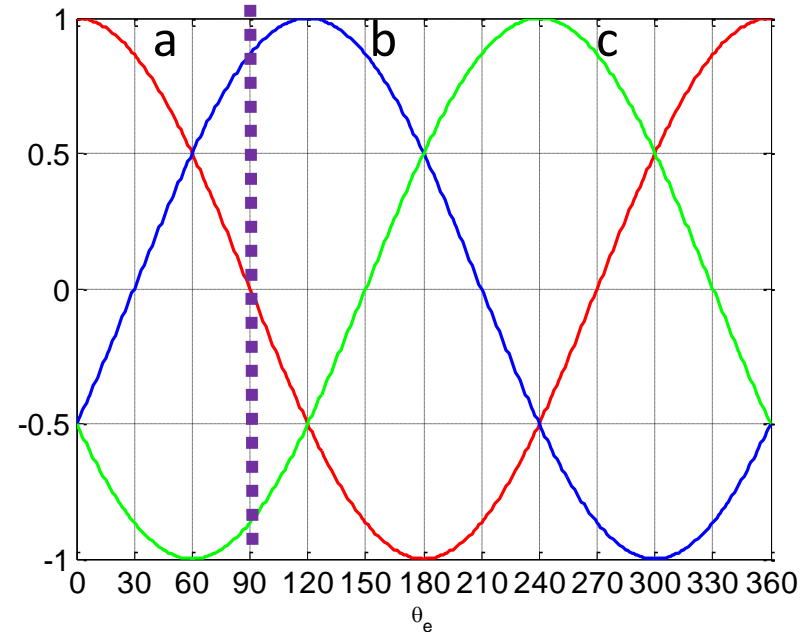
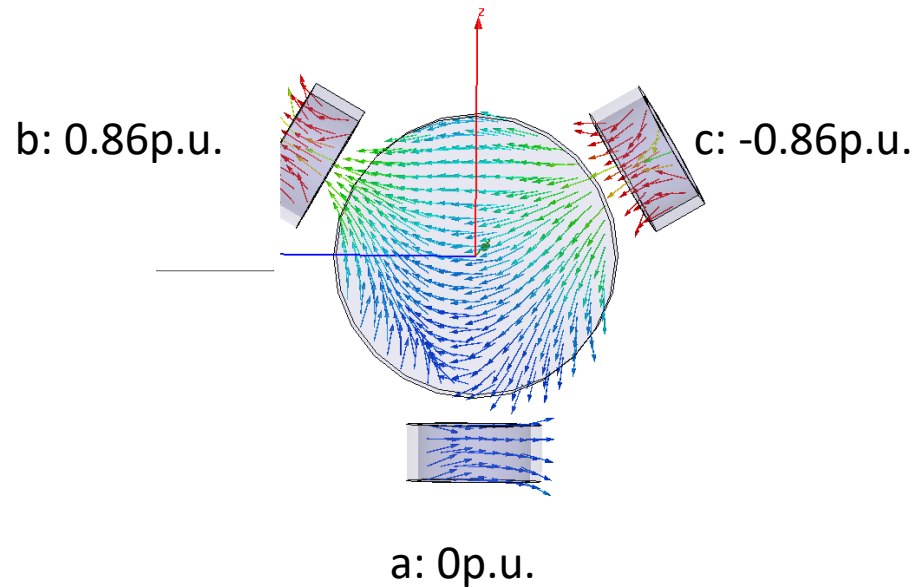
direction of flux through rotor

