23-Synchronous Generators

ECEGR 3500

Text: 12.5

Electrical Energy Systems

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Overview

- Excitation
- Induced frequency
- Induced EMF
- Equivalent Circuit
- Armature Reaction
- Power Relationship
- Approximate Power Relationship

Introduction

- AC machines (generators, motors) have similar stators
- Rotors are different
  - Induction
  - Synchronous

In AC generators, the emf is induced in a stationary coil. The term "armature" refers to the windings in which the emf is induced and current flows when connected to a load.

Introduction

Advantages of armature windings in the stator:

- Larger coils can be used since they are located in the stator
- High power rated slip rings can be avoided
- Easier to cool stator than rotor
- Easier to construct the armature winding if it is in the stator
- Easier to electrically insulate the stator

Stator

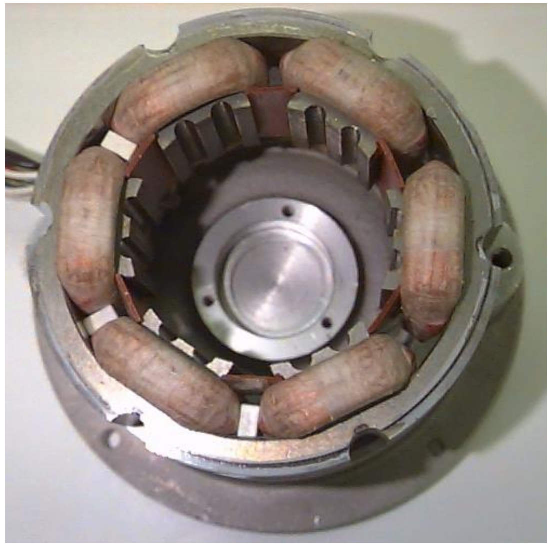
- Houses armature windings
- Contains large gauge coils (low resistance)
- Conductors are symmetrically arranged to form a balanced poly-phase winding
- Induced emf can be in kV range
- Power ratings can be in MVA range

Armature Windings

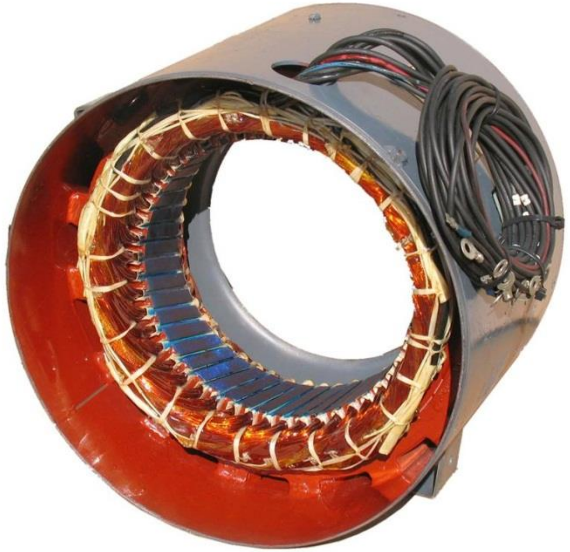
- Common for the armature (stator) windings to be three-phase
- Windings are identical, but displaced by 120° electrical
- Can be delta or wye connected (generators)
  - wye is common if higher voltage is needed
    - neutral point is grounded
- Windings are commonly double layer
  - Equal number of slots and windings

Salient vs Cylindrical

- Early machines used salient pole stators
  - Salient poles still used in rotors
- Modern machines use cylindrical stators

