22-Induction Motors

ECEGR 3500

Text: 12.2

Electrical Energy Systems

Professor Henry Louie

Overview

- Introduction
- Applications
- Three-Phase Induction Motor Rotation
- Slip Speed

Introduction

- Practical three-phase induction motor invented by Nikola Tesla in 1883
- Induction machines do not require brushes or slip rings
- Induction motors are very common
  - 1/3 of electrical energy consumption
  - Induction generators often used in wind turbines
- "singly-fed" machine (only stator is connected to power)
  - Induction generators may be "doubly fed"

Applications

- Large range of power ratings
  - Several watts to 40,000 hp
- Can be single phase, two phase, three phase…
  - >5 hp usually three phase
- Typical applications:
  - Washers, dryers, blenders, electric vehicles, fans, pumps

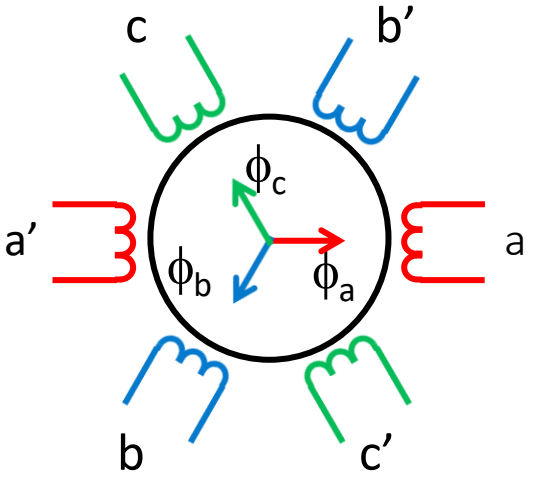
Applications

- Advantages:
  - Low cost
  - Simple and rugged construction
  - Low maintenance
  - Appealing torque-speed characteristics
- Disadvantages:
  - Consumes reactive power (lagging)
  - Speed cannot be easily controlled if connected to fixed-frequency AC source

Applications

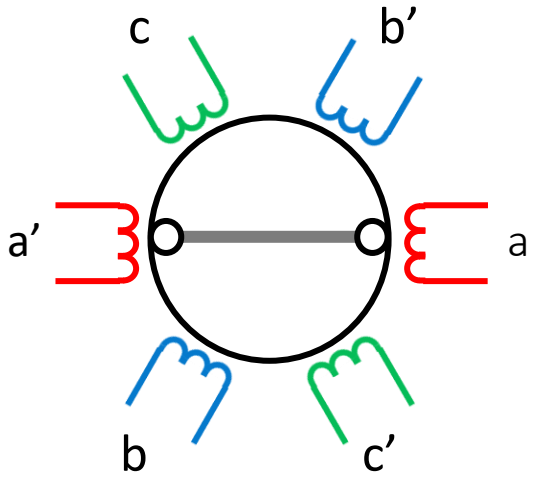
Induction Motor

Recall: revolving magnetic field established by connecting armature (stator) windings to three-phase AC source



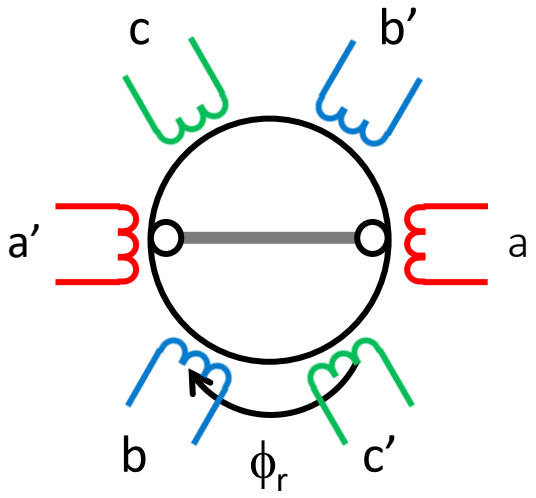
Induction Motor

Consider a single rotor coil with shorted terminals



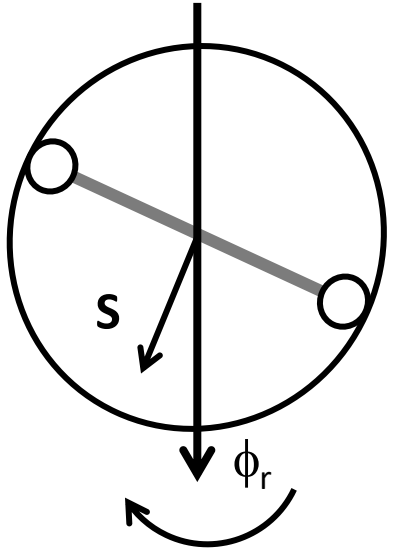
Induction Motor

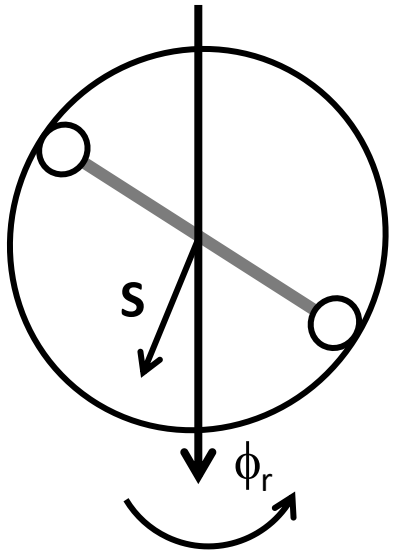
- Assume rotor is locked in place (cannot rotate)
- Net flux through the coil varies with time
  - Voltage induced in rotor, current flows



Induction Motor

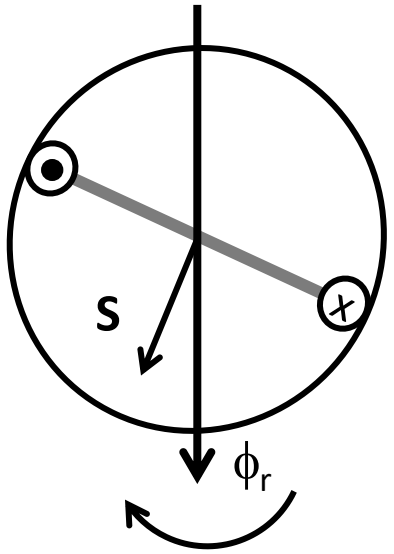
Determine the direction of the induced current for the shown rotational directions of flux.



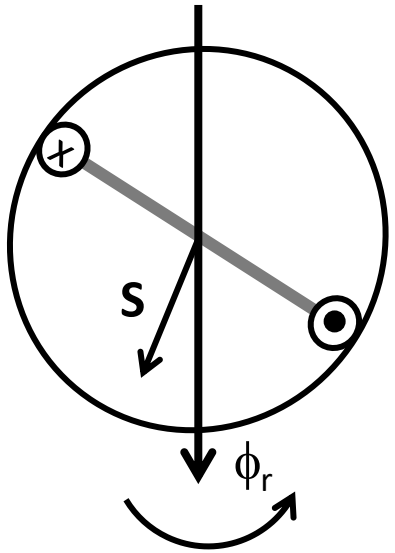


Exercise

Determine the direction of the torque on the conductors.



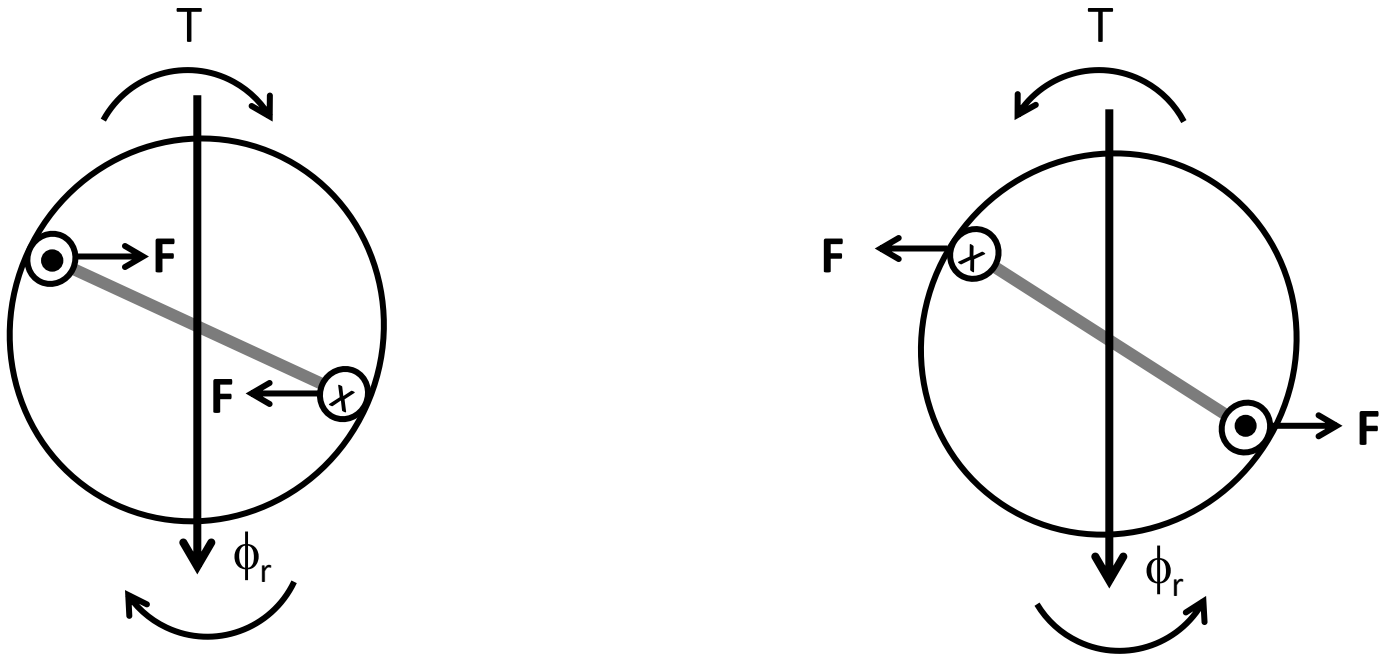
$\mathbf{S} \bullet \mathbf{B}$ increasing



$\mathbf{S} \bullet \mathbf{B}$ decreasing

Exercise

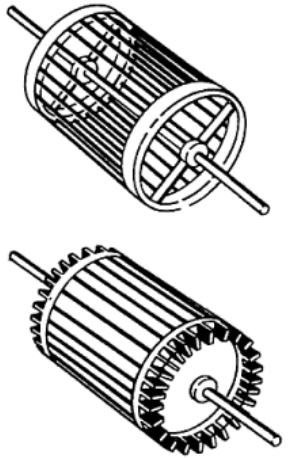
Determine the direction of the torque on the conductors.



