Emission of SARS-CoV-2 Virions from Saliva Droplets in Air and Some Comments on Face Masks – Part II (AMENDED II October 26, 2023)

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Abstract

In this study, the effect of various volume fractions of SARS-CoV-2 virions on the release rate, the distance traveled (and time spent) for full evaporation is quantitatively addressed. We also comment on the challenges of making a filter for virions and propose definitive experiments to evaluate the efficacy of masking materials and masking systems. This study is a continuation of our initial paper which addressed the settling of both "dry" and "wet" virions and reviewed the efficacy of masking. Up to about 10^7 virions in a 100 μm saliva droplet has no discernible effect on the settling of the droplet. The release rate of virions increases with increasing initial volume fraction of virions and decreases over time at a given volume fraction until all virions are emitted. Importantly, it was quantitively shown that for equivalent masses, how much more rapidly a population of small saliva droplets emits virions into the surroundings. As in our previous work, we were unable to find any instance in which currently available masks could reliably filter viruses. Suggestions are made for scientifically valid approaches, including a critical experiment using radioactive isotopes to track the interaction of the SARS-CoV-2 virions with the masking material and the mask as a system.

Subject Areas: Fluid Mechanics, Filtration, Bio-Engineering, Bio-Physics, Bio-Mechanics, COVID-19, SARS-CoV-2, Masking.

Key Words: Stokes Law, Size Distribution, Droplet Evaporation, Effect of Virion Concentration in Saliva Droplets, Face Masks, SARS-CoV-2, COVID-19

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Outline of Paper

Symbols Used

v = settling velocity of terminal velocity of COVID-19 particle [m/sec].

ρ_v, ρ_a = density of the falling particle and media containing particle respectively [kg/m^3].

g = acceleration due to gravity [m/sec^2].
R = radius of settling particle [m]. Sometimes in [nm] or [μm] depending on context.
D_o, D = initial and instantaneous diameter at any time of settling droplet [m]
μ = viscosity of the media in which the particle is falling (in this case, air) [kg/m/sec].
C = basic saliva evaporation rate under defined conditions [m^2/sec].
C^* = effective saliva evaporation rate at a given volume fraction of virions [m^2/sec].
A = total surface area of virus-containing droplet [m^2].
A_s = surface area of saliva droplet containing virions.
f_A, f_V = area and volume fractions of virions in saliva droplet.
N_o = absolute initial number of virions in a given droplet
$Ncalc$ = calculated absolute initial number of virions in a given droplet.
N_V = number of virions per unit volume of saliva droplet [#/m^3].
Δρ = difference in density between virus or virus-droplet and air [kg/m^3].
RH = relative humidity [%]
P = porosity [%]
d_f, d_v = fiber and virion diameters [mm or nm]

I. Introduction

In this study, which follows from our preceding paper on the properties, settling, and mediation of SARS-CoV-2, as well as masking, we consider the following:

- The effects of the volume fraction of virions in saliva on the shedding of virions.
- The effects of volume fraction of virions in a typical saliva droplet formed by sneezing on the distance and time to complete evaporation.
- A proposed experiment using radioactive virions to trace the path of virions and measure both the efficacy of masking material and actual masking systems.

We conclude by showing that using current technology, SARS-CoV-2 can be fully excluded from human beings.

II. Properties and Calculations of Settling Distances and Times of SARS-CoV-2 Virions

A. The Relation between Area and Volume Fraction.

In this section the effect of the volume fraction of SARS-COV-2 virions on settling and emission is considered. Since the virions affect the amount of saliva exposed for evaporation, essentially the critical processes take place at the surface while practical measurements are usually expressed in volume fractions, the relation between area fraction and volume fraction is important. A random distribution of virions is assumed; for calculational purposes both the virions and saliva droplets are assumed to be spherical, Figure 1.

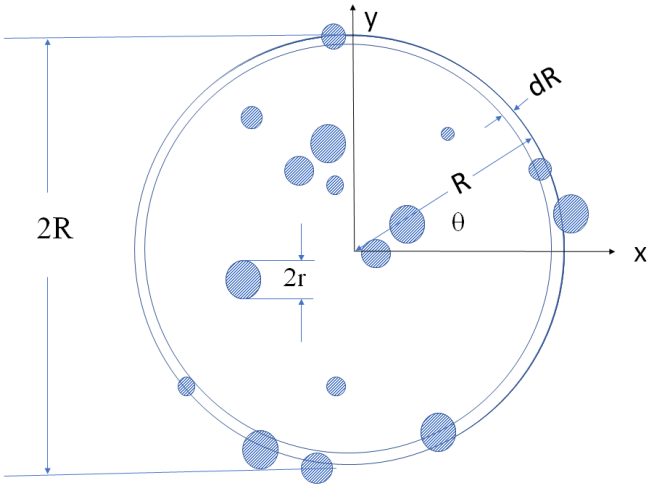

Fig. 1. Maximum cross-section of saliva droplet of current radius R containing SARS-CoV-2 virions of maximum radius r. Note that the virions do not appear to be of equal radius in the cross-section shown. Note also that the schematic is not to scale. As mentioned in the previous paper[1], the virions are so small that they would not even show up as the tiniest "dot" in this illustration were it drawn to scale.