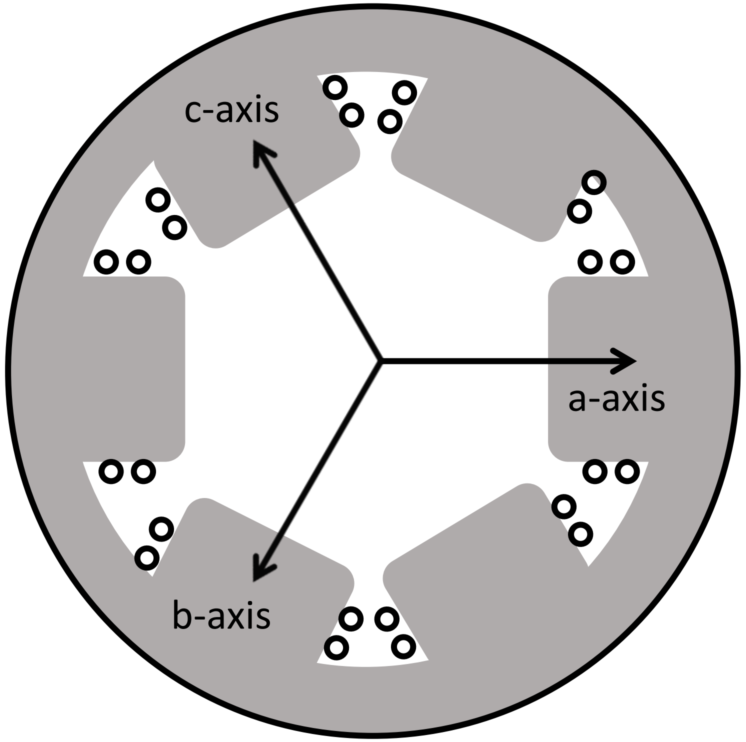


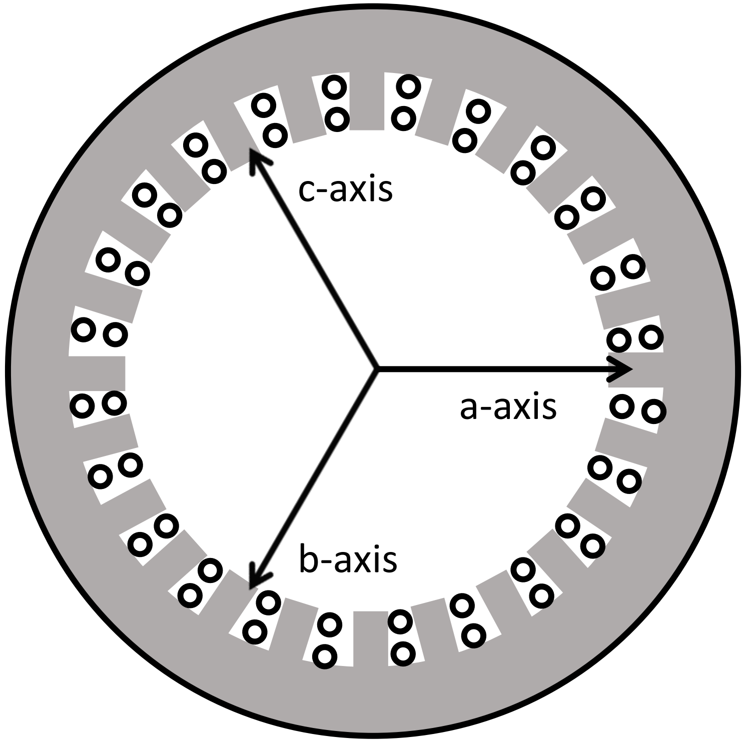
salient pole



cylindrical

Salient-Pole vs Cylindrical

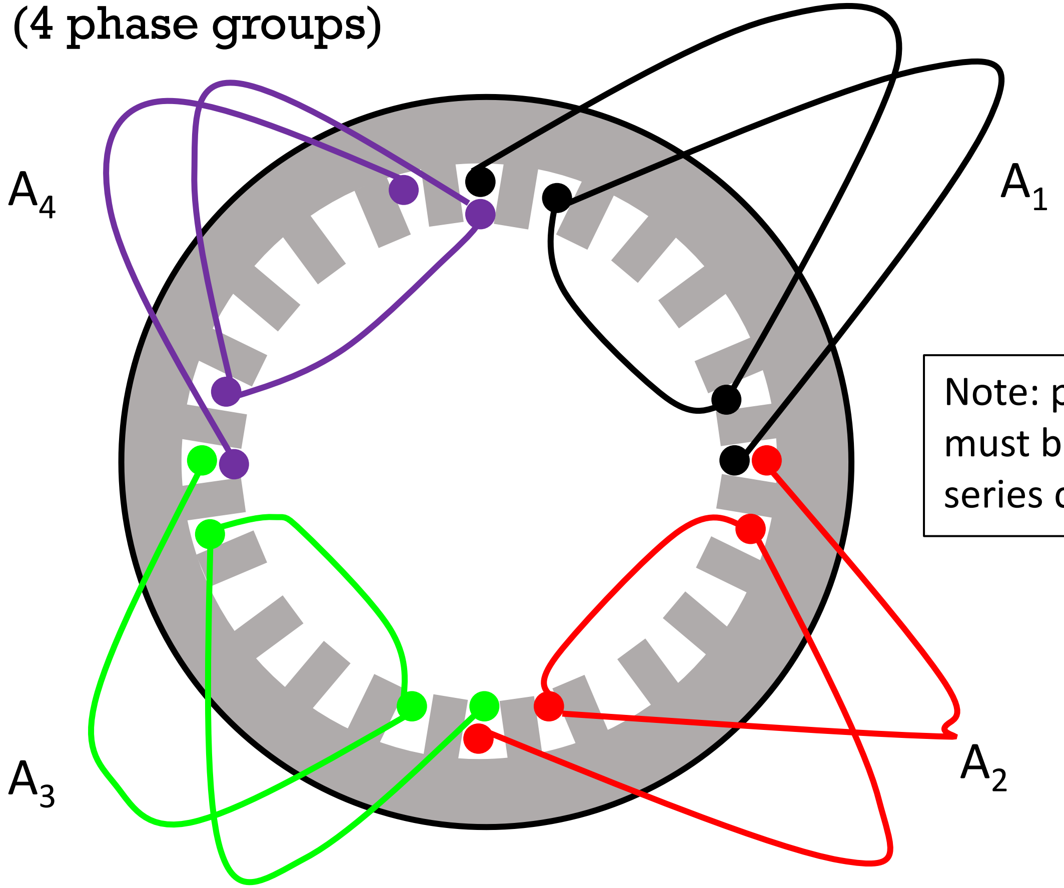




Armature Windings

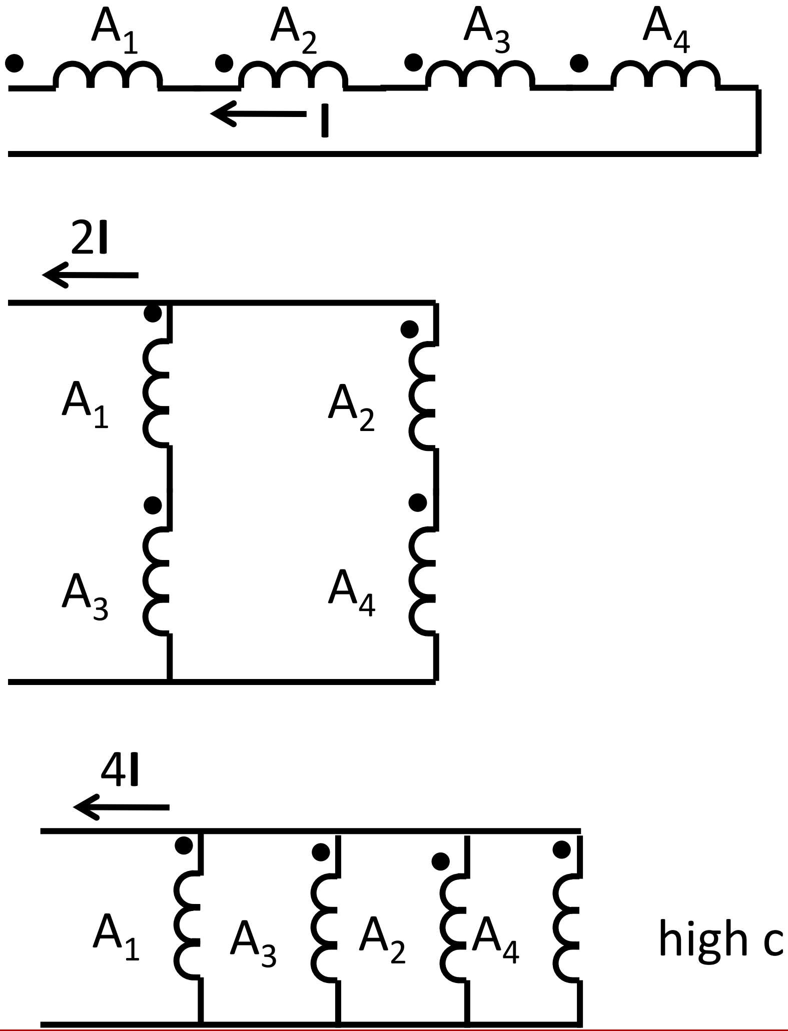
- Coils in each phase group are connected in series
- A-phase windings shown (4 phase groups)

four-poles,
24 slots



Note: phase group coils must be series connected.