Torque is in the same direction as rotating field

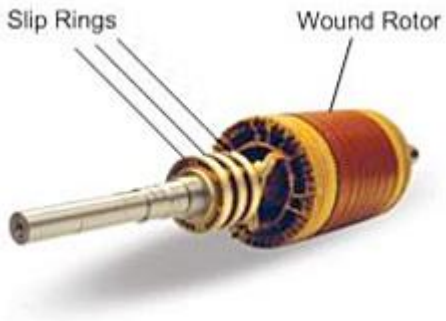
Squirrel Cage Induction Motors

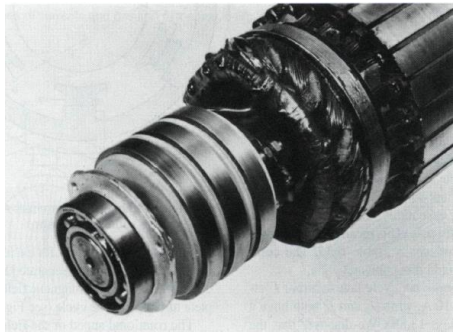
- Most common induction motor type
- Rotor made of solid conductors shorted through end rings
  - Low resistance
  - Promotes high current flow
  - No external connection



Wound Rotor Induction Motors

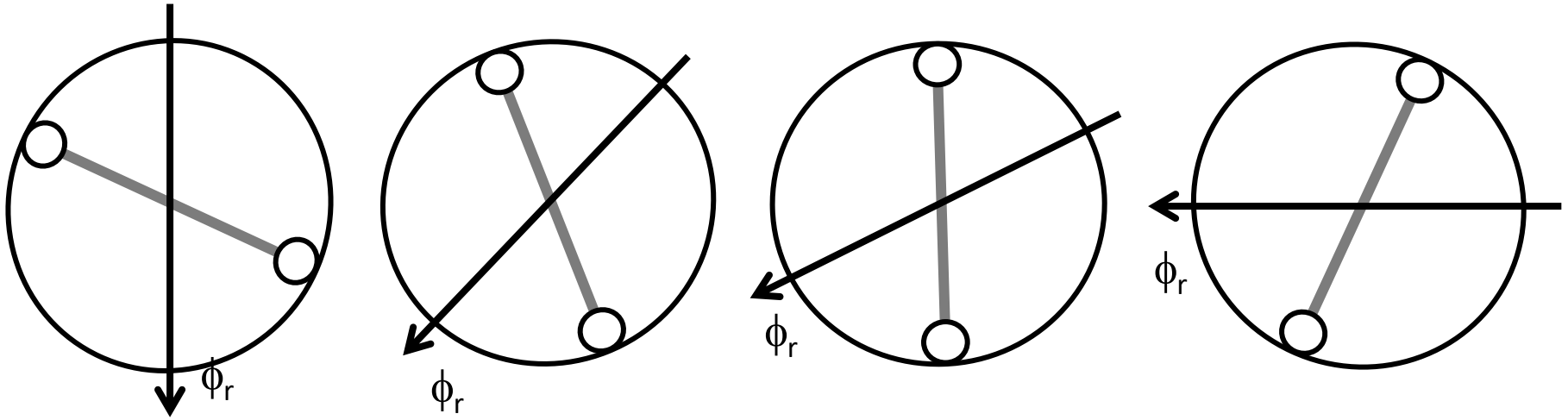
- Rotor wound with phases
- Slip rings used to connect rotor to external stationary circuit
- Greater control of motor
  - often used in induction generators





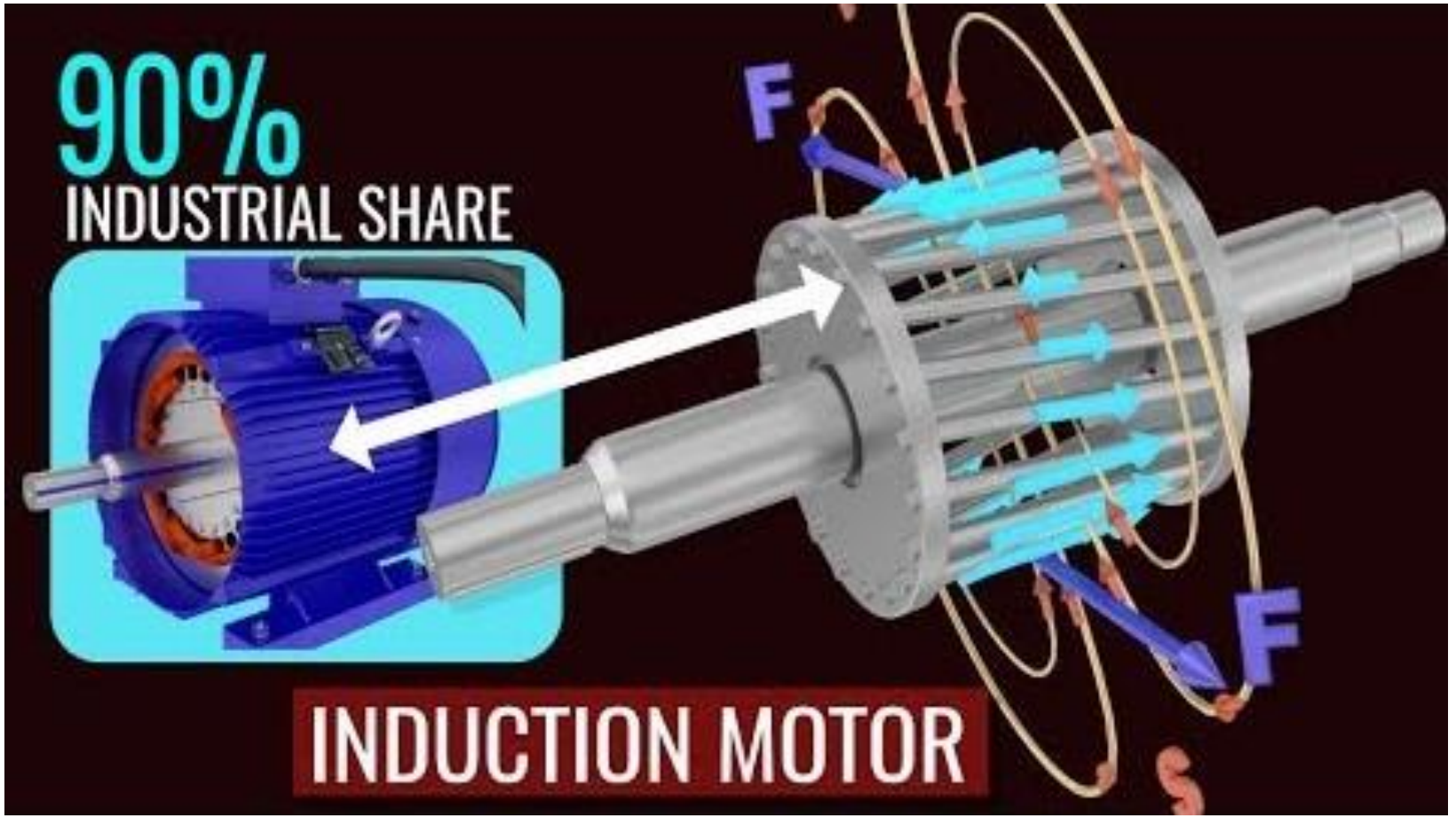
Induction Motor

- Now assume rotor rotates at synchronous speed N_s (same speed as rotating field)
- Does any current flow?



Three-Phase Induction Motor

- The rotor must rotate at a different speed than the synchronous speed
  - rotor speed < synchronous (motor)
  - rotor speed > synchronous (generator)
- If it rotated at the same speed, the flux through the closed would be constant and no emf would be induced
- Rate of change of flux through coil is difference between field speed and mechanical speed
- For this reason, induction motors are known as asynchronous motors


90%
INDUSTRIAL SHARE
F
F
INDUCTION MOTOR

Three-Phase Induction Motor

- The rotor speed is dependent on the load (torque)
- As load increases the rotor will start to slow down
- As it slows down, the rate of change of the flux through the closed loop increases, resulting in greater current and hence greater applied torque
- The rotor will speed up until the load torque equals the applied torque

Slip Speed

- Slip speed: difference in synchronous speed and rotor speed
  - Relative speed of the revolving flux ahead of the rotor

$$N_r = N_s - N_m$$

$$\omega_r = \omega_s - \omega_m$$

  - N_r: slip speed (rpm)
  - N_m: rotor speed (rpm)
  - ω_r: slip speed (rad/s)
  - ω_m: rotor speed (rad/s)

Slip

Slip of a motor is:

$$s = \frac{\omega_r}{\omega_s} = \frac{\omega_s - \omega_m}{\omega_s} = \frac{N_s - N_m}{N_s}$$

Note: slip is often expressed as a percent

0.5 slip = 50% slip

Exercise

Compute the synchronous speed of a 4-pole, 50 Hz three phase induction motor. What is the percent slip if the rotor rotates at 1200 RPM?

Exercise

Compute the synchronous speed of a 4-pole, 50 Hz three phase induction motor. What is the percent slip if the rotor rotates at 1200 RPM?