The area fraction of virions intersecting the surface is denoted by f_A. Then the total area occupied by the virions is:

$$A_t^{vir} = 4\pi R^2 f_A \quad \ldots \text{(Eq. 1)}$$

The differential volume of virions associated with this area is obtained by simply multiplying Eq. (1) by *dR*:

$$dV_t^{vir} = 4\pi R^2 f_A dR \quad \ldots \text{(Eq. 2)}$$

The total volume of virions is obtained by integration of Eq. (2):

$$V_t^{vir} = \int_0^R 4\pi R'^2 f_A dR' = \frac{4}{3}\pi R^3 f_A \quad \ldots \text{(Eq. 3)}$$

Where R' is a dummy variable of integration. Equation (3) is rearranged to yield:

$$\frac{V_t^{vir}}{\frac{4}{3}\pi R^3} = f_V = f_A \quad \ldots \text{(Eq. 4)}$$

Equation (4) shows that the volume fraction is equal to the surface fraction. This is a well-known result which was first developed (using a different approach) by Delasse [2] in his study of the mineral constituents of rocks He stated:

Supposons que le volume occupé par *la roche* soit rapporté à un système de coordonnées, et soit *p* la surface occupée par l'un des minéraux constituants dans une section formée par un plan parallèle aux *xy*; pour obtenir exactement le volume occupé par ce minéral dans la roche, il faudrait pouvoir connaître les valeurs successives de *p*, quand on mène une série de plans infiniment rapprochés et parallèles à *xy*; l'intégrale $\int p\,dz$ donnerait alors l'expression du volume cherché.

The above extract was taken from p. 380 of Delasse's paper [2]. An idiomatic translation is:

"*Let us suppose that the volume occupied by the rock is put in terms of an [xyz] coordinate system and that p is the surface [area] occupied by one of the mineral constituents in a planar section parallel to the xy plane; in order to obtain the exact volume occupied by the mineral in the rock, it would be necessary to know the successive values of p for a series of planes infinitely close to one another and parallel to the xy plane; the integral of p with respect to z would then give the expression for the desired volume.*" [3] .

Perhaps a more satisfying approach is to use spherical co-ordinates as shown in Fig. 2.

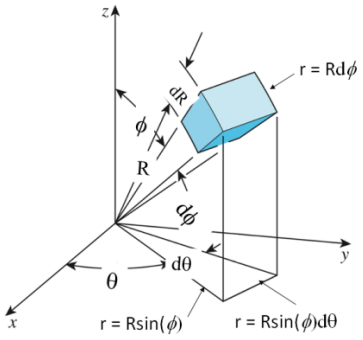

Fig. 2. Definition of spherical coordinate system.

In this representation, the differential surface area of the virions is:

$$dA_{sp}^{vir} = f_A R^2 \sin(\varphi) d\varphi d\theta \qquad \ldots \text{Eq. (5)}$$

The total volume of virions is given by multiplying Eq. (5) by dR and integrating over appropriate limits to get the volume of 1/8th of the sphere and then multiplying by 8. This gives:

$$V_t^{vir} = 8\int_0^R R'^2 dR' \int_0^{\frac{\pi}{2}} \sin(\varphi) d\varphi \int_0^{\frac{\pi}{2}} d\theta = \frac{4}{3}\pi R^3 f_A \qquad \ldots (6)$$

which is exactly the result previously developed, Equations (3) and (4). In this paper we shall use f_A and f_V interchangeably. Although f_A is more aligned with the physical process of evaporation at the surface, results are presented in terms of f_V since it is easier to determine experimentally and is conventionally used for concentration.

B. Calculation of Settling Times and Distances of CoVID-19 Virions

In the prior paper it was assumed that each droplet contains a single SARS-COV-2 virion and three cases were considered:

- Falling of the "dry" virus in dry air
- Falling of saliva-covered "wet" virus ignoring evaporation.
- Falling of saliva-covered "wet" virus with simultaneous evaporation.

In the following sections, the effect of multiple SARS-CoV-2 virions in an evaporating saliva droplet at 23° C and 30% RH are considered in terms of:

(1) the distance vs time and,
(2) the virion shedding rate as a function of time when the virion falls in dry air
(3) initial diameter of the saliva droplets in a sneeze.