Winding Connections



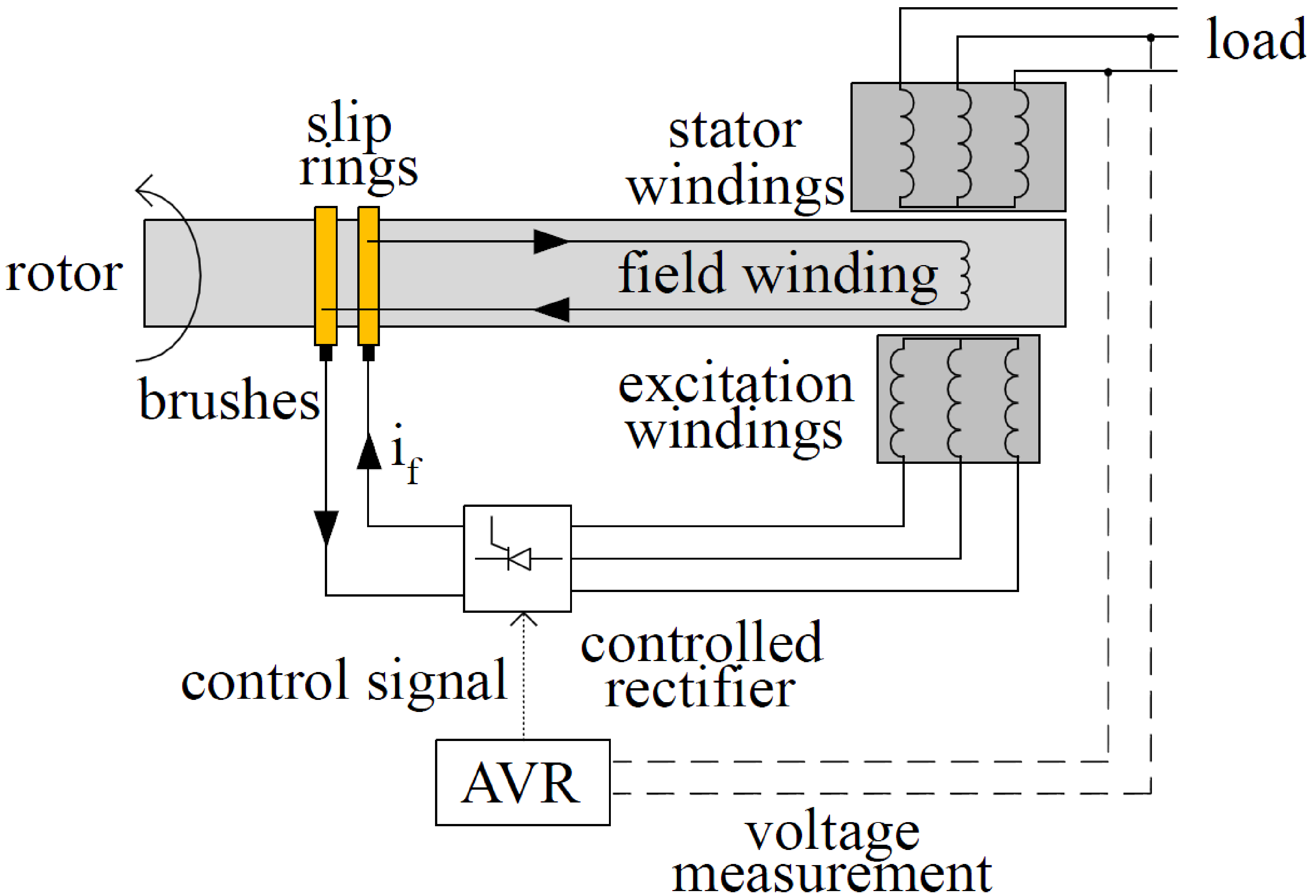
low current, high voltage

high current, low voltage

Excitation

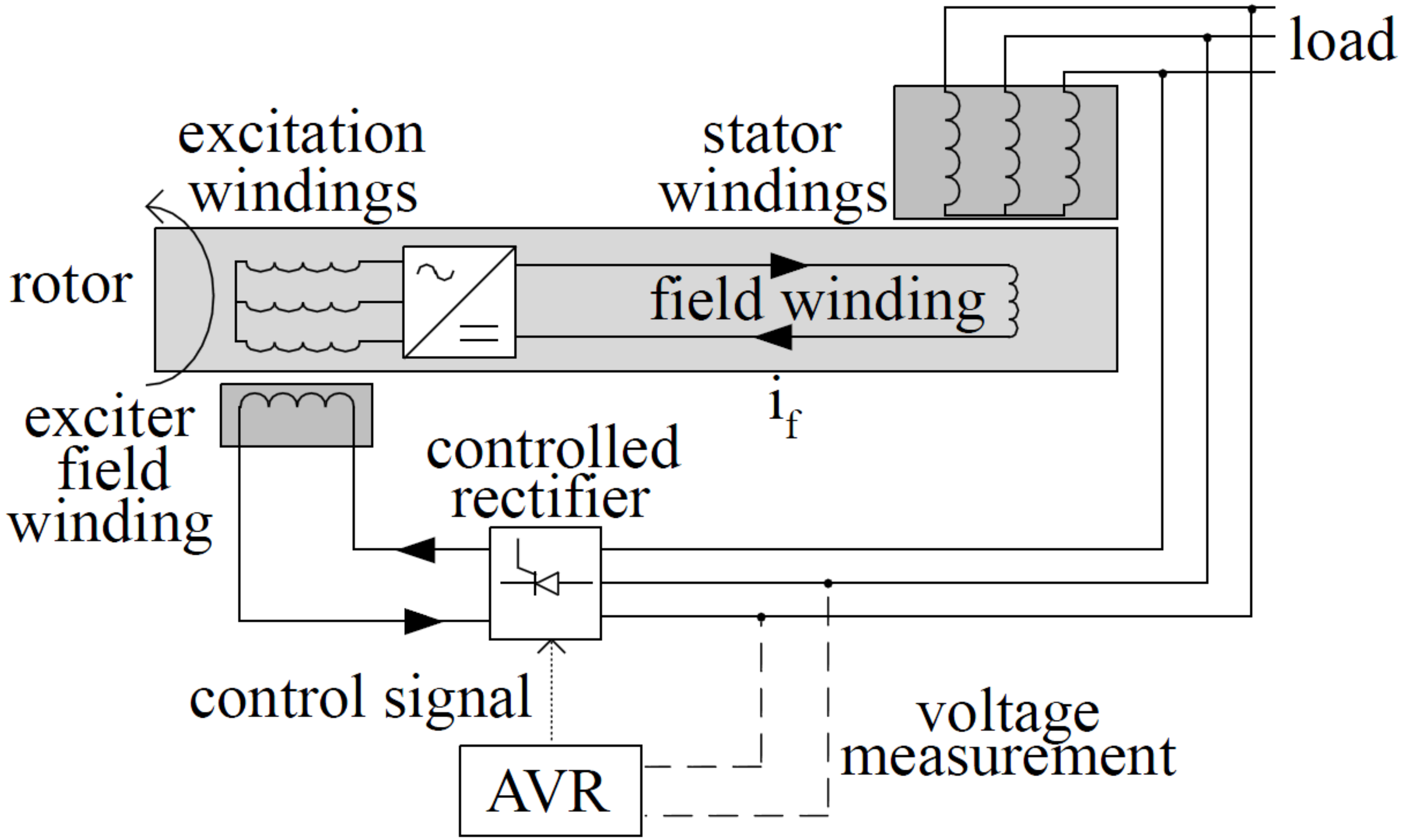
- Synchronous generators (motors) require revolving magnetic field
  - Permanent magnet
  - Field winding (dc)
- Exciter: supplies current to field winding (i_f)
  - DC generator
  - Brushless generator
  - Power rating: <3% of generator rating
- Field current is related to ϕ_p by k_f
- Automatic Voltage Regulator (AVR) controls the current to the field winding to maintain the desired terminal voltage

Excitation



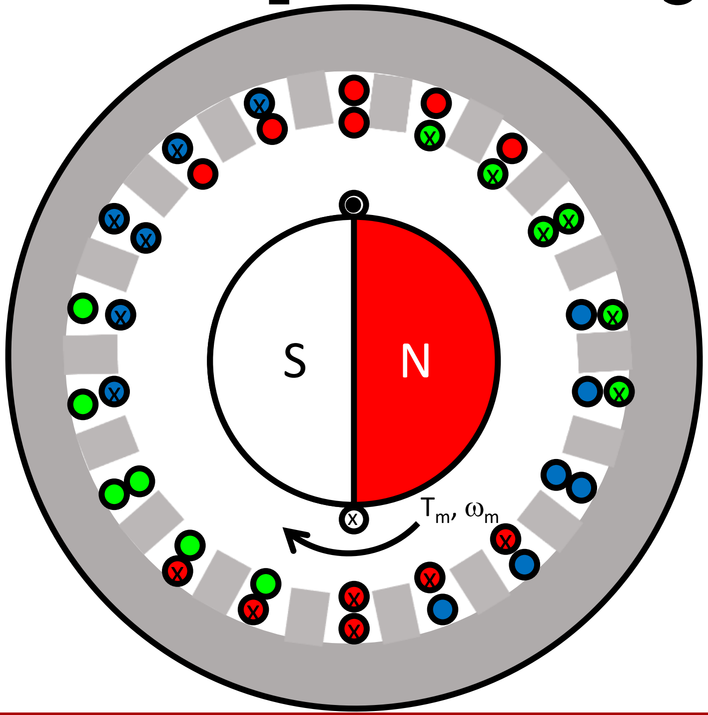
Voltage: 125 to 600VDC
Automatically controlled
(terminal voltage magnitude, reactive power)