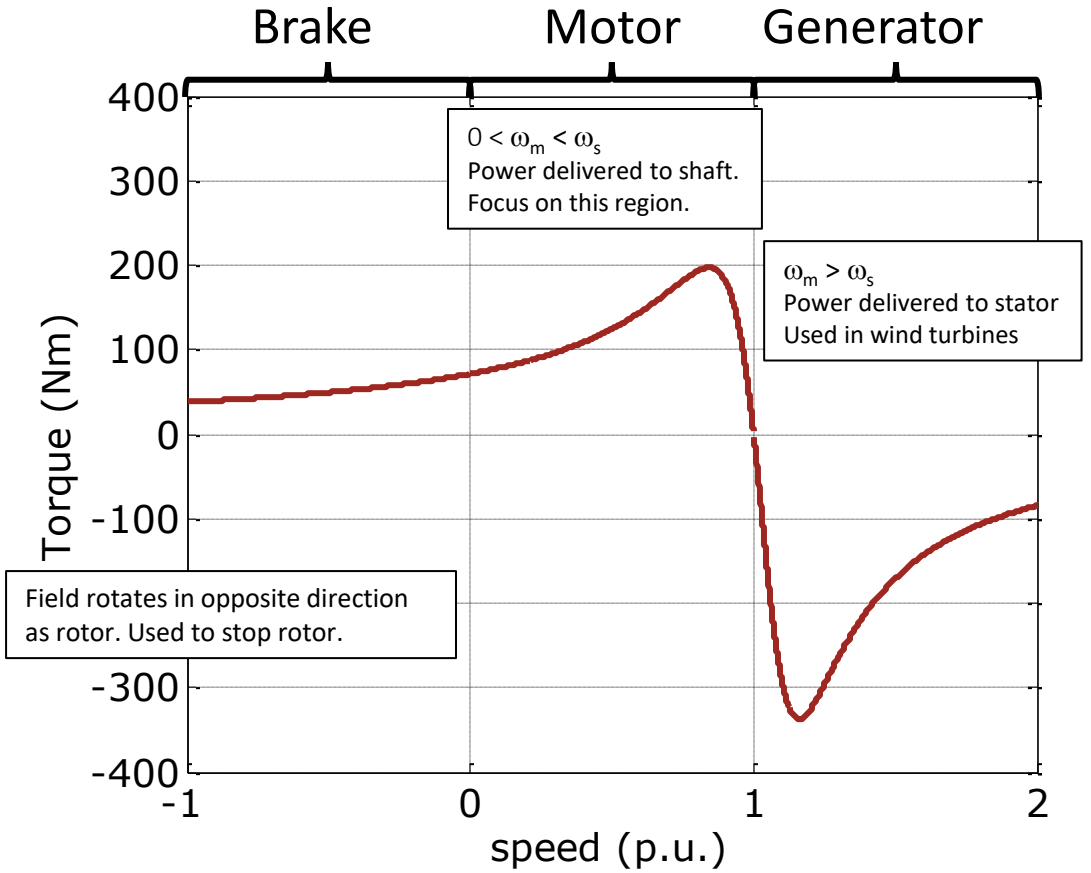
$$N_s = \frac{120f}{P} = \frac{120(50)}{4} = 1500 \text{ rpm}$$

$$s = 100 \times \frac{N_s - N_m}{N_s} = 20\%$$

Induced Voltage

- Voltage induced in rotor depends on rate of change of flux
- The closer to synchronous speed the rotor rotates, the smaller the change in flux
- Induced voltage in the rotor decreases in proportion to the slip:
  - If the slip is zero, then the induced voltage is zero
  - If the slip is 0.5, then the induced voltage is half of that when the slip is 1.0

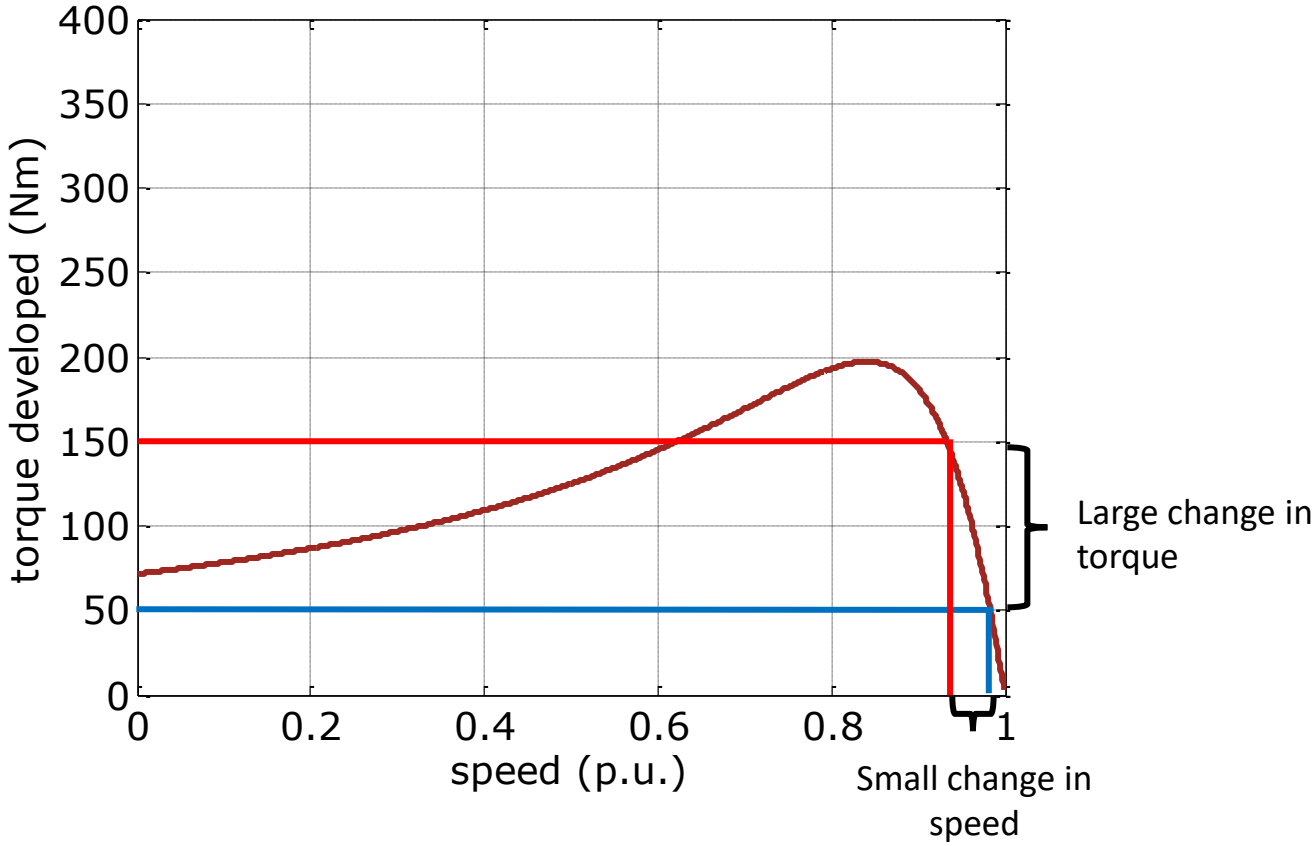
General Characteristics



General Characteristics

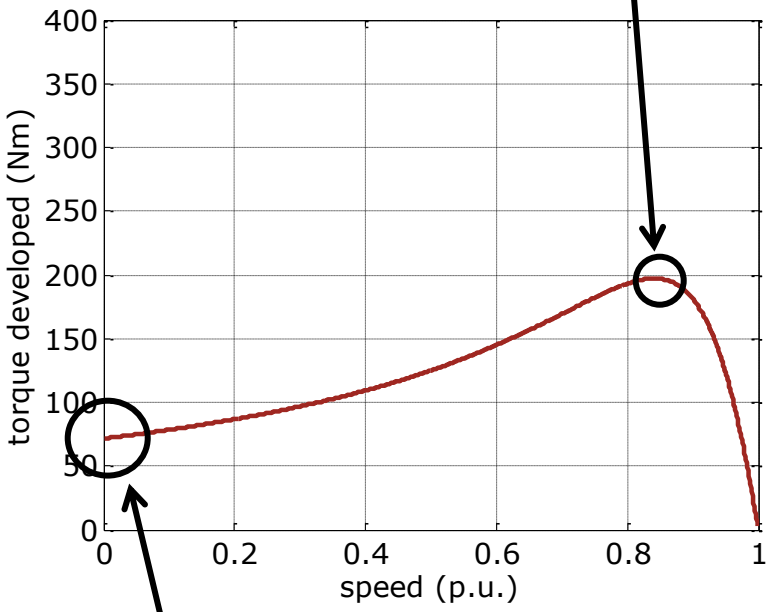
Slip is generally low < 5%

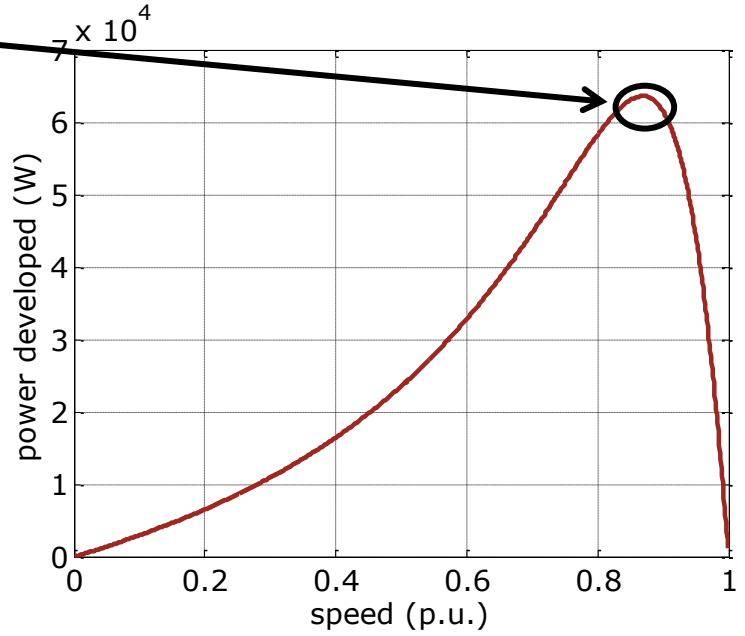
Induction motors are nearly *constant speed* in this region