Conceptual Illustration $\theta_e = 60^\circ$



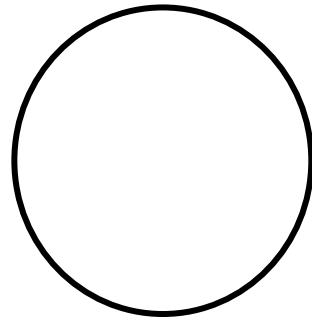
direction of flux through rotor

Conceptual Illustration $\theta_e = 90^\circ$

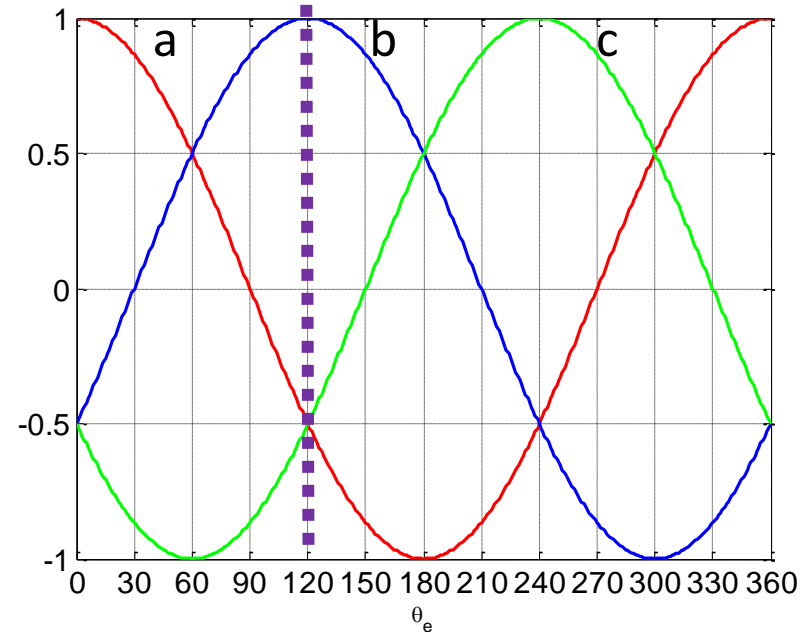


direction of flux through rotor

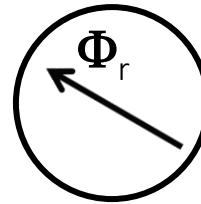
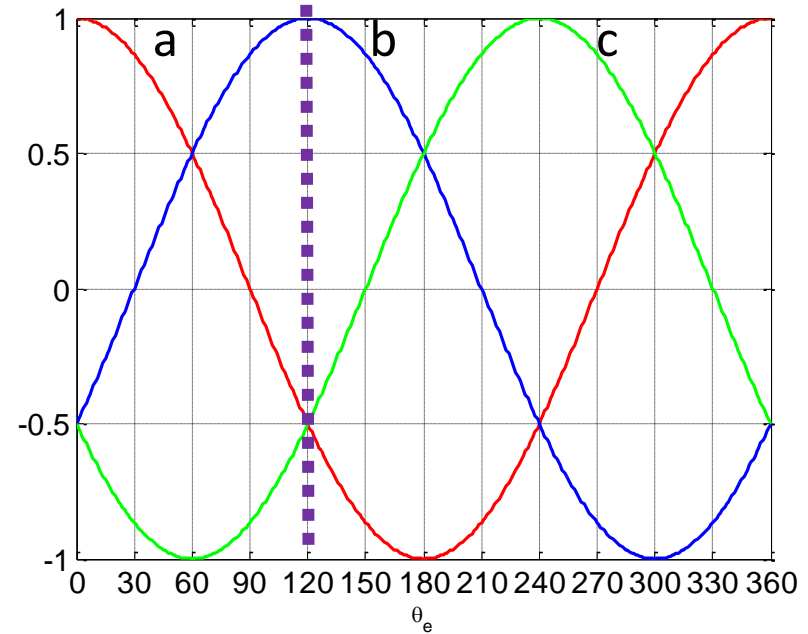
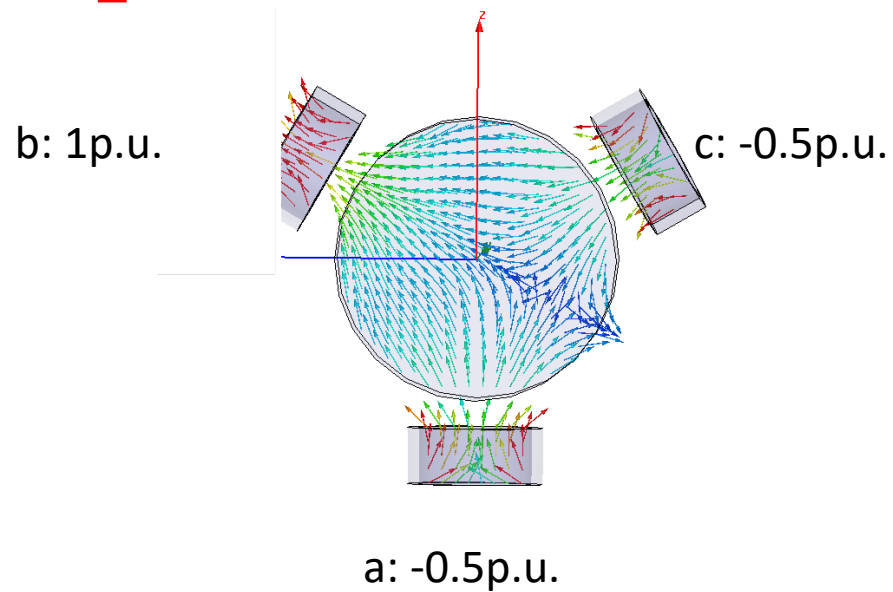
Conceptual Illustration $\theta_e = 120^\circ$



What direction will the flux through the rotor be when $\theta_e = 120^\circ$?