Brushless Excitation



Induced Frequency

- 2-pole synchronous generator
- Balanced three-phase voltage induced

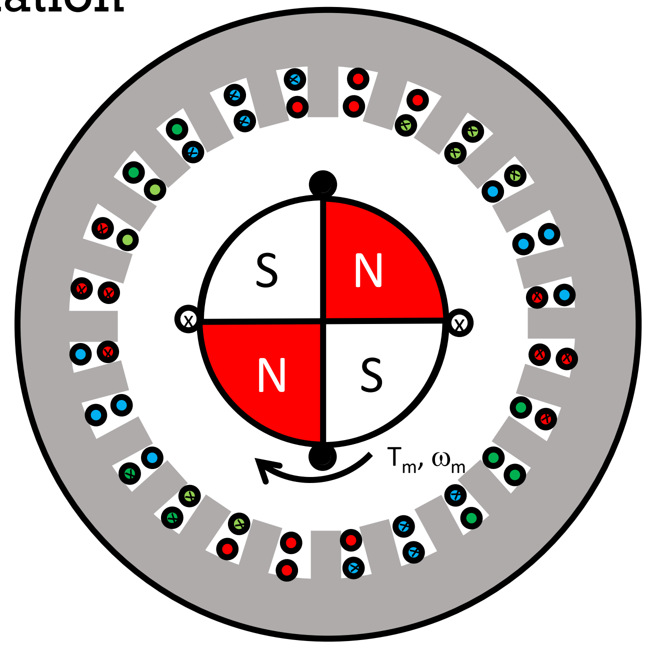


Two-pole machine:
electrical frequency =
mechanical frequency

Induced Frequency

- 4-pole synchronous generator
- Each coil "sees" two Norths and two Souths per rotation
  - Two electrical sinewaves for each mechanical rotation
- In general:

$$f = \frac{f_m P}{2} = \frac{N_m P}{120}$$



Exercise

What is the fastest speed (in rpm) a generator shaft may rotate at and still produce 60Hz AC voltage?

Exercise

What is the fastest speed (in rpm) a generator shaft may rotate at and still produce 60Hz AC voltage?

$$f = \frac{f_m P}{2} = \frac{P}{4\pi}\omega_m$$

$$60 = \frac{P}{4\pi}\omega_m$$

$$\frac{60 \times 4\pi}{2} = \omega_m$$

$$\omega_m = 377\,\mathrm{rad}/\mathrm{s} = 60\mathrm{Hz} = 3600\ \mathrm{rpm}$$

Prime mover shaft may rotate at higher speeds if a gearbox couples it to generator shaft

Induced EMF

- We next examine the induced emf in a synchronous generator
- Flux linking a single stator coil (ϕ_c): $\phi_c = \phi_p k_p \cos(\omega t)$
- Induced voltage in coil with N_c turns: $e_c = N_c \phi_p \omega k_p \sin(\omega t)$
- Maximum induced voltage: $E_m = N_c \phi_p \omega k_p$

k_p: "pitch factor". A scalar value (<1) accounting for the span of the coil (1 for full pitch)
Φ_p: flux per pole (Wb)

Induced EMF

- RMS value of the induced emf:

$$E_m = N_c \phi_p \omega k_p$$

$$|\mathbf{E}_c| = \frac{1}{\sqrt{2}} E_m = \frac{1}{\sqrt{2}} 2\pi f N_c k_p \phi_p = 4.44 f N_c k_p \phi_p$$

- Induced voltage in a phase group, accounting for the number of coils in series, pitch factor and the distribution factor, is:

$$|\mathbf{E}_{pg}| = n k_d E_c = 4.44 n N_c k_p k_d f \phi_p$$

$$|\mathbf{E}_{pg}| = 4.44 n N_c k_w f \phi_p$$

$$k_w \triangleq k_p k_d \quad \text{(winding factor)}$$

k_d: "distribution factor". A scalar value (<1) accounting for any overlap of the coils of the same phase (1 if the coils are in the same slot)

Induced EMF

- If a generator has "a" parallel paths and P poles, then the emf per phase is:

$$|\mathbf{E}_a| = \frac{P}{a} 4.44 n N_c k_w f \phi_p$$

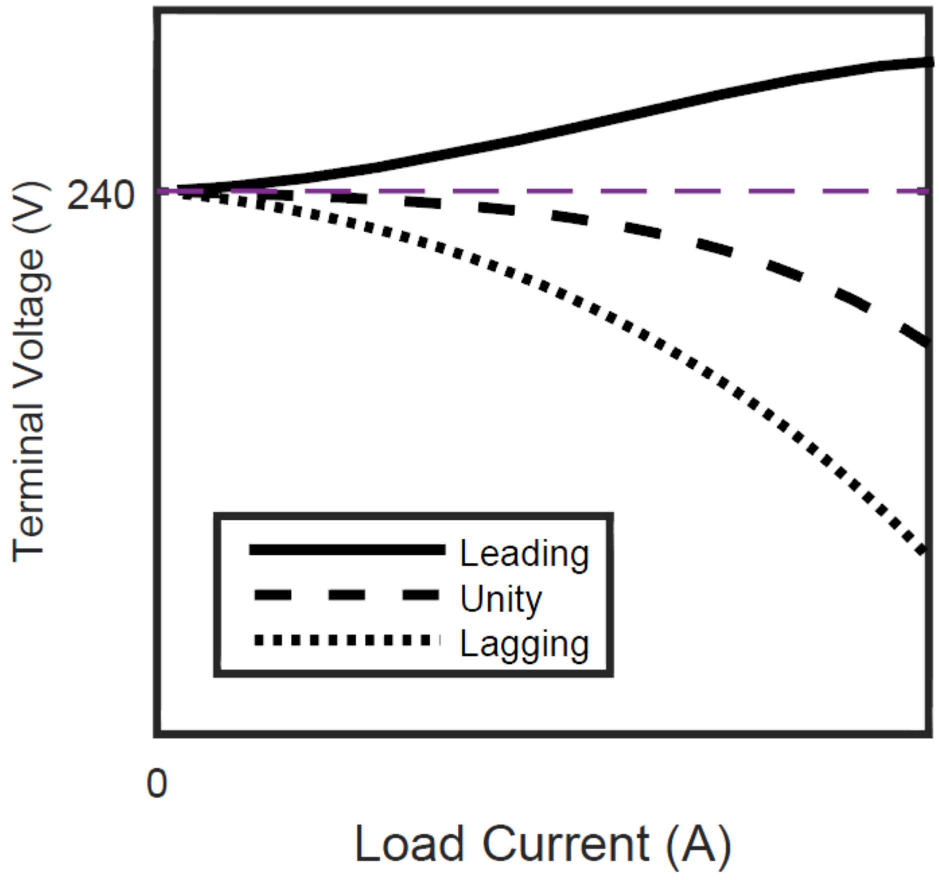
$$N_e \triangleq \frac{P n N_c k_w}{a}$$

  - N_e: effective turns per phase

- We can then write: $|\mathbf{E}_a| = 4.44 N_e f \phi_p$

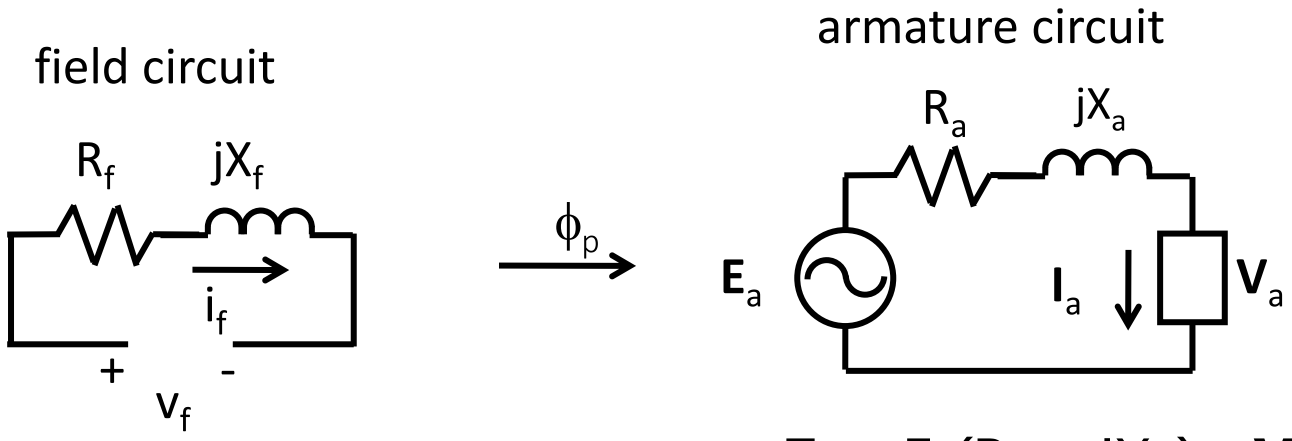
Equivalent Circuit

- Generator terminal voltage ($\mathbf{V}_a$) of a synchronous generator depends upon the load
  - Terminal voltage may be greater or lesser than induced emf
  - Will usually be higher when the power factor is leading
  - Assumes generator is not grid-connected
- Terminal voltage is affected by:
  - Armature resistance voltage drop
  - Armature leakage reactance voltage drop
  - Armature reaction