General Characteristics

Unique maximum torque and power operating points



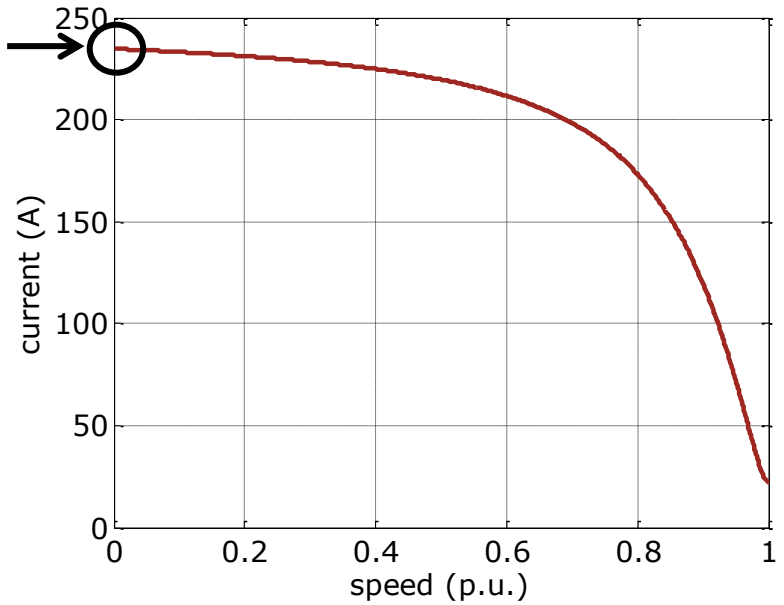


Non-zero starting torque

Recall:
power = torque x speed

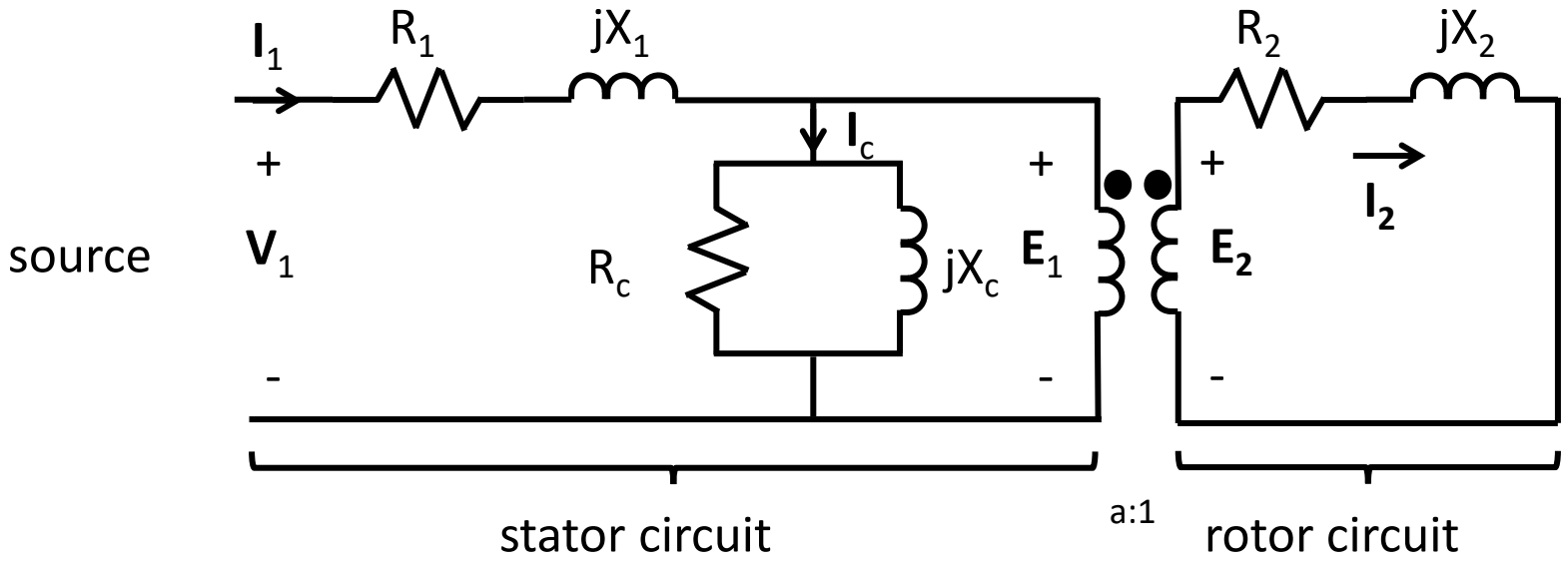
General Characteristics

High starting current

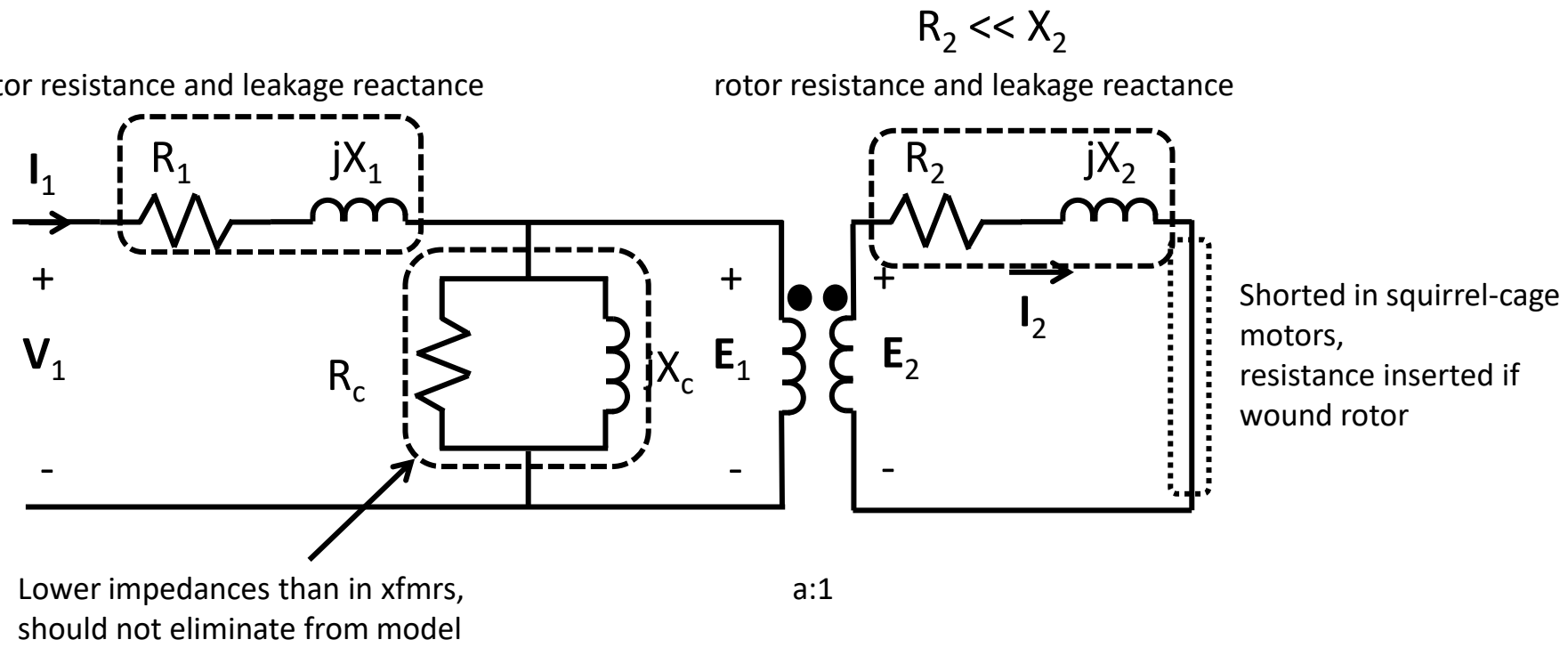


Blocked Rotor Model

- Consider an induction motor with a blocked rotor (rotor is mechanically prevented from moving)
  - s = 1
- Identical to a transformer
- Equivalent per-phase circuit (assuming Y rotor, Y stator)



Blocked Rotor Model

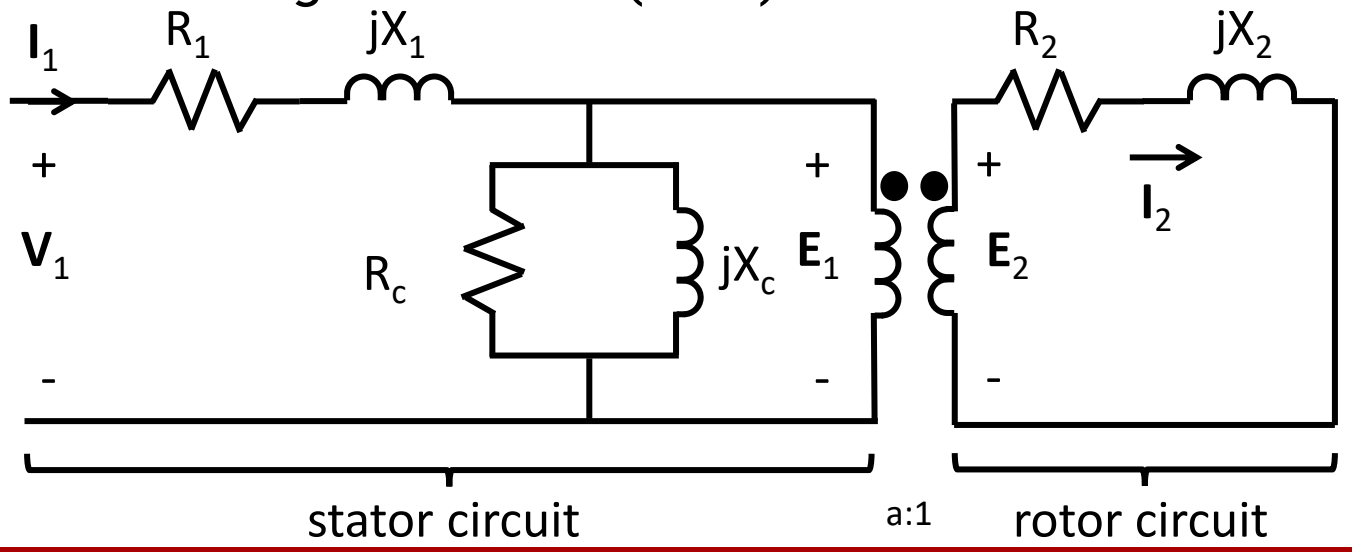


Blocked Rotor Model

- Secondary (rotor) circuit is shorted (unless wound rotor)
  - Large stator current will flow under blocked rotor or start-up conditions

$$\mathbf{I}_2 = \frac{\mathbf{E}_2}{R_2 + jX_2}$$

- What happens as rotor begins to rotate (s < 1)?



Rotating Induction Motor Model

- Behavior is now like a rotating transformer
  - Primary operates at frequency f
  - Secondary operates at frequency sf
- Secondary induced voltage is reduced
  - Let: $\mathbf{E_r} = s\mathbf{E_2}$
- Secondary leakage reactance decreases
  - Let: $jX_r = sjX_2$
- Real power is delivered to the mechanical load
  - Function of slip and rotor impedance
  - Desirable to model in circuit

The subscript "r" means it is a rotor variable that accounts for rotor speed. Not that the rotor resistance is not speed dependent.

Exercise

- An induction motor has a synchronous speed of 1800 rpm. When it operates at 1700 rpm, the induced voltage in the rotor is 106V. Do you expect the induced voltage in the rotor to be greater than 106V or less than 106V if the rotor rotates at 1275 rpm?