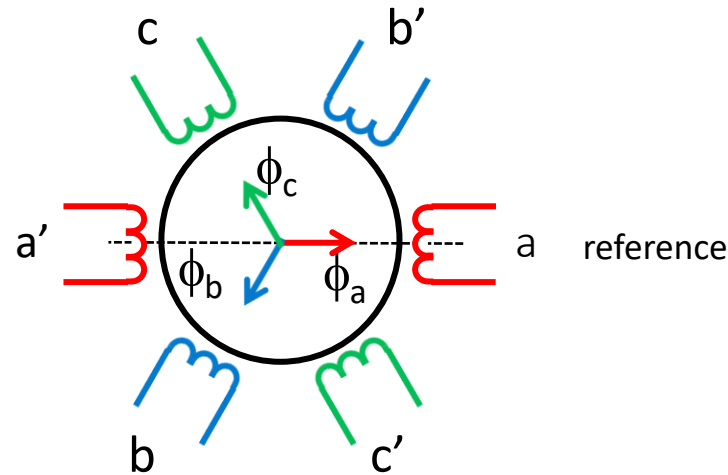
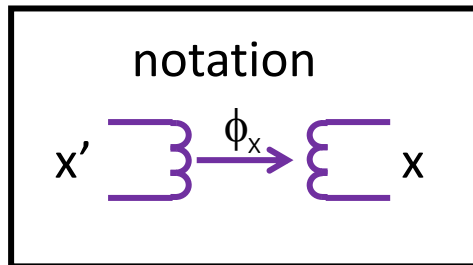
Conceptual Illustration $\theta_e = 120^\circ$



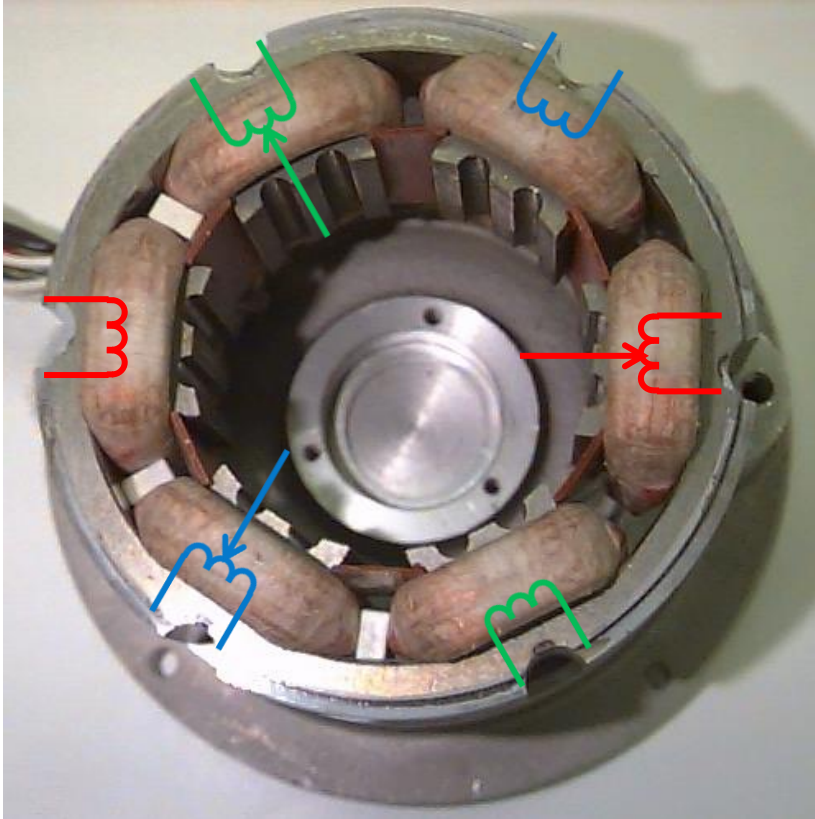
direction of flux through rotor

Two-Pole Three-Phase Revolving Field

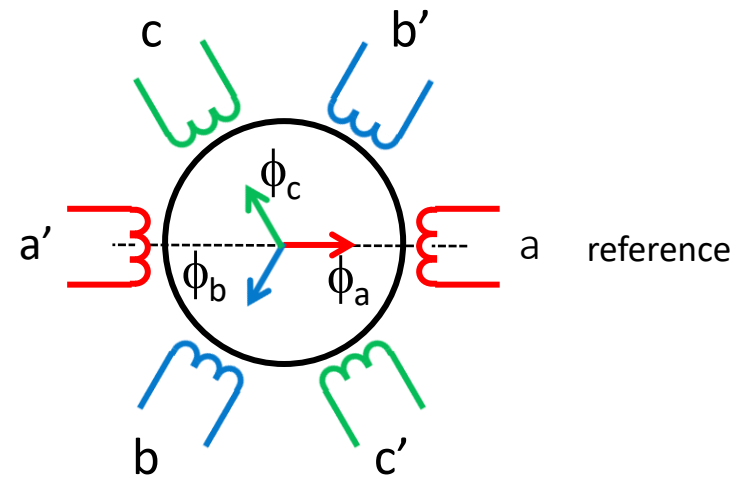
- Coils are spatially separated by 60°
 - Do not confuse the spatial direction with the phase of the flux
- We will analyze how the flux varies with time
- Note: if flux is negative, the direction is opposite as shown



Three-Phase AC Motor



Note: salient windings are shown.
Large motors use cylindrical windings.