1. Distance vs Time for SARS-COV-2 Virions in an Evaporating Saliva Droplet Containing N_o Virions

As in the previous paper [1], all calculations are based upon modifying Stokes Law [4,5] which is:

$$v = \frac{dx}{dt} = K[D(t)]^2 \quad \ldots \text{(Eq. 7)}$$

Where $K = \frac{g\,\Delta\rho}{18\mu}$ . . . (Eq. 8)

And
- v = terminal velocity
- ρ_v, ρ_a = density of the falling particle and media containing particle respectively
- $\Delta\rho$ = $\rho_v - \rho_a$
- g = acceleration due to gravity
- $D(t)$ = diameter of settling particle at any instant in time t
- μ = viscosity of the media in which the particle is falling (in this case, air)

In reference [1] numerical calculations were carried out in the SI system using numbers shown Table 1 of that paper. For convenience, appropriate sections of that table and other are included here.

Table 1. Basic Properties Used in Stokes Law Calculations. Data Obtained From [6,7,8,9]

Saliva Density ρ_v (kg/m^3)	Dry Air Density* ρ_a (kg/m^3)	Density Difference $\Delta\rho$ (kg/m^3)	Accel. of Gravity g (m/sec^2)	Air Viscosity μ (kg/sec/m)	$K=\Delta\rho g/18\mu$ ($m^{-1}s^{-1}$)	Initial Droplet Radius R (m)
1.012E+03	1.184	1.011E+03	9.8	1.849E-05	2.98E+07	5.00E-05

As in the prior paper, the key to obtaining the position as a function of time lies in computing the diameter of the virion-containing droplet as a function of time using evaporation data reproduced here from [1] as Figure 3.

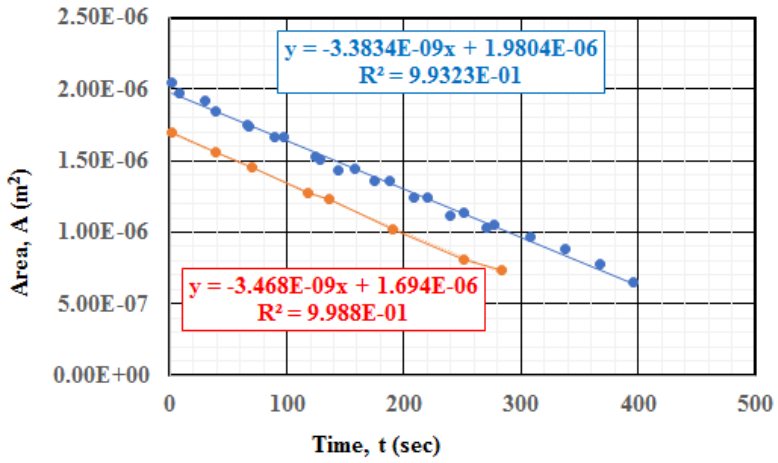

Fig. 3. Evaporation rate of saliva droplets at RT and 30% RH for 0.81 mm droplets (blue) and 0.78 mm droplets (red) as shown in [1] and obtained from [10]. Note the high degree of correlation of the best-fit lines with the data.

To calculate the continually slowing settling rate, due to the droplet becoming smaller from evaporation, we first examine the extent to which the droplet changes size over time. To do this assume a spherical droplet of diameter *D(t)* at any time t and constant surface evaporation rate for pure saliva (in m^2/s) of magnitude *C* for a given set of conditions (i.e., temperature, pressure, relative humidity).

If the surface fraction of a spherical droplet covered by virions is f_A, then the area of saliva on the droplet surface at any time is:

$$A_s(t) = \pi D^2(t)\left(1 - f_A\right) \quad \ldots \text{(Eq. 9)}$$

From which:

$$\frac{dA_s}{dt} = C = 2\pi D(1 - f_A)\frac{dD}{dt} \quad \ldots \text{(Eq. 10)}$$

Assuming that *C* is in fact constant, as appears to be the case from Fig. 3, Eq. (10) can be integrated from the initial size *Do* at *t =0* to the size *D* at any time *t* with the result:

$$D = \sqrt{D_o^2 + \frac{Ct}{\pi\left(1 - f_A\right)}} = \sqrt{D_o^2 + \frac{C^*t}{\pi}} \quad \ldots$$

Eq. (11)

Where $C^* = \dfrac{C}{(1 - f_A)}$ . . . Eq. (12)

C^* may be viewed as the *effective* evaporation rate for a given area fraction of virions covering the surface of the virion-impregnated saliva droplet under given conditions.