Equivalent Circuit

- Equivalent circuit
  - R_a: per-phase armature resistance (Ohm)
  - X_a: armature leakage reactance (Ohm)

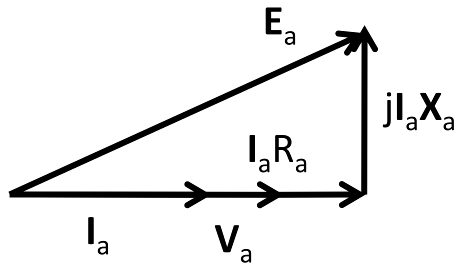


$$\mathbf{E}_a = \mathbf{I}_a(R_a + jX_a) + \mathbf{V}_a$$

Equivalent Circuit

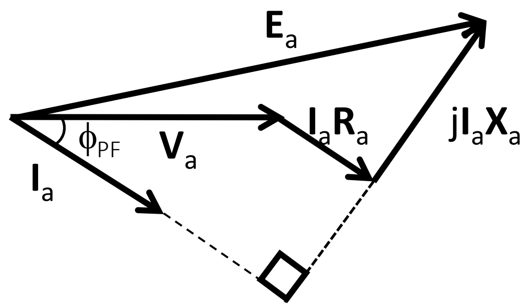
Phasor diagrams (compare magnitude of $\mathbf{E}_a$, $\mathbf{V}_a$)

$$\mathbf{E}_a = \mathbf{I}_a(R_a + jX_a) + \mathbf{V}_a$$



Unity power factor

$\mathbf{V}_a$: reference
$\mathbf{I}_a$: in phase $\mathbf{V}_a$ (unity PF)
$\mathbf{I}_a R_a$: in phase with $\mathbf{I}_a$
$j\mathbf{I}_a X_a$: 90° out of phase from $\mathbf{I}_a$
$\mathbf{E}_a > \mathbf{V}_a$

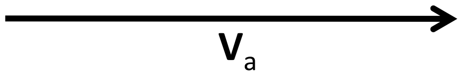


Lagging power factor

$\mathbf{V}_a$: reference
$\mathbf{I}_a$: lags $\mathbf{V}_a$ (by ϕ_{PF})
$\mathbf{I}_a R_a$: in phase with $\mathbf{I}_a$
$j\mathbf{I}_a X_a$: 90° out of phase from $\mathbf{I}_a$
$\mathbf{E}_a > \mathbf{V}_a$

Exercise

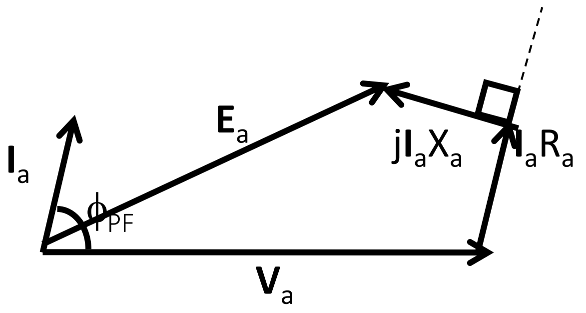
Draw the phasor diagram for a synchronous generator with a leading PF



Leading power factor

Exercise

Draw the equivalent circuit for a leading PF



Leading power factor

$$\mathbf{E}_a = \mathbf{I}_a(R_a + jX_a) + \mathbf{V}_a$$

V$_a$: reference
I$_a$: leads **V**$_a$ (by ϕ_{PF})
I$_a$R$_a$: in phase with **I**$_a$
j**I**$_a$X$_a$: 90^o out of phase from **I**$_a$
E$_a$ < **V**$_a$

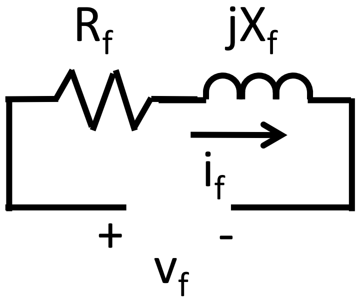
Possible for induced voltage to be greater than terminal voltage

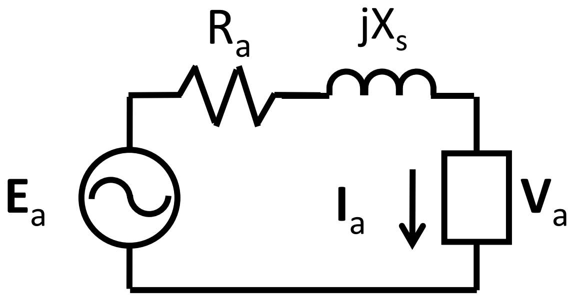
Armature Reaction

- Flux in the armature is from two sources:
  - field winding
  - armature current (when connected to a load)
- Fluxes interact with each other
- Resulting distortion can have a profound effect on the operation of the machine

Equivalent Circuit

- Model armature reaction by the "synchronous reactance"
  - X_s : accounts for leakage reactance and armature reaction

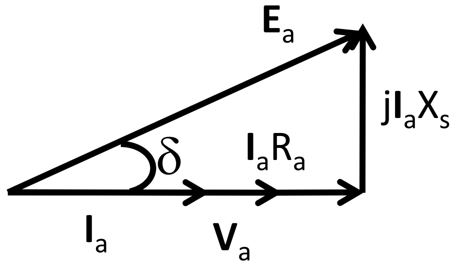




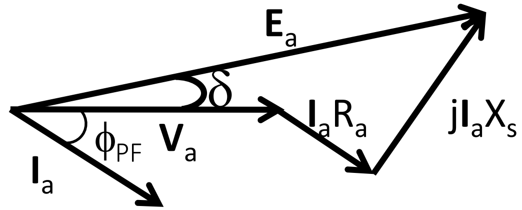
$$\mathbf{E}_a = \mathbf{I}_a(R_a + jX_s) + \mathbf{V}_a$$

Equivalent Circuit

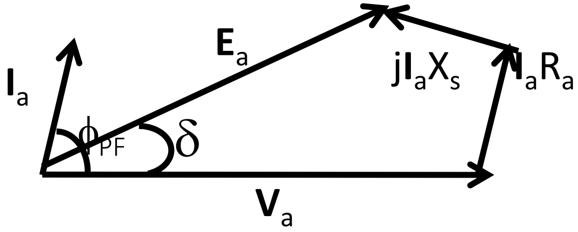
- Phasor diagrams of new per-phase circuit
  - δ: angle between $\mathbf{E}_a$ and $\mathbf{V}_a$ (induced voltage and terminal voltage), known as the *power angle* or *torque angle*