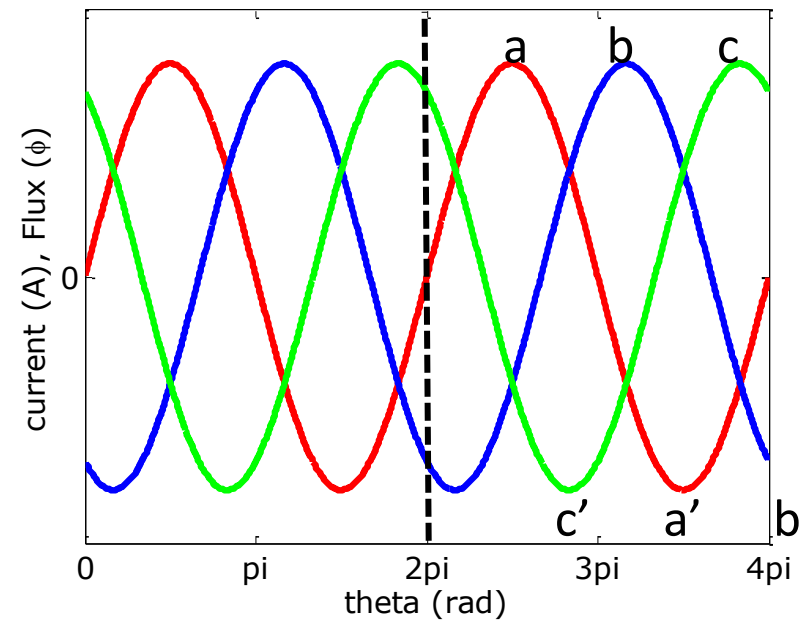
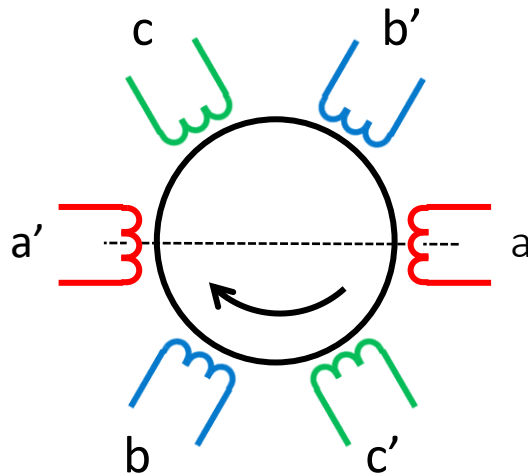


Rotating Magnetic Field

- Want to analyze the net flux as seen by the rotor
- General approach:
 - Consider the flux at 0, 60 and 120 degrees in time
 - Compute the a, b, c phase flux magnitudes
 - Determine resulting flux by adding a, b, c phase flux
 - Generalize results

Rotating Magnetic Field

- Maximum flux occurs in the following sequence
 - a, c', b, a', c, b' and so on
 - same relative ordering of coils around stator



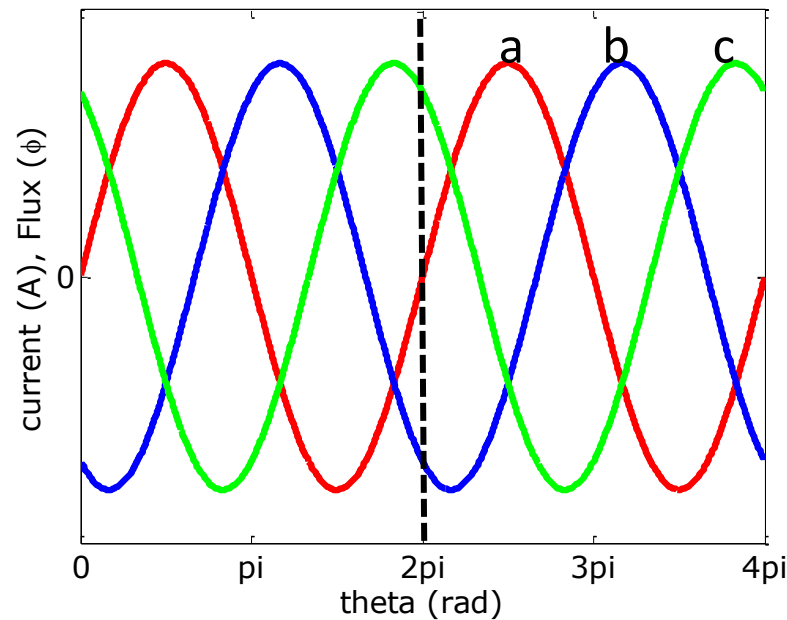
Rotating Magnetic Field

Let $\omega t = 0$. The magnitudes are:

$$|\Phi_a| = \phi_{\max} \sin(\omega t) = 0$$

$$|\Phi_b| = \phi_{\max} \sin(\omega t - 120^\circ) = \left| -\frac{\sqrt{3}}{2} \phi_{\max} \right|$$

$$|\Phi_c| = \phi_{\max} \sin(\omega t + 120^\circ) = \frac{\sqrt{3}}{2} \phi_{\max}$$