Exercise

- An induction motor has a synchronous speed of 1800 rpm. When it operates at 1700 rpm, the induced voltage in the rotor is 106V. Do you expect the induced voltage in the rotor to be greater than 106V or less than 106V if the rotor rotates at 1275 rpm?
- Greater than 106V.

$$s_a = 100 \times \frac{N_s - N_{m,a}}{N_s} = 100 \times \frac{1800 - 1700}{1800} = 5.6\%$$

$$s_b = 100 \times \frac{N_s - N_{m,b}}{N_s} = 100 \times \frac{1800 - 1275}{1800} = 29.2\%$$

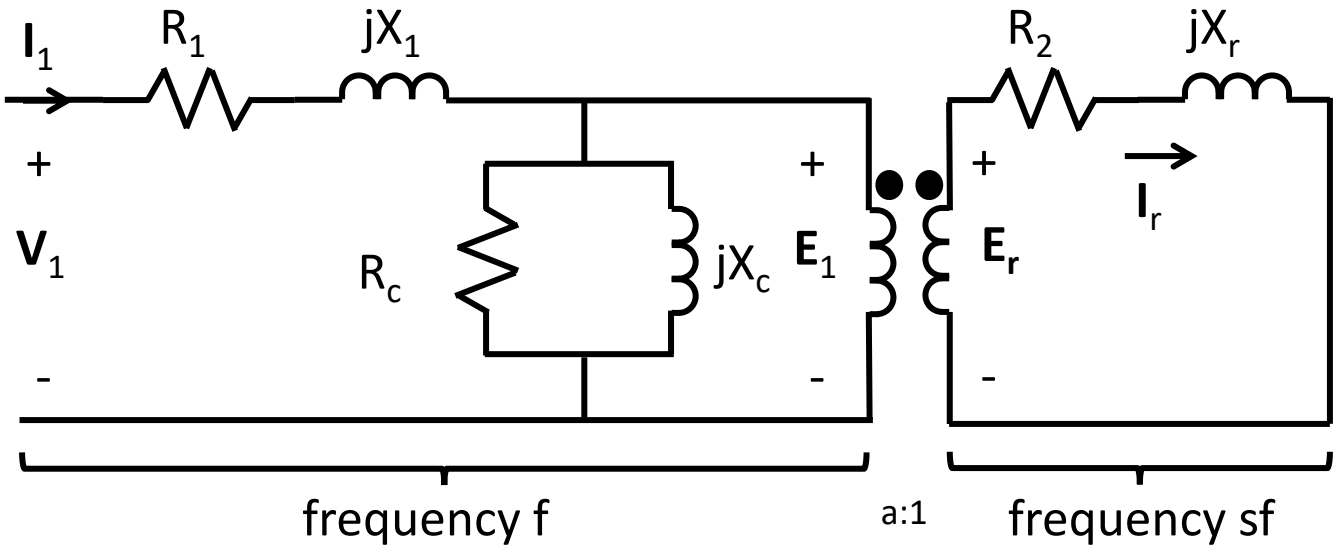
$$E_r = sE_2$$

$$s_b E_2 > s_a E_2$$

Induction Motor Model

- Note: $\mathbf{V}_1, \mathbf{E}_1 \mathbf{I}_1$, and $\mathbf{E}_2, \mathbf{I}_2$ have different frequencies (f, sf)
- Solving secondary (rotor) circuit:

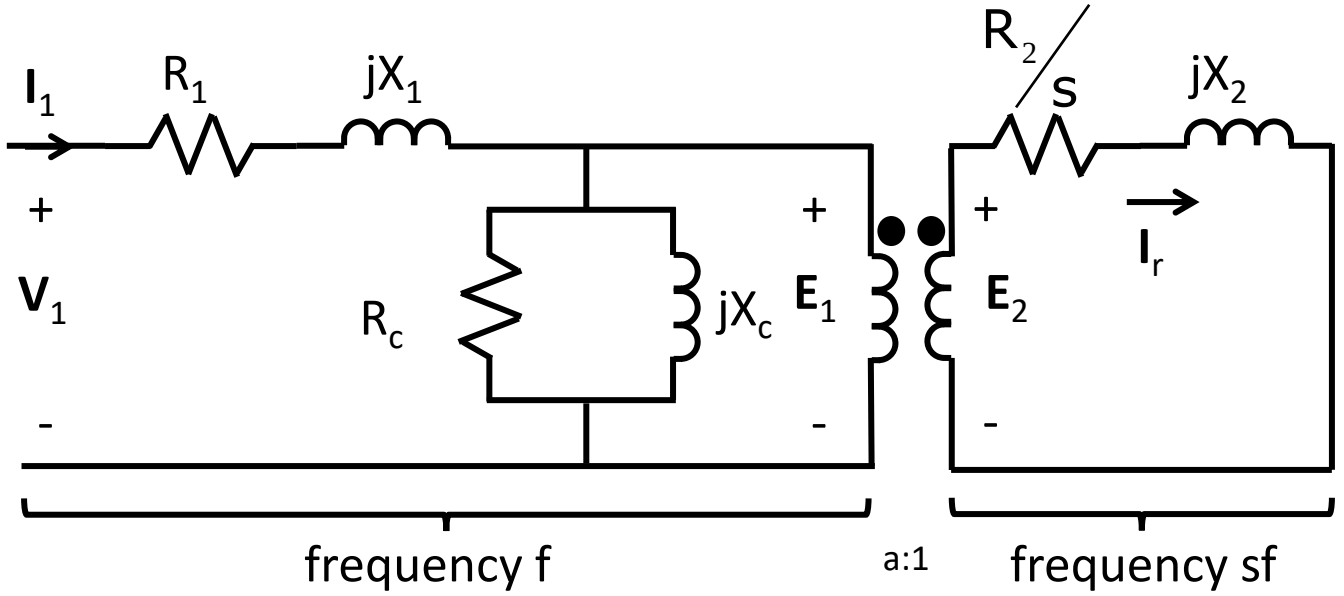
$$\mathbf{I}_r = \frac{\mathbf{E}_r}{R_2 + jX_r} = \frac{s\mathbf{E}_2}{R_2 + jsX_2}$$



Induction Motor Model

- Customary to write rotor current as:

$$\mathbf{I}_r = \frac{s\mathbf{E}_2}{R_2 + jsX_2} = \frac{\mathbf{E}_2}{R_2/s + jX_2}$$ (divide numerator and denominator by s)



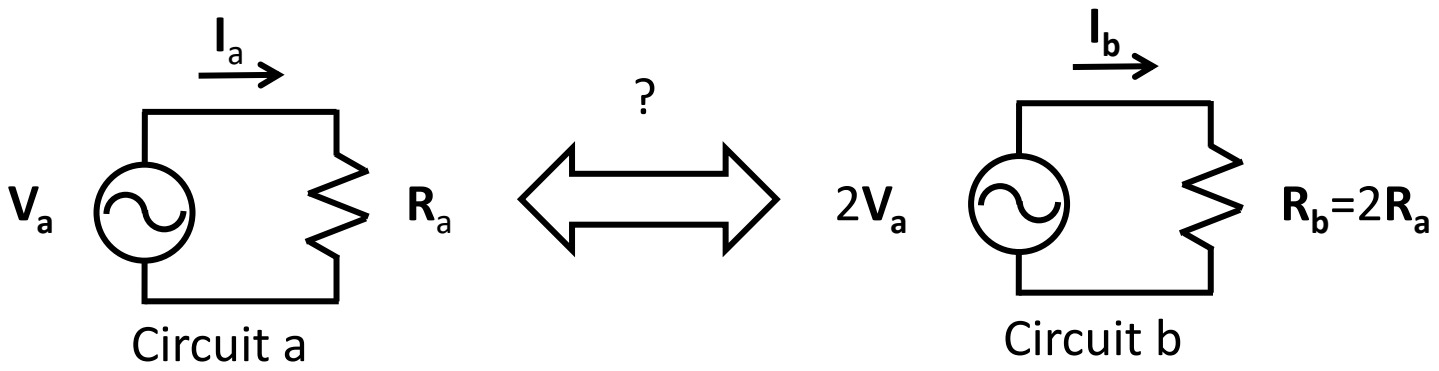
Compare this to the model on the previous slide

An Observation

- Examine the previous algebraic operation:

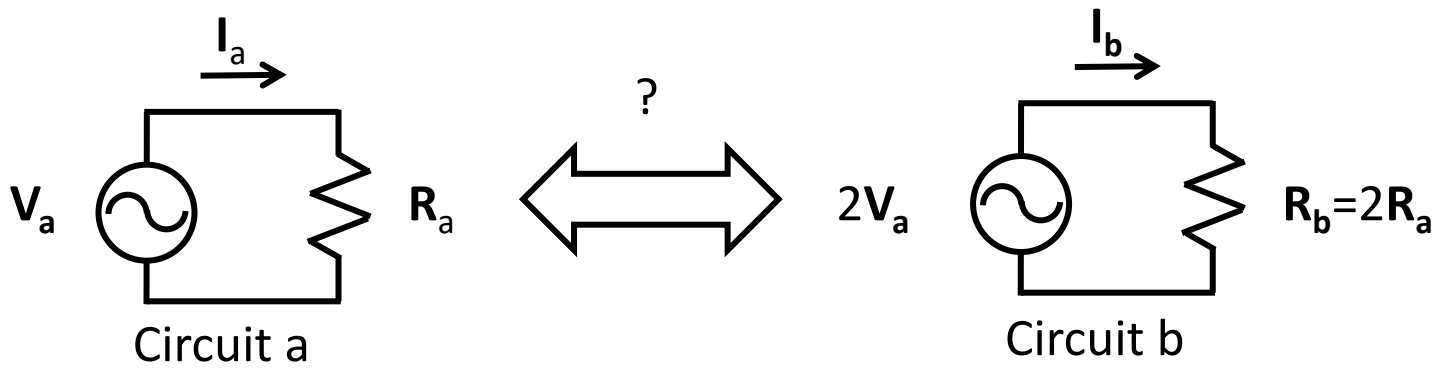
$$\mathbf{I}_r = \frac{s\mathbf{E}_2}{R_2 + jsX_2} = \frac{\mathbf{E}_2}{R_2/s + jX_2}$$ (divide numerator and denominator by s)

- From Ohm's Law, we see that the current is unchanged ($I_a = I_b$), but are the circuits equivalent?
- Example:



An Observation

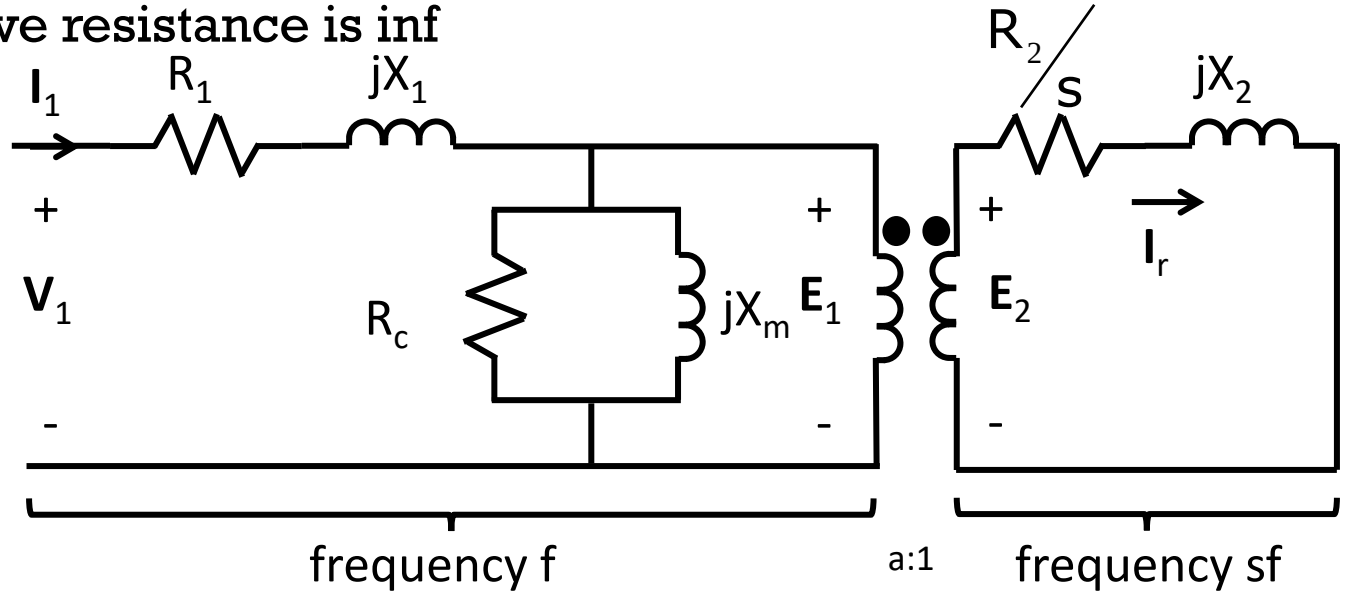
- Current is the same, but power is not
  - Circuit a: $P_a = I_a^2 R_a$
  - Circuit b: $P_b = I_b^2 R_b = I_a^2 2R_a \neq P_a$
- Division by s does not preserve power
- Since s < 1, the new equivalent circuit is consuming power <u>in addition to rotor copper loss</u>



Effective Resistance

- R_2/s is the effective resistance

- Hypothetical, accounts for rotor copper losses and mechanical power developed by the rotor
- If s =1, effective resistance is the rotor resistance
- If s = 0, effective resistance is inf



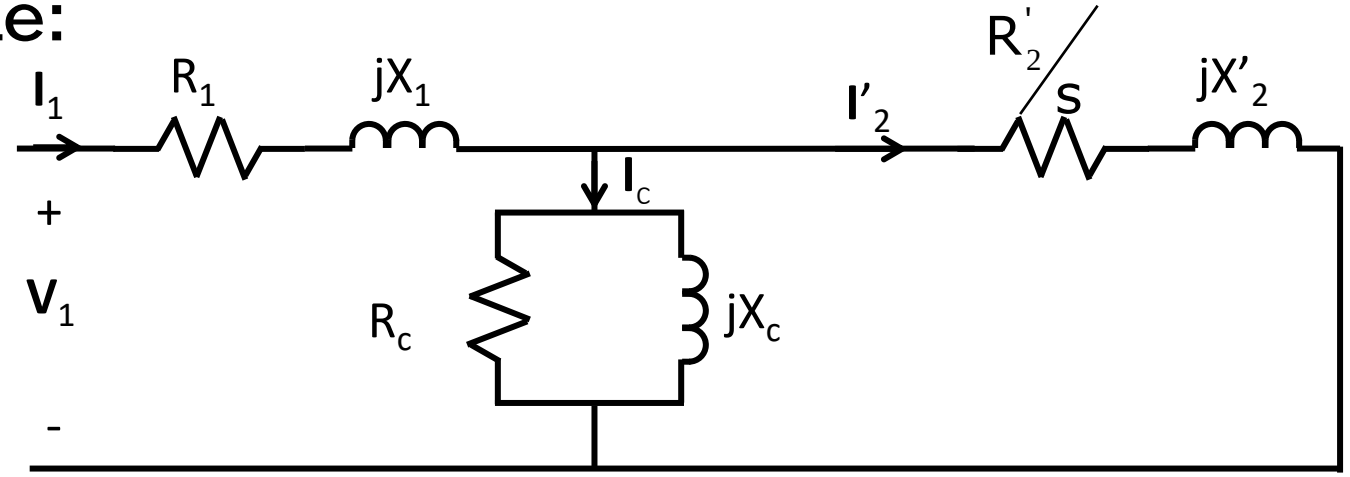
Induction Motor Model

- More convenient to eliminate ideal xfmr in circuit model
- Refer secondary side to primary (stator)
- Transformation ratio: $a = \frac{N_1 k_{\omega 1}}{N_2 k_{\omega 2}}$ assume $k_{\omega 1}$, $k_{\omega 2}$ are equal (machine constants)
- Quantities become:

$$R_2' = a^2 R_2$$

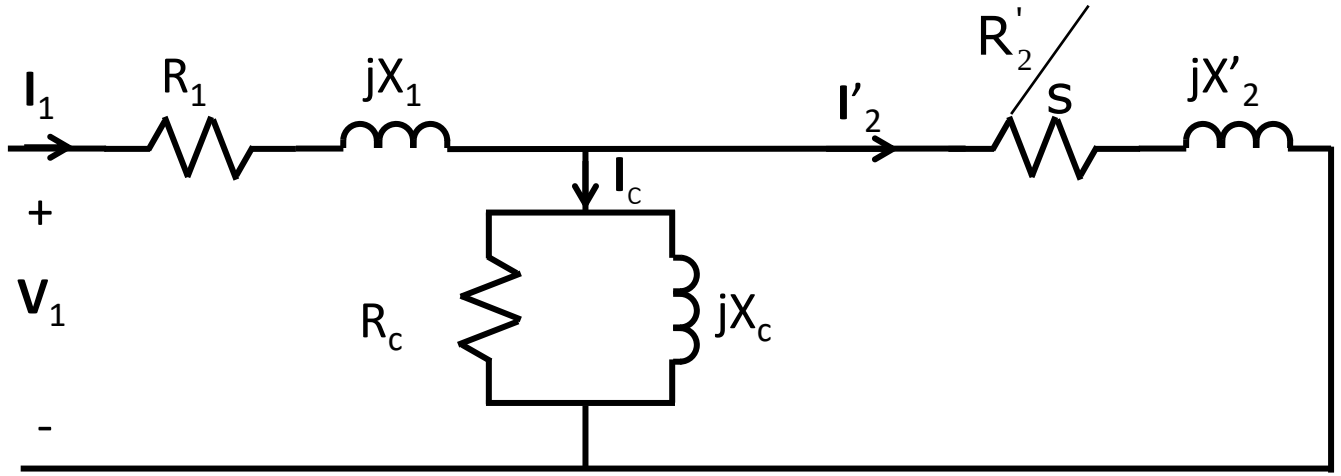
$$X_2' = a^2 X_2$$

$$\mathbf{I}_2' = \frac{\mathbf{I}_r}{a}$$



Induction Motor Model

- Per-phase stator current: $\mathbf{I}_1 = \mathbf{I}_c + I'_2$
- Low speeds (s~1): high current
- High speed (s ~0): low current



Induction Motor Power

- Per-phase power delivered to the rotor: $P_{r,\phi} = \frac{|\mathbf{I}_r|^2 R_2}{s} = \frac{|\mathbf{I}_2'|^2 R_2'}{s}$
  - Copper loss
  - Power developed (available to shaft)

- Per-phase rotor copper loss: $P_{r,cu,\phi} = |\mathbf{I}_r|^2 R_2 = |\mathbf{I}_2'|^2 R_2'$

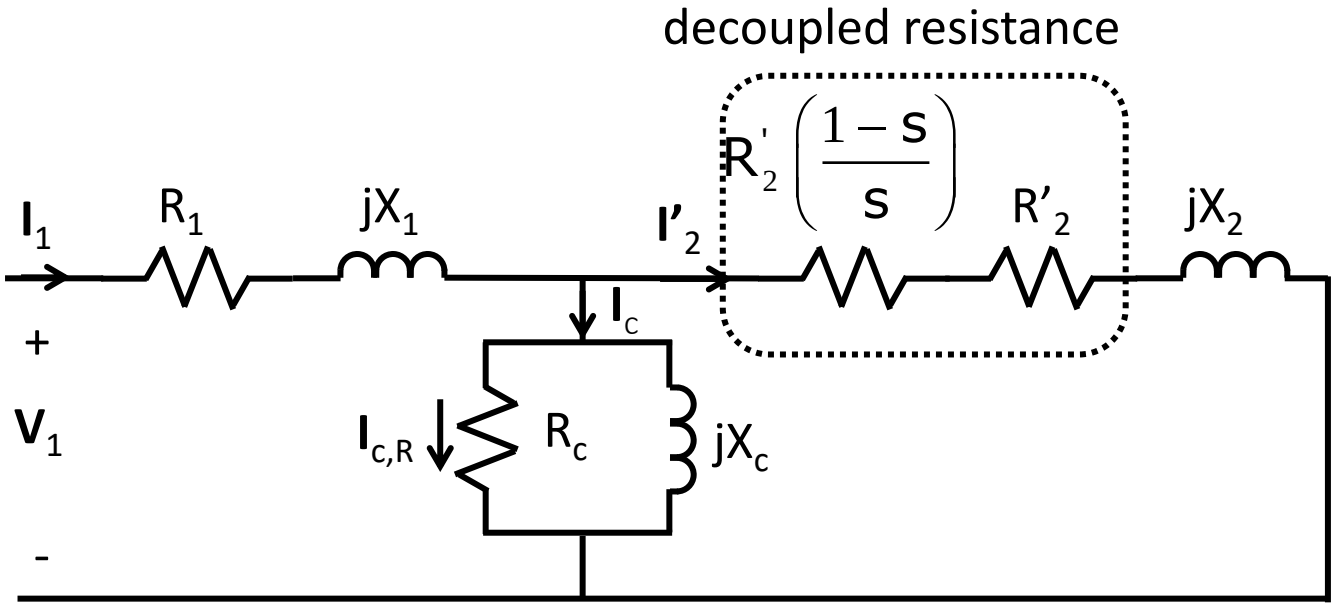
- Per-phase power developed: $P_{d,\phi} = \underbrace{\frac{|\mathbf{I}_r|^2 R_2}{s}}_{P_{r,\phi}} - \underbrace{|\mathbf{I}_r|^2 R_2}_{P_{r,cu,\phi}} = |\mathbf{I}_r|^2 R_2 \left(\frac{1-s}{s}\right)$