Unity power factor



Lagging power factor

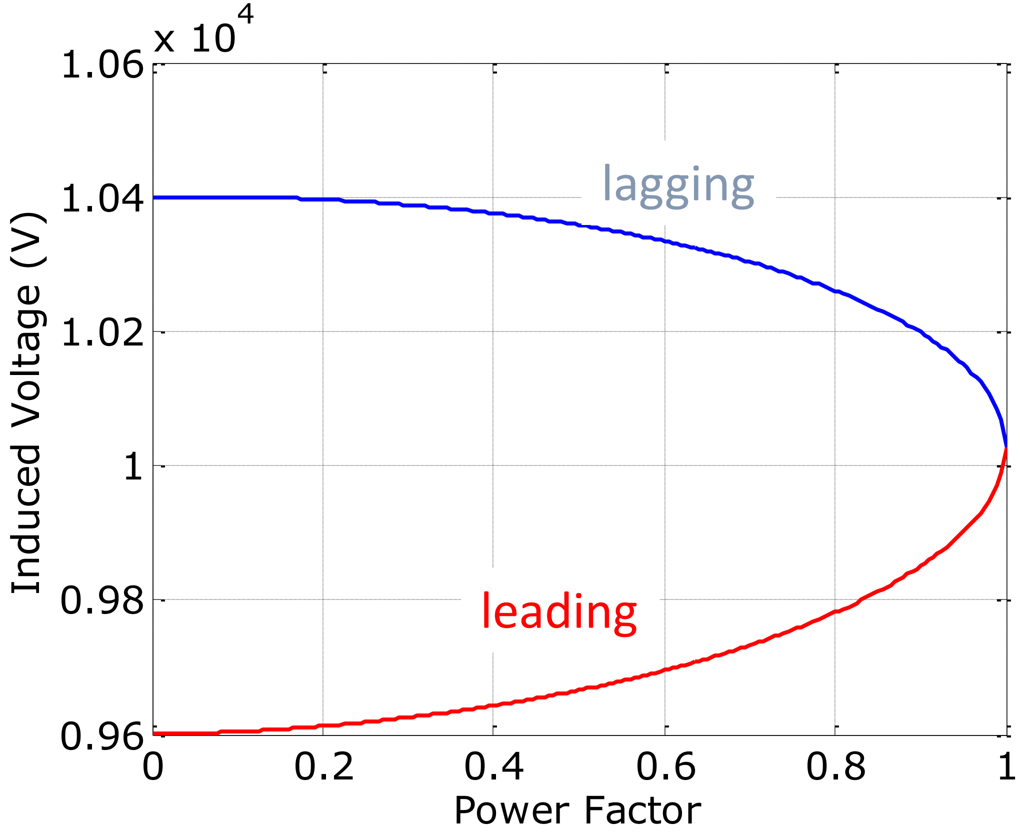


Leading power factor

δ is measured from $\mathbf{V}_a$ to $\mathbf{E}_a$
δ is positive for generators

Equivalent Circuit

E_a as a function of power factor.
Terminal voltage (V_a) held constant at 10kV.



Example

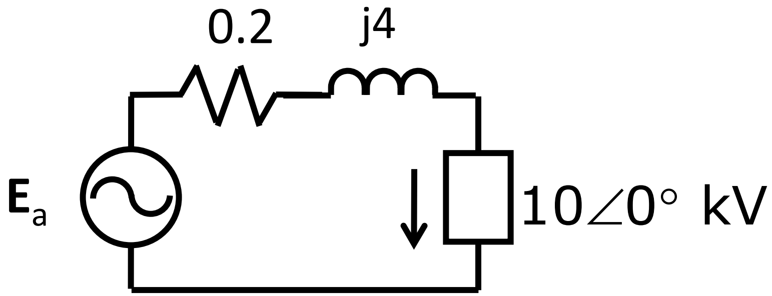
A synchronous generator has a per-phase synchronous impedance of $0.2 + j4\Omega$. The generator supplies a per-phase load current of 100A at a lagging power factor of 0.866 lagging. The per-phase terminal voltage is 10kV.

Compute the per-phase induced voltage.

Compute the power angle.

Example

- Per phase armature current: $\mathbf{I}_a = 100\angle -30^\circ$ A
- Solving the circuit: $\mathbf{E}_a = 10.2\angle 1.8^\circ$ kV
- Power angle: 1.8 degrees



$$\mathbf{E}_a = \mathbf{I}_a(R_a + jX_s) + \mathbf{V}_a$$

Voltage Regulation

- The voltage regulation of a synchronous generator is:

$$VR = \frac{|\mathbf{E}_a| - |\mathbf{V}_a|}{|\mathbf{V}_a|} \times 100$$

  - $\mathbf{E}_a$: induced emf, also the no-load terminal voltage
  - $\mathbf{V}_a$: terminal voltage at full load (V)

Power Relationships

- Mechanical power supplied to the shaft of a synchronous generator by the prime mover
  - steam turbine
  - combustion turbine
  - dc motor
  - others

Power Relationships

- Mechanical power in:

  $$P_{in,m} = T_s \omega_s$$

  - T_s: shaft torque (Nm)
  - ω_s: shaft speed (rad/s)
- Total power in: $P_{in} = T_s \omega_s + v_f i_f$
- Electrical power out: $P_o = 3|\mathbf{V}_a||\mathbf{I}_a|\cos\phi_{PF} = 3\,\mathrm{Re}\{\mathbf{V}_a \mathbf{I}_a^*\}$
- Copper losses: $P_{cu} = 3|\mathbf{I}_a|^2 R_a$

Power Relationships

- Power output: $P_o = 3|\mathbf{V}_a||\mathbf{I}_a|\cos\phi_{PF} = 3\,\mathrm{Re}\{\mathbf{V}_a\mathbf{I}_a^*\}$
  - Requires knowledge (usually computation) of armature current
- Desired to have an equivalent expression of generator power output without having to compute armature current

Power Expressions

- From the equivalent circuit:

$$\mathbf{I}_a = \frac{\mathbf{E}_a - \mathbf{V}_a}{R_a + jX_s} = \frac{\mathbf{E}_a - \mathbf{V}_a}{\mathbf{Z}_s}$$

- Power output:

$$P_o = 3\,\mathrm{Re}\{\mathbf{V}_a\mathbf{I}_a^*\} = 3\,\mathrm{Re}\left\{\frac{\mathbf{V}_a\mathbf{E}_a^* - |\mathbf{V}_a|^2}{\mathbf{Z}_s^*}\right\}$$

$$= 3\,\mathrm{Re}\left\{\frac{\mathbf{V}_a\mathbf{E}_a^*\mathbf{Z}_s}{|\mathbf{Z}_s|^2} - \frac{|\mathbf{V}_a|^2\,\mathbf{Z}_s}{|\mathbf{Z}_s|^2}\right\} = 3\,\mathrm{Re}\left\{\frac{\mathbf{V}_a\mathbf{E}_a^*\mathbf{Z}_s}{|\mathbf{Z}_s|^2} - \frac{|\mathbf{V}_a|^2 R_a}{|\mathbf{Z}_s|^2} - j\frac{|\mathbf{V}_a|^2 X_s}{|\mathbf{Z}_s|^2}\right\}$$

  - Above expansion uses:

$$\mathbf{Z}_s = |\mathbf{Z}_s|\angle\theta_z$$

$$\mathbf{Z}_s^* = |\mathbf{Z}_s|\angle-\theta_z$$

Recall that dividing by a phasor means dividing by the magnitude and subtracting the angle

$$\frac{1}{\mathbf{Z}_s^*} = \frac{1}{\mathbf{Z}_s^*}\frac{\mathbf{Z}_s^*}{\mathbf{Z}_s^*} = \frac{|\mathbf{Z}_s|\angle-\theta_z}{|\mathbf{Z}_s|^2\angle-2\theta_z} = \frac{|\mathbf{Z}_s|\angle\theta_z}{|\mathbf{Z}_s|^2} = \frac{\mathbf{Z}_s}{|\mathbf{Z}_s|^2}$$