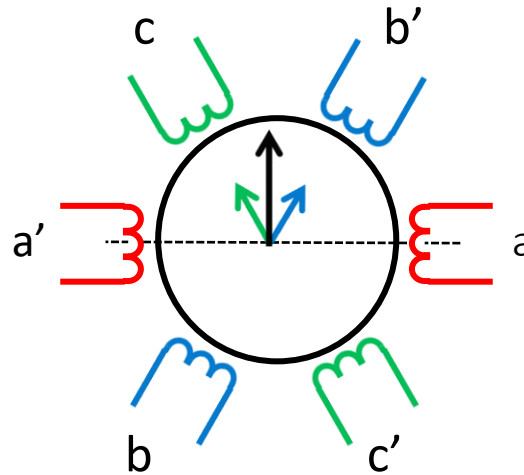


Rotating Magnetic Field

- At $\omega t = 0$ the flux is as shown
- The resulting flux, Φ_r , is found through vector addition

$$\Phi_r = \Phi_a + \Phi_b + \Phi_c$$

$$= 0 + \frac{-\sqrt{3}}{2} \phi_m \angle 240^\circ + \frac{\sqrt{3}}{2} \phi_m \angle 120^\circ = 1.5 \phi_m \angle 90^\circ$$



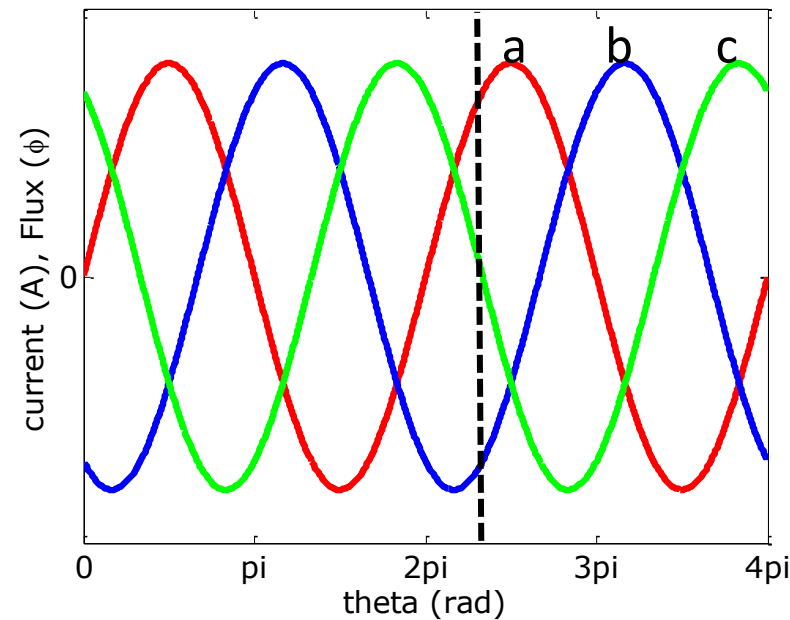
$$\begin{aligned}\Phi_a &= 0 \angle 0^\circ \\ \Phi_b &= -\frac{\sqrt{3}}{2} \phi_{\max} \angle 240^\circ \\ \Phi_c &= \frac{\sqrt{3}}{2} \phi_{\max} \angle 120^\circ\end{aligned}$$

Rotating Magnetic Field

At $\omega t = 60^\circ$: $|\Phi_a| = \phi_{\max} \sin(\omega t) = \frac{\sqrt{3}}{2} \phi_{\max}$

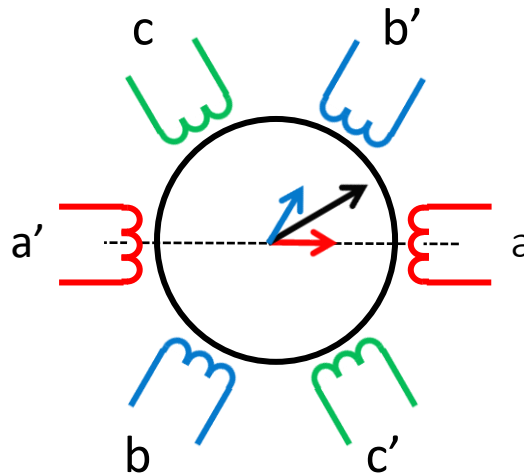
$$|\Phi_b| = \phi_{\max} \sin(\omega t - 120^\circ) = \left| -\frac{\sqrt{3}}{2} \phi_{\max} \right|$$

$$|\Phi_c| = \phi_{\max} \sin(\omega t + 120^\circ) = 0$$



Rotating Magnetic Field

$$\begin{aligned}\Phi_r &= \Phi_a + \Phi_b + \Phi_c \\ &= \frac{\sqrt{3}}{2} \phi_m \angle 0^\circ + \frac{-\sqrt{3}}{2} \phi_m \angle 240^\circ + 0 = 1.5 \phi_m \angle 30^\circ\end{aligned}$$