Effective Resistance

- Effective resistance can be decoupled into copper loss resistance, and resistance representing mechanical power

$$\frac{R_2'}{s} = \underbrace{R_2'}_{\text{rotor copper}} + \underbrace{R_2'\left(\frac{1-s}{s}\right)}_{\text{load resistance}}$$

Effective Resistance



Induction Motor Power

- Total input power to induction motor: $P_{in} = 3 |\mathbf{V}_1| |\mathbf{I}_1| \cos\phi$
- Total stator copper loss: $P_{s,cu} = 3 |\mathbf{I}_1|^2 R_1$
- Total stator core loss: $P_c = 3 |\mathbf{I}_{c,R}|^2 R_c \approx 3 \frac{|\mathbf{V}_1|^2}{R_c}$
- Remaining power must be delivered to rotor (air-gap power):

$$P_{ag} = P_{in} - P_m - P_c = \frac{3 |\mathbf{I}_2|^2 R_2}{s} = \frac{3 |\mathbf{I}_2'|^2 R_2'}{s} = 3P_{r\phi}$$

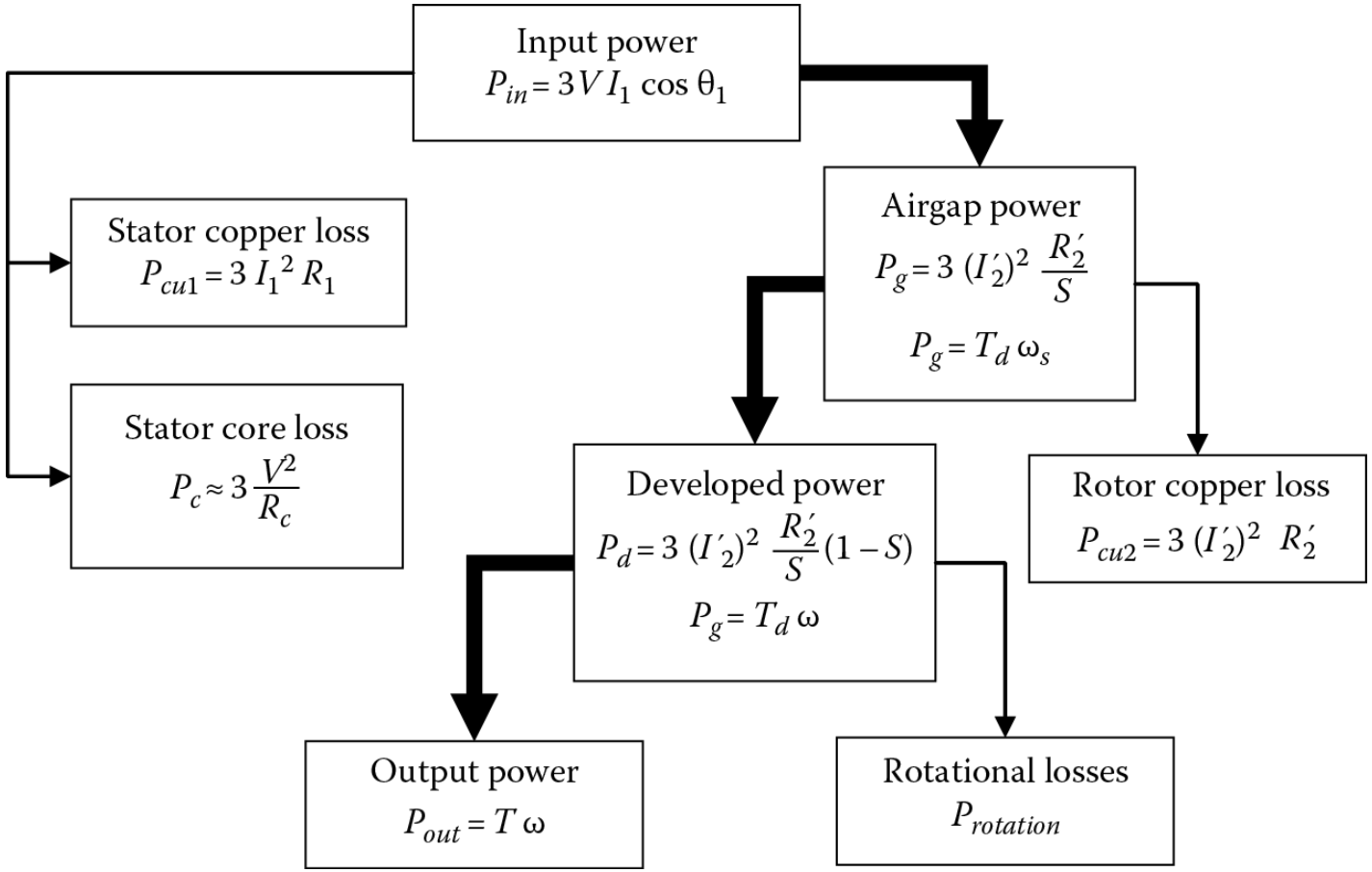
Induction Motor Power

- Total rotor copper loss: $P_{r,cu} = 3P_{r,cu,\phi} = 3 | \mathbf{I}_2 |^2 R_2 = 3 | \mathbf{I}_2' |^2 R_2' = sP_{ag}$
  - Percentage of air gap power consumed by electrical losses is s
- Mechanical <u>power developed</u> by the motor:

$$P_d = P_{ag} - P_{r,cu} = \frac{3 | \mathbf{I}_2 |^2 (1-s)R_2}{s} = \frac{3 | \mathbf{I}_2' |^2 (1-s)R_2'}{s} = (1-s)P_{ag}$$

- Subtracting rotational losses yields total <u>power output</u> by the motor: $P_{out} = P_d - P_{rot}$

Induction Motor Power Relationships



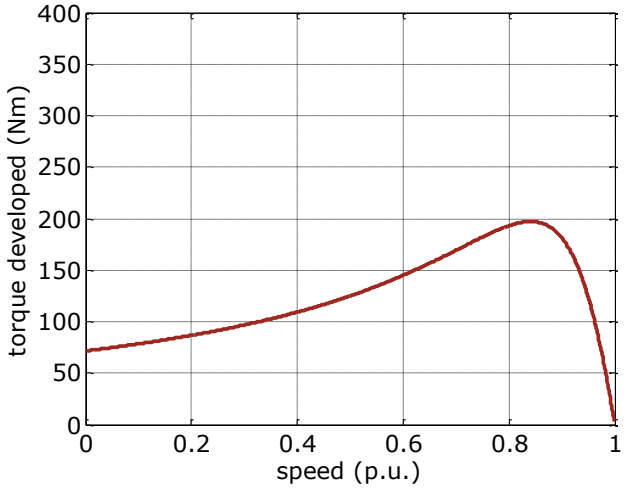
Induction Motor Torque

- The total developed torque is: $T_d = \frac{P_d}{\omega_m} = \frac{P_{ag}}{\omega_s} = 3 | \mathbf{I}_2 |^2 \frac{R_2}{s\omega_s}$

- The total output torque is: $T_{out} = \frac{P_{out}}{\omega_m}$

Induction Motor Torque

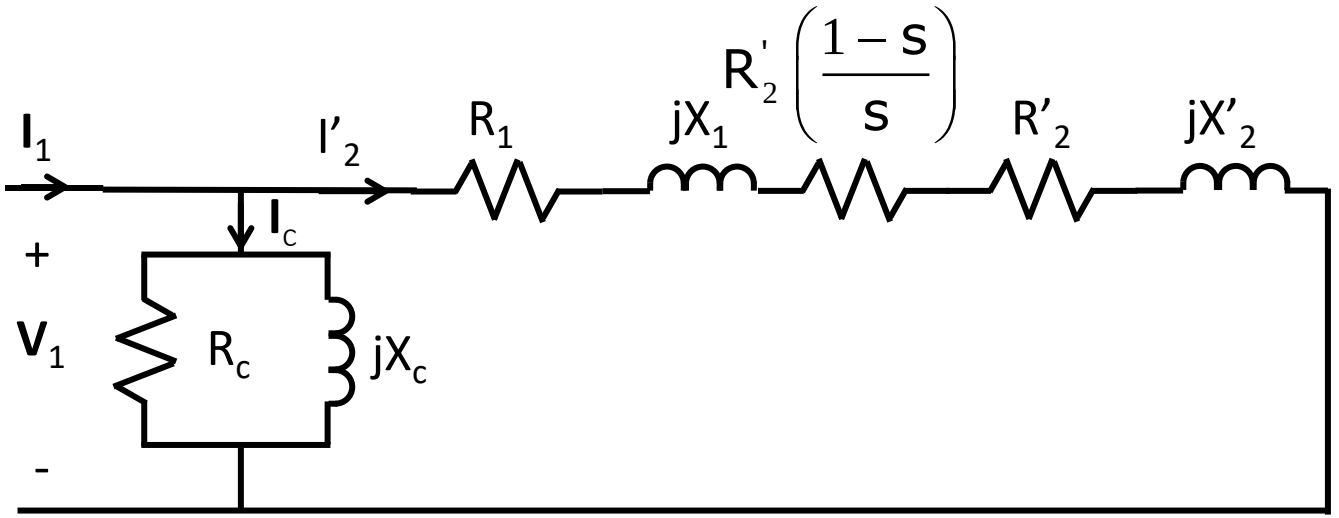
- Torque is non-zero at starting (s = 1)
- Torque is zero at synchronous speed (s = 0)
  - $\mathbf{I}_2$ is zero



Approximate Circuit

Note error in
Fig. 12.15 of digital book.
Should be "(1-s)", not "(-s)"