Power Expressions

Continuing:

$$P_o = 3\,\mathrm{Re}\left\{\frac{\mathbf{V}_a\mathbf{E}_a^*\mathbf{Z}_s}{|\mathbf{Z}_s|^2} - \frac{|\mathbf{V}_a|^2 R_a}{|\mathbf{Z}_s|^2} - j\frac{|\mathbf{V}_a|^2 X_s}{|\mathbf{Z}_s|^2}\right\}$$

$$= 3\,\mathrm{Re}\left\{\frac{\mathbf{V}_a\mathbf{E}_a^*\mathbf{Z}_s}{|\mathbf{Z}_s|^2}\right\} - \frac{|\mathbf{V}_a|^2 R_a}{|\mathbf{Z}_s|^2}$$

$$= 3\,\mathrm{Re}\left\{\frac{\mathbf{V}_a\mathbf{E}_a^*(R_a + jX_s)}{|\mathbf{Z}_s|^2}\right\} - \frac{|\mathbf{V}_a|^2 R_a}{|\mathbf{Z}_s|^2}$$

$$= 3\,\mathrm{Re}\left\{\frac{|\mathbf{V}_a||\mathbf{E}_a|(\cos\delta - j\sin\delta)(R_a + jX_s)}{|\mathbf{Z}_s|^2}\right\} - \frac{3|\mathbf{V}_a|^2 R_a}{|\mathbf{Z}_s|^2}$$

$$= \frac{3|\mathbf{V}_a||\mathbf{E}_a|}{|\mathbf{Z}_s|^2}(R_a\cos\delta + X_s\sin\delta) - \frac{3|\mathbf{V}_a|^2 R_a}{|\mathbf{Z}_s|^2}$$

Important result

Note:

$\mathbf{E}_a = |\mathbf{E}_a| \angle\delta$

$\mathbf{V}_a = |\mathbf{V}_a| \angle 0° = |\mathbf{V}_a|$

Power Relationships

- Power balance equation:

$$P_{in} = T_s\omega_s + i_f v_f = 3|\mathbf{V}_a||\mathbf{I}_a|\cos\phi_{PF} + 3|\mathbf{I}_a|^2 R_a + i_f v_f + P_r + P_{sl}$$

  - P_r: rotational losses (W)
  - P_{sl}: stray load losses (W)

- Constant losses grouped as: $P_c = i_f v_f + P_r + P_{sl}$

Power Relationship

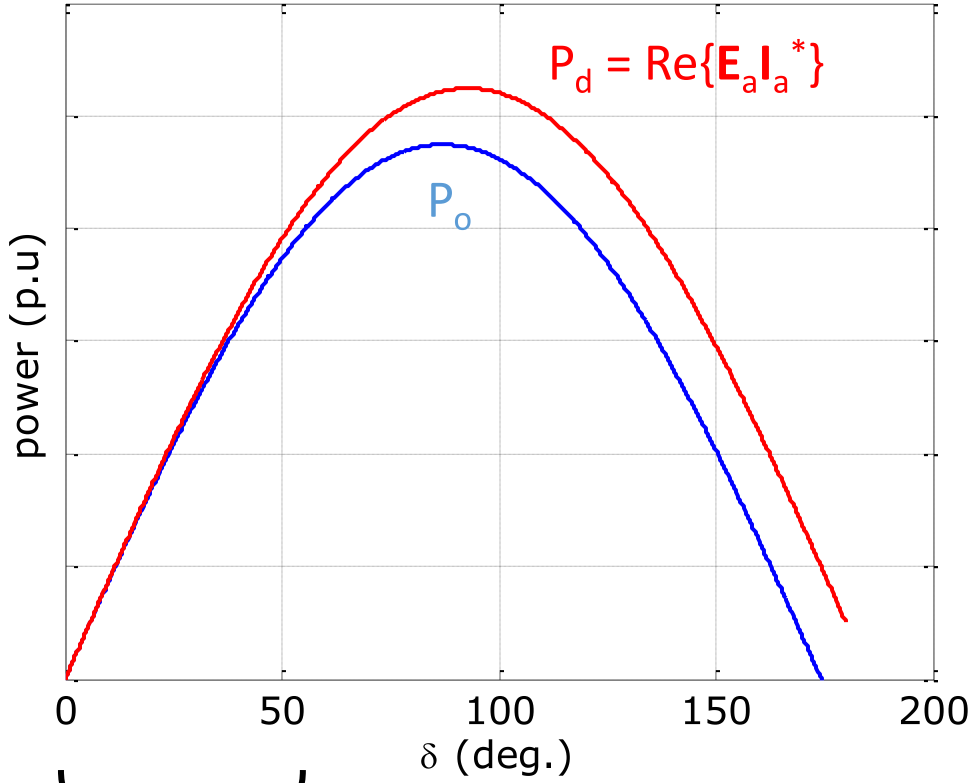
- Generator efficiency:

$$\eta = \frac{3|\mathbf{V}_a||\mathbf{I}_a|\cos\phi_{PF}}{3|\mathbf{V}_a||\mathbf{I}_a|\cos\phi_{PF} + 3|\mathbf{I}_a|^2 R_a + P_c}$$

- For maximum efficiency:

$$3|\mathbf{I}_a|^2 R_a = P_c$$

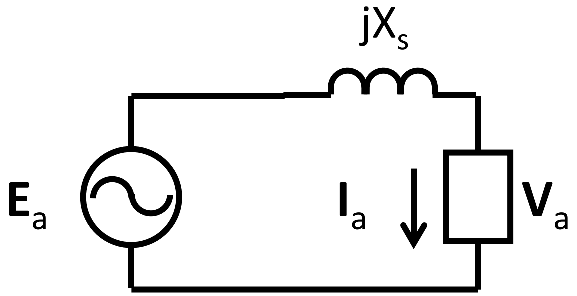
Power Relationship

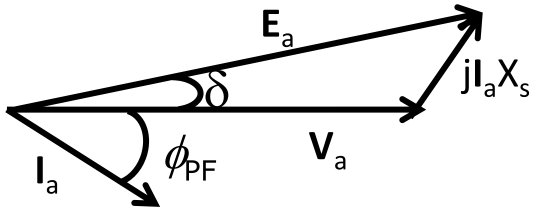


Armature copper
losses negligible

Approximate Power Relationship

- Armature resistance is small
- Common to ignore it





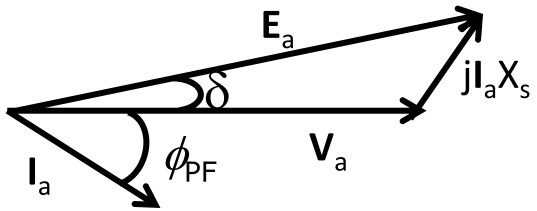
Example lagging PF load

Approximate Power Relationship

Computing the real power output:

$$\mathbf{E}_a = |\mathbf{E}_a|\angle\delta = |\mathbf{E}_a|\cos\delta + j|\mathbf{E}_a|\sin\delta$$
$$\mathbf{I}_a = |\mathbf{I}_a|\angle -\phi_{PF} = |\mathbf{I}_a|\cos\phi_{PF} - j|\mathbf{I}_a|\sin\phi_{PF}$$
$$\mathbf{V}_a = |\mathbf{V}_a|\angle 0 = |\mathbf{V}_a| + j0$$

Euler's Identity



Example lagging PF load

$$\mathbf{V}_a = \mathbf{E}_a - j\mathbf{I}_a X_s$$

$$\mathbf{I}_a = \frac{\mathbf{E}_a - \mathbf{V}_a}{jX_s} = \frac{|\mathbf{E}_a|\cos\delta - |\mathbf{V}_a|}{jX_s} + \frac{j|\mathbf{E}_a|\sin\delta - 0}{jX_s}$$
$$= \frac{|\mathbf{E}_a|\sin\delta}{X_s} - j\frac{|\mathbf{E}_a|\cos\delta - |\mathbf{V}_a|}{X_s}$$

$$|\mathbf{I}_a|\cos\phi_{PF} = \frac{|\mathbf{E}_a|\sin\delta}{X_s} \quad \text{(equating real parts)}$$

$$P_o = 3|\mathbf{V}_a||\mathbf{I}_a|\cos\phi_{PF} = \frac{3|\mathbf{V}_a||\mathbf{E}_a|\sin\delta}{X_s}$$