$$\Phi_a = \frac{\sqrt{3}}{2} \phi_{\max} \angle 0^\circ$$

$$\Phi_b = -\frac{\sqrt{3}}{2} \phi_{\max} \angle 240^\circ$$

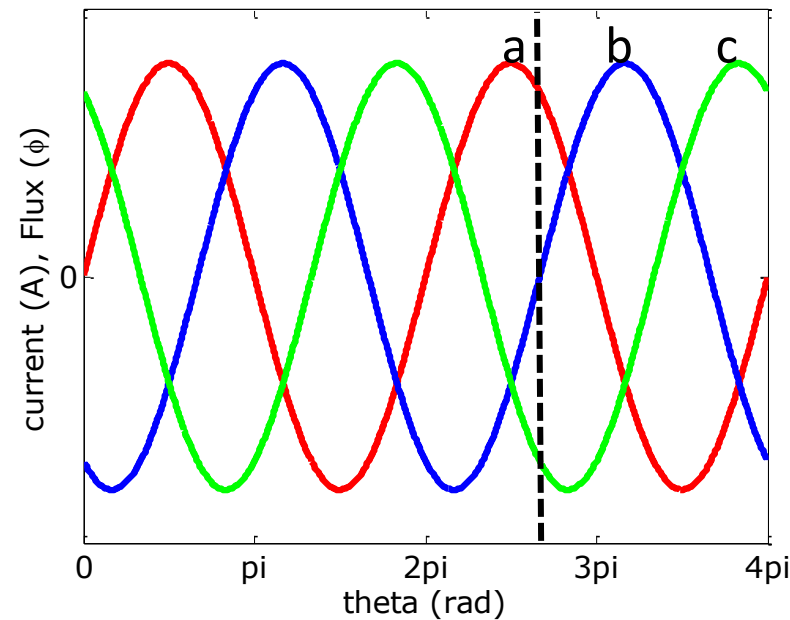
$$\Phi_c = 0 \angle 120^\circ$$

Rotating Magnetic Field

Let $\omega t = 120^\circ$ $|\Phi_a| = \phi_{\max} \sin(\omega t) = \frac{\sqrt{3}}{2} \phi_{\max}$

$|\Phi_b| = \phi_{\max} \sin(\omega t - 120^\circ) = 0$

$|\Phi_c| = \phi_{\max} \sin(\omega t + 120^\circ) = -\frac{\sqrt{3}}{2} \phi_{\max}$

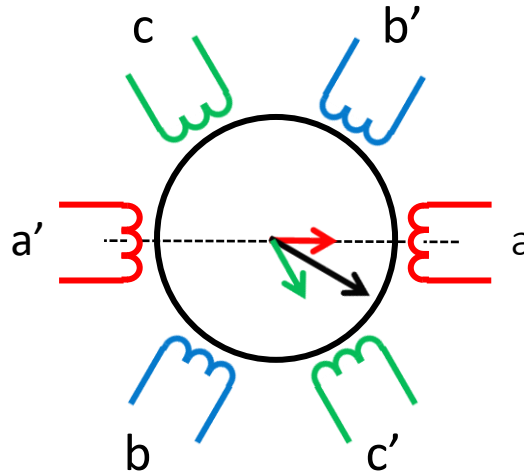


Rotating Magnetic Field

- At $\omega t = 60^\circ$ the flux is as shown
- The resulting flux is found through vector addition