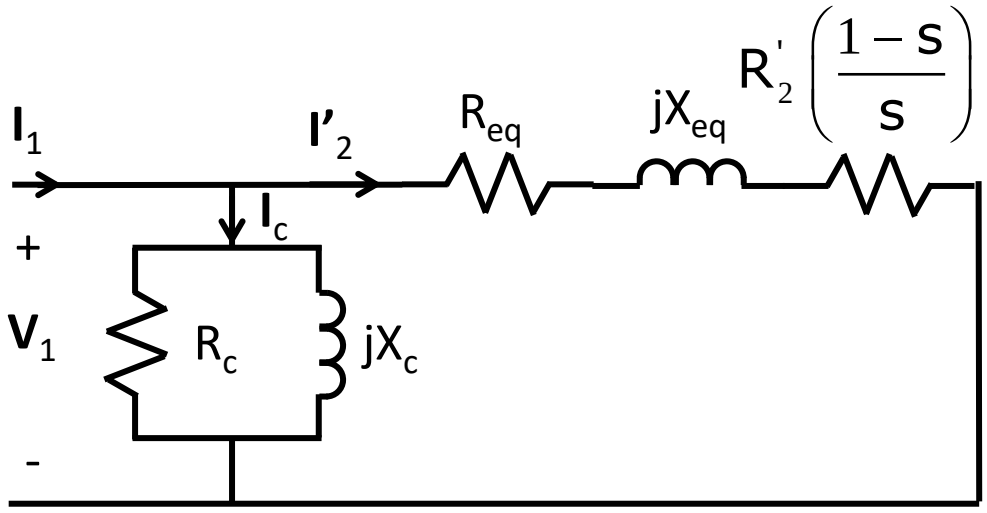
- R_1, X_1 are designed to be minimized
- R_c, X_c designed to be maximized
- Per-phase circuit can be simplified (approximated)



Approximate Circuit

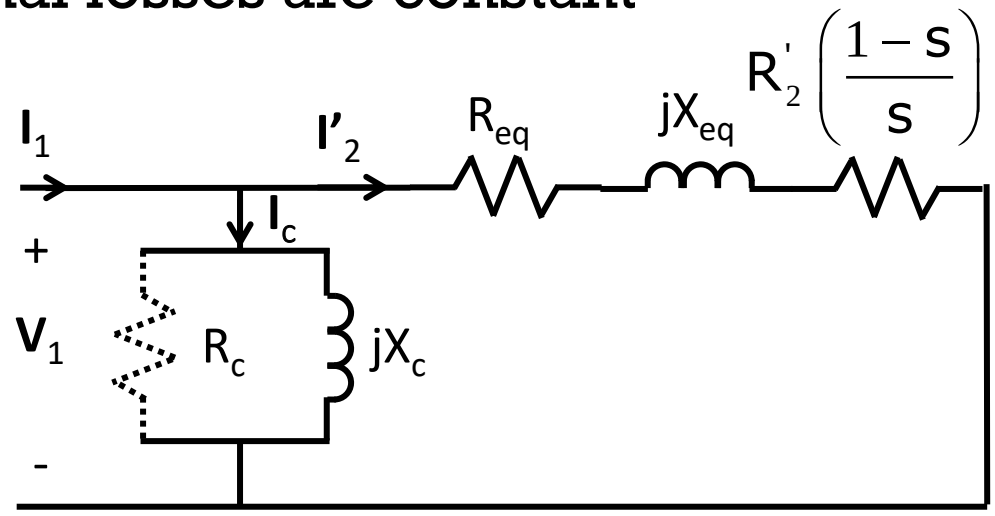
Combine R_1, X_1 with R'_2, X'_2

- $R_{eq} = R_1 + R'_2$
- $X_{eq} = X_1 + X'_2$



Approximate Circuit

- Further simplifications:
  - replace core resistance with constant power loss
  - Assume rotational losses are constant



Example

- A 60 Hz, 480-V, wye-connected induction motor has the following constants (referred to the stator)
  - $R_1 = 0.322\ \Omega$; $R'_2 = 0.196\ \Omega$; $X_1 = 0.675\ \Omega$; $X'_2 = 0.510\ \Omega$; $X_c = 12.5\ \Omega$
  - Rotational and core losses: 1850 W
- The motor operates at a slip of 3% (3492 RPM). Compute the stator current and the power factor of the motor using the approximate model.

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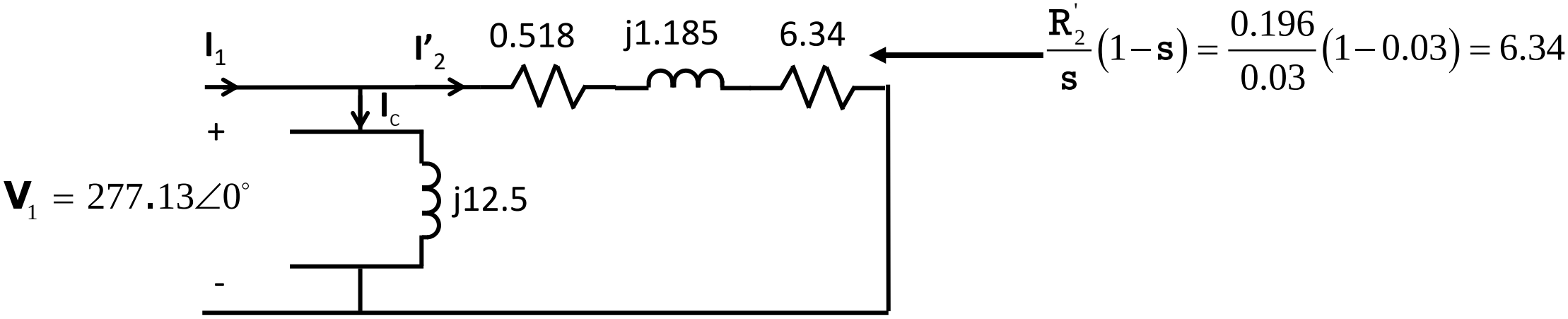


Example

- Compute stator current by KCL: $\mathbf{I}_1 = \mathbf{I}_c + \mathbf{I}'_2$

$$\mathbf{I}_1 = \frac{277.13\angle 0^\circ}{12.5\angle 90^\circ} + \frac{277.13\angle 0^\circ}{0.518 + 6.34 + j1.185} = 22.17\angle -90^\circ + 39.83\angle -9.8^\circ$$

$$= 48.78\angle -36.42^\circ \text{ A}$$

$\mathbf{I}_c$ $\mathbf{I}'_2$

- Compute the power factor:

PF: $\cos(-36.42^0) = 0.805$ (lagging)

How does this compare if the exact circuit model is used?

Example

Consider the motor in the previous example.

Compute the input power and stator copper losses.

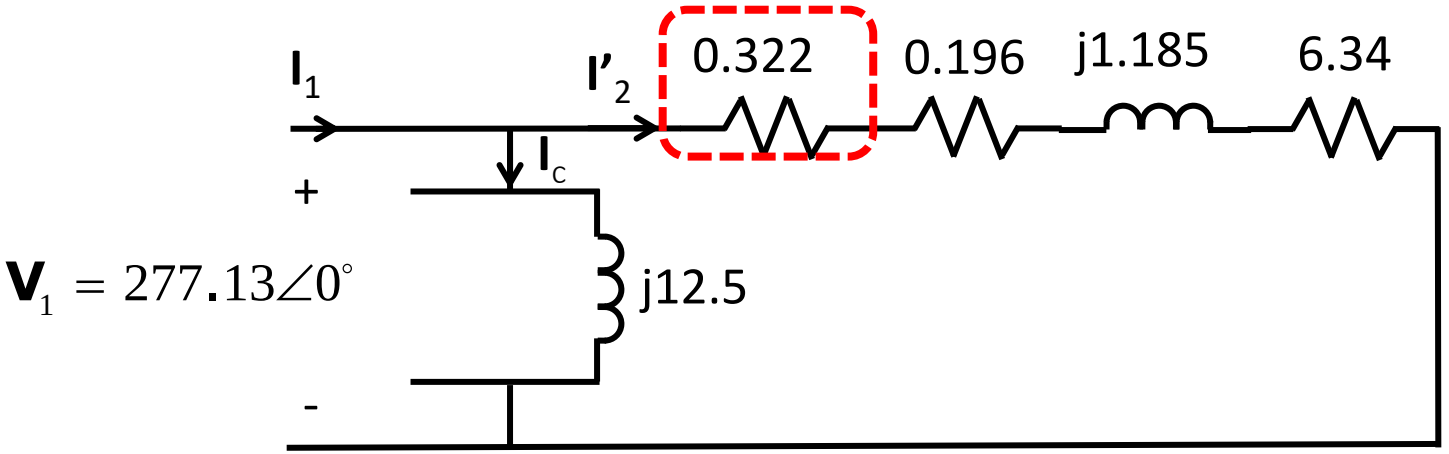
Example

- The stator and copper losses are:

$$P_{input} = 3 \mid \mathbf{V}_1 \mid \mid \mathbf{I}_1 \mid \cos\phi = 3 \times 277 \times 48.78 \times 0.805 = 32,634 \text{ W}$$

$$P_{s,cu} = 3 \mid \mathbf{I}_2' \mid^2 R_1 = 3 \mid 39.83 \mid^2 0.322 = 1,533 \text{ W}$$

Note: using the Approximate model, it is $\mathbf{I}'_2$ that flows through R_1, not $\mathbf{I}_1$.



Example

Consider the motor in the previous example.

Compute the air gap power, power developed and power output of the motor.

Example

Consider the motor in the previous example.

$$P_{ag} = P_{in} - P_{s,cu} = 31,101 \text{ W}$$
$$P_d = (1 - s)P_{ag} = 30,168 \text{ W}$$
$$P_{out} = P_d - P_{rot} = 28,318 \text{ W}$$

Note: rotor copper losses are 933 W

Efficiency: P_{out}/P_{in} = 86.77%

Example

Consider the motor in the previous example.

Compute the torque developed and the output torque.

Example

Consider the motor in the previous example.

Compute the torque developed and the output torque.

$$\omega_m = \frac{3492 \times 2\pi}{60} = 365.8 \text{ (rad/s)}$$

$$T_d = \frac{P_d}{\omega_m} = 82.49 \text{ Nm}$$

$$T_{out} = \frac{P_{out}}{\omega_m} = 77.43 \text{ Nm}$$

Summary

- Induction motors are the "workhorse" of the industry
- Synchronous revolving field induces current in rotor circuit
- Rotor rotates in direction of revolving field
- Rotor rotates at different speed than field
- Percent difference in speed is called "slip"
  - Higher the slip, the slower the rotation of the rotor