Important result

Approximate Power Relationship

- Synchronous generator power output (approximate)

$$P_o = 3|\mathbf{V}_a||\mathbf{I}_a|\cos\phi_{PF} = \frac{3|\mathbf{V}_a||\mathbf{E}_a|\sin\delta}{X_s}$$

- Assumes:
  - Armature resistance is zero
  - Constant speed
  - Constant field current
  - Cylindrical rotor

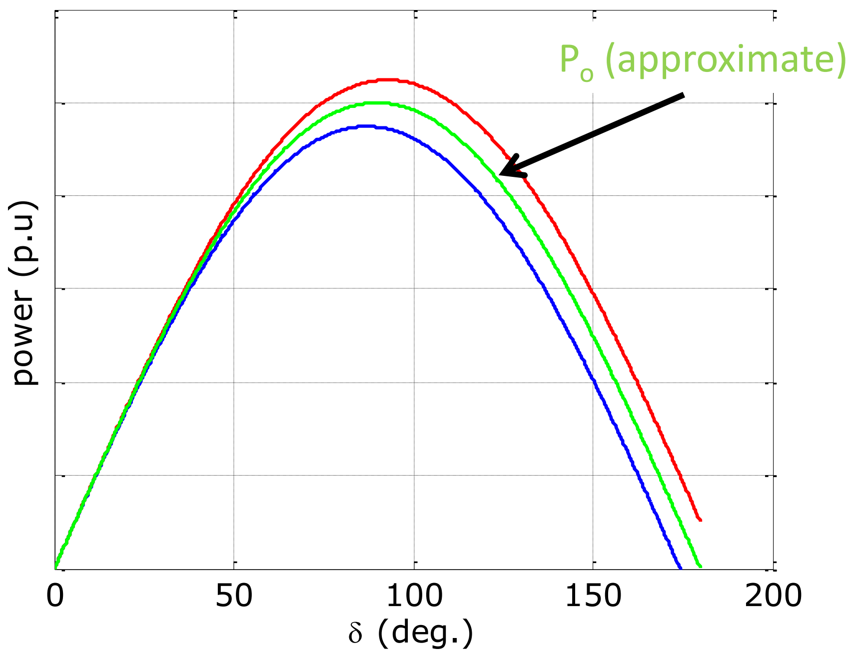
Approximate Power Relationship

- Power-angle relationship:

$$P_o = \frac{3|\mathbf{V}_a||\mathbf{E}_a|\sin\delta}{X_s}$$

- Maximum power:

$$P_{dm} = \frac{3|\mathbf{V}_a||\mathbf{E}_a|}{X_s}$$



Power Relationship

- Torque developed (approximate):

$$T_d = \frac{P_d}{\omega_s} = \frac{3|\mathbf{V}_a||\mathbf{E}_a|\sin\delta}{\omega_s X_s}$$

- Maximum torque (approximate):

$$T_{dm} = \frac{3|\mathbf{V}_a||\mathbf{E}_a|}{X_s\omega_s}$$

- Maximum power and torque occur at $\delta = 90°$

Example

A 2-pole synchronous generator has a per-phase terminal voltage of 7.5 kV, a per-phase induced voltage of 7.9 kV and a synchronous reactance of 1Ω. If the power angle is 15 degrees, compute the total real power delivered to the load. Assume the rotational losses are 1MW.

Example

A 2-pole synchronous generator has a per-phase terminal voltage of 7.5 kV, a per-phase induced voltage of 7.9 kV and a synchronous reactance of 1Ω. If the power angle is 15 degrees, compute the total real power delivered to the load. Assume the rotational losses are 1MW.

$$P_o = \frac{3|\mathbf{V}_a||\mathbf{E}_a|\sin\delta}{X_s} = 46\text{MW}$$

Rotational losses are not electric, so we do not need to subtract them.

Power Expressions

Several different forms of round-rotor power output:

$$P_o = 3\,|\mathbf{V}_a|\,|\mathbf{I}_a|\cos\phi_{PF}$$

$$= 3\,\mathrm{Re}\{\mathbf{V}_a \mathbf{I}_a^*\}$$

$$= \frac{3\,|\mathbf{E}_a|\,|\mathbf{V}_a|}{|\mathbf{Z}_s|^2}(R_a\cos\delta + X_s\sin\delta) - \frac{3\,|\mathbf{V}_a|^2 R_a}{|\mathbf{Z}_s|^2}$$

$$P_o = \frac{3\,|\mathbf{V}_a|\,|\mathbf{E}_a|\sin\delta}{X_s} \quad \text{(valid only if } R_a \text{ can be ignored)}$$

Power Relationship Summary

$P_{in} = T_s\omega_s + v_f i_f$ (total input power)

$\downarrow$

$\rightarrow$ $v_f i_f$ (field winding loss)

$P_{inm} = T_s\omega_s$ (input mechanical power)

$\rightarrow$ $P_r + P_{sl}$ (rotational and stray load loss)

$P_d = 3\,\mathrm{Re}\{\mathbf{E}_a \mathbf{I}_a^*\}$ (developed electrical power)

$\rightarrow$ $P_{cu} = 3\,|\mathbf{I}_a^2|\,R_a$ (copper loss in armature)

$\downarrow$

$$P_o = 3\,|\mathbf{V}_a|\,|\mathbf{I}_a|\cos\phi_{PF} = 3\,\mathrm{Re}\{\mathbf{V}_a \mathbf{I}_a^*\} \text{ (output electrical power)}$$

$$= \frac{3\,|\mathbf{E}_a|\,|\mathbf{V}_a|}{|\mathbf{Z}_s|^2}(R_a\cos\delta + X_s\sin\delta) - \frac{3\,|\mathbf{V}_a|^2 R_a}{|\mathbf{Z}_s|^2}$$

Power Relationship Example

Let:
v_f=400V
i_f = 250A
$P_r + P_{sl}$= 2MW
$\mathbf{Z}_s = 0.2 + j4\Omega$
$\delta = 30^o$
$\mathbf{V}_a$ = 10kV
$|\mathbf{E}_a|$ = 11kV

$$P_{in} = T_s\omega_s + v_f i_f = 44.21\text{MW}$$

$$V_f i_f = 0.1\text{MW}$$

$$P_{inm} = T_s\omega_s = 44.11\text{MW}$$

$$P_r + P_{sl} = 2\text{MW}$$

$$P_d = 3\,\mathrm{Re}\{\mathbf{E}_a \mathbf{I}_a^*\} = 42.11\text{MW}$$

$$P_{cu} = 3\,|\,\mathbf{I}_a^2\,|\,R_a = 1.14\text{MW}$$

$$P_o = \frac{3\,|\,\mathbf{E}_a\,||\,\mathbf{V}_a\,|}{|\,\mathbf{Z}_s\,|^2}(R_a\cos\delta + X_s\sin\delta) - \frac{3\,|\,\mathbf{V}_a\,|^2 R_a}{|\,\mathbf{Z}_s\,|^2} = 40.97\text{MW}$$

Summary

- Exciters are used to supply DC current to the rotor of synchronous generators
- Frequency of induced voltage increases with the number of poles for a fixed mechanical speed
- Leakage reactance and armature reaction can be combined into X_s, the synchronous reactance
- Approximate power delivered by a synchronous generator is:

$$P_o = 3|\mathbf{V}_a||\mathbf{I}_a|\cos\phi_{PF} = \frac{3|\mathbf{V}_a||\mathbf{E}_a|\sin\delta}{X_s}$$