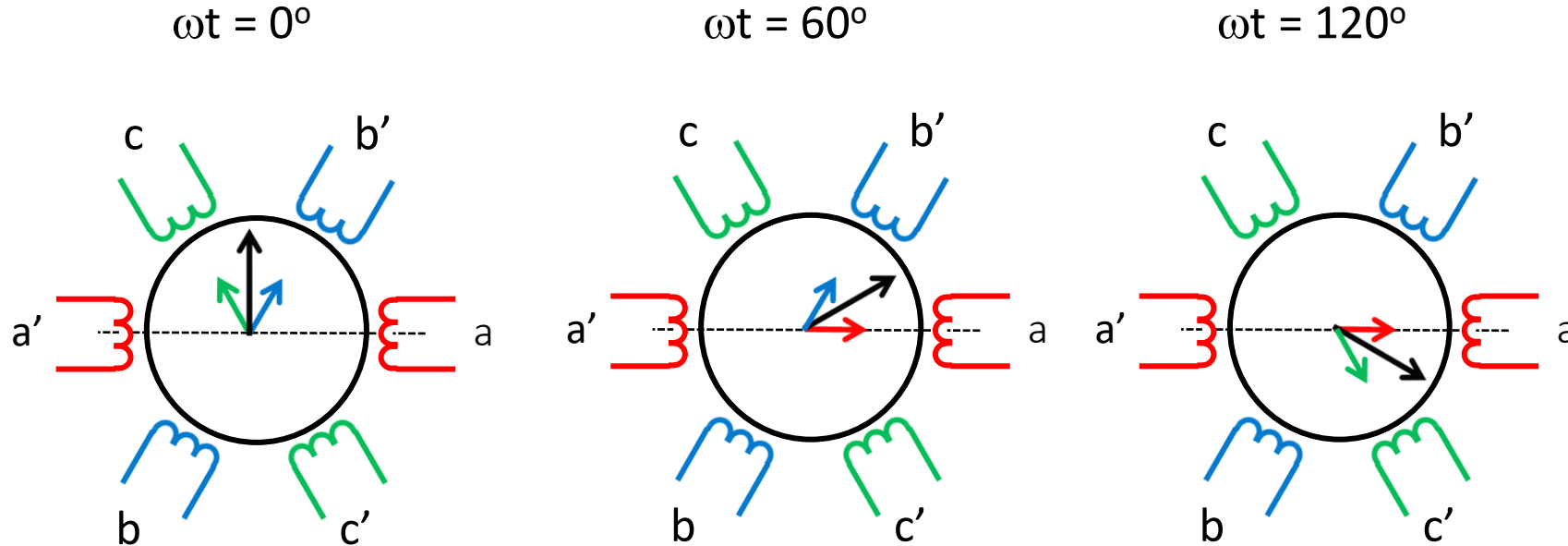
$$\Phi_r = \Phi_a + \Phi_b + \Phi_c$$

$$= \frac{\sqrt{3}}{2} \phi_m \angle 0^\circ + 0 + -\frac{\sqrt{3}}{2} \phi_m \angle 120^\circ = 1.5 \phi_m \angle -30^\circ$$



$$\begin{aligned}\Phi_a &= \frac{\sqrt{3}}{2} \phi_{\max} \angle 0^\circ \\ \Phi_b &= 0 \angle 240^\circ \\ \Phi_c &= -\frac{\sqrt{3}}{2} \phi_{\max} \angle 120^\circ\end{aligned}$$

Rotating Magnetic Field



resulting flux vector rotates CW in time

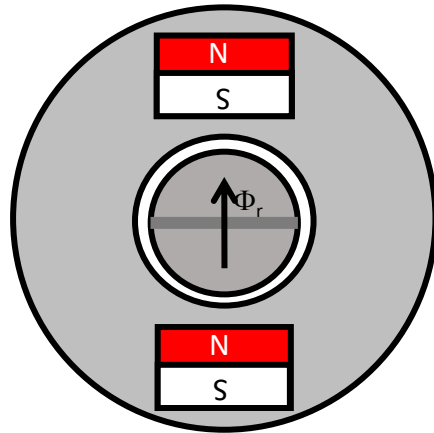
Rotating Magnetic Field

Observations:

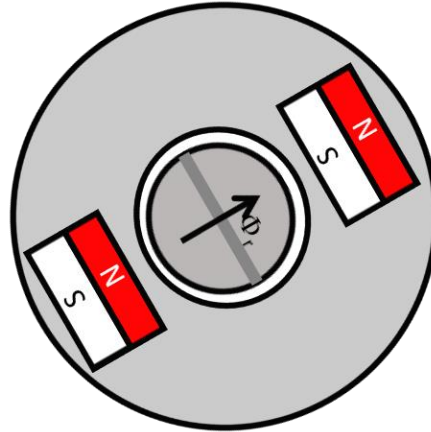
- Resulting flux magnitude is constant
- Direction of the resulting flux rotates with time
- 120° phase shift in the time domain has shifted the spatial orientation of the flux 120°
- To make the field rotate in the opposite direction (counter clockwise) switch any two phases (e.g. b and c phases)

Rotating Magnetic Field

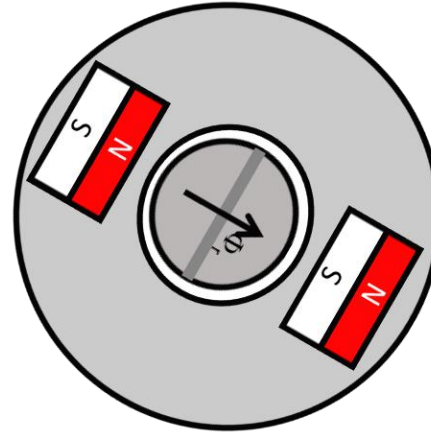
Conceptually like rotating magnets around the periphery



$$\omega t = 0^\circ$$



$$\omega t = 60^\circ$$



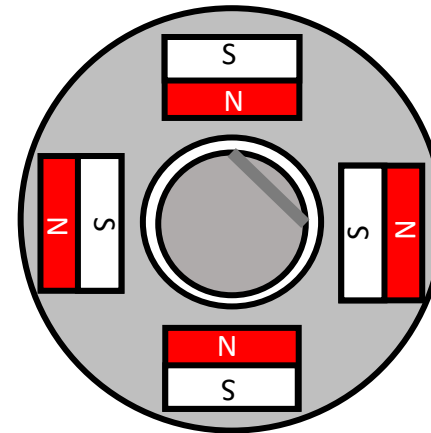
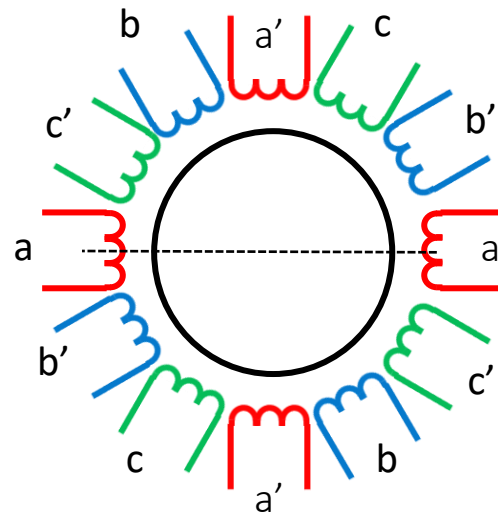
$$\omega t = 120^\circ$$

➤ Magnetic Field Rotational Speed

- For a 2-pole motor, one full rotation of the magnetic field occurs after one complete electrical cycle
- How does a 4-pole motor affect the rotational speed of the magnetic field?

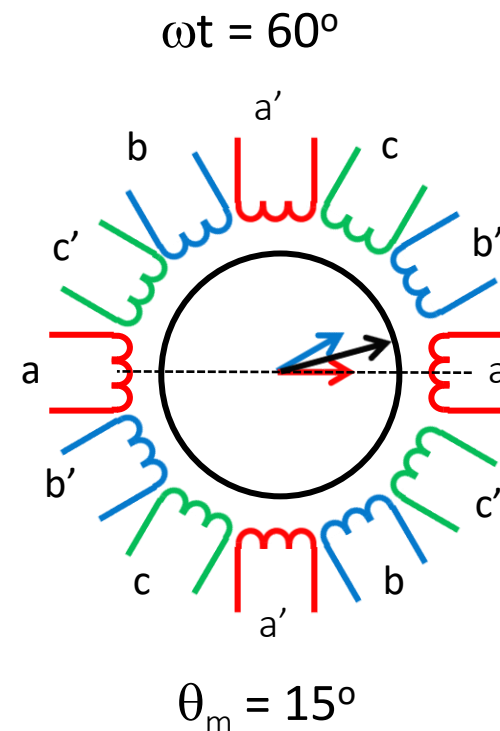
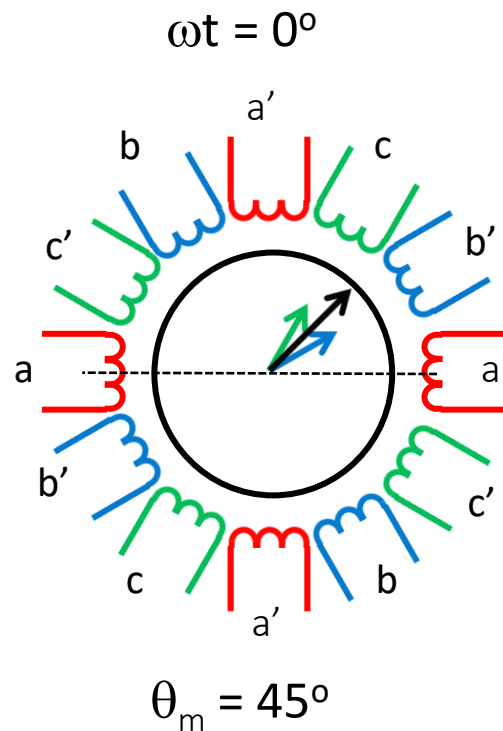
➤ Magnetic Field Rotational Speed

- Coils separated by 30 degrees
 - a, c', b, a', c, b' ordering is preserved
- Four poles, examine one pole-pair



→ Magnetic Field Rotational Speed

60 degrees time shift resulted in spatial rotation of 30 degrees



➤ Magnetic Field Rotational Speed

- For a 4 pole-motor one full rotation of the magnetic field requires two complete electrical cycles
- To generalize: $T_s = \frac{P}{2} T$
 - T_s : period of the flux rotation (s)
 - T : period of the AC waveform (s)
 - P : number of poles

Note: do not confuse “T” for period, with “T” for torque.

➤ Magnetic Field Rotational Speed

- Also $n_s = \frac{1}{T_s} = \frac{2f}{p}$
 - n_s : speed of the revolving field (revolutions/s)
 - f : frequency of the AC waveform (Hz)
- n_s is known as the synchronous speed

Note: this and previous equations relate frequency of applied source with rotation of magnetic field, not the actual rotation of the rotor.

» Exercise

Write N_s , the synchronous speed in revolutions per minute (RPM) and radians per second (ω_s) as a function of the number of poles and frequency f

» Exercise

- Find N_s , the synchronous speed in revolutions per minute (RPM) and radians per second (ω_s)

$$N_s = \frac{120f}{p} \text{ (RPM)}$$

$$\omega_s = \frac{4\pi f}{p} = \frac{2}{p} \omega \text{ (rad/s)}$$

→ Example

An 6-pole AC motor is connected to 50 Hz source. What is the synchronous speed of the motor in rpm?

→ Example

An 6-pole AC motor is connected to 50 Hz source. What is the synchronous speed of the motor in rpm?

$$N_s = \frac{120 \times 50}{6} = 1000 \text{ rpm}$$

Summary

- Magnetic field rotates with constant magnitude
- The resulting flux is $0.5n$ times the single phase flux, where n is the number of phases
- The synchronous speed is inversely proportional to the number of poles and proportional to the frequency of the applied source