For complete evaporation it is reasonable to take $D \sim 0$ since the final step is the escape of the last virion regardless of how many virions are in the droplet. The size ratio of a typical virion (~100x10^{-9} m) to a typical saliva drop (100x10^{-6} m) is 1/10000. With this reasonable approximation, the maximum time to complete evaporation is:

$$t^{max} = -\frac{\pi D_o^2}{C^*} \quad \text{. . . (Eq. 13)}$$

The Stokes Law evaporation-modified settling law is obtained by inserting Eq. 11 into Eq. 7.[11]:

$$v = \frac{dx}{dt} = K\left(D_o^2 + \frac{C^* t}{\pi}\right) \quad \text{. . . (Eq. 14)}$$

Where all quantities have been previously defined. The preceding results demonstrate that the presence of virions throughout the droplet can effectively be taken into account by using the results of the previous paper and replacing C by C^*.

Equation 14 is the time-mediated Stokes Law for a given volume fraction of virions. It is seen that the velocity becomes essentially zero when the droplet is fully evaporated.

Equation 14 may be re-arranged and integrated from x = 0 (position at start of settling) at t=0 to any position x at time t with the result:

$$x = KD_0^2 t + (KC^*/2\pi)t^2 \quad \text{. . . (Eq. 15)}$$

(a) Application to 100 μm Saliva Droplets at Various Volume Fractions of Virions

Equation (15) is plotted in Fig. 4 for 100 μm diameter sneeze droplets at different volume fractions of virions. The data used in creating Fig. 4 is shown in Tables 2 and 3.

Table 2. Data and Conditions Used to Construct Fig. 4

Saliva Density ρ_v (kg/m^3)	Dry Air Density* ρ_a (kg/m^3)	Density Difference $\Delta\rho$ (kg/m^3)	Accel. of Gravity g (m/sec^2)	Air Viscosity μ (kg/sec/m)	K=$\Delta\rho$g/18μ (m^{-1}s^{-1})	Initial Droplet Radius R (m)	**Volume Fraction of Virions**	**Initial Droplet Diameter D (μm)**	**Evaporation Rate C* (m^2/sec)**
1.012E+03	1.184	1.011E+03	9.8	1.849E-05	2.98E+07	5.00E-05	0.00	100	**-3.42E-09**
							0.05		**-3.60E-09**
							0.25		**-4.56E-09**
							0.50		**-6.84E-09**

The form of the curves is as expected:

- The velocity, dx/dt, decreases monotonically to zero with time.
- The higher the volume fraction of virions, the more rapid the evaporation; these conditions thus favour shorter times and distances to complete evaporation.

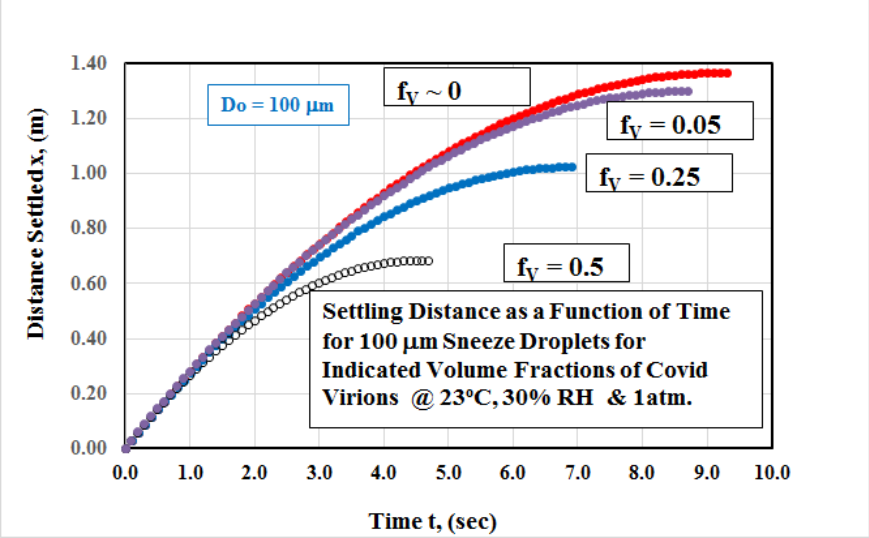

Figure 4. Distance settled as a function of time for saliva droplets of 100 µm initial diameter containing different volume fractions of virions sneezed at 23°C, in 30% RH air and 1 atmosphere. At maximum distance and zero velocity, (i.e. dx/dt =0) the virions have all been released and are "dry".

The time for complete evaporation of the virion-containing sneeze drop is obtained by setting dx/dt = 0 in Eq. (15), yielding the same result as Eq. (11). Equation (11) may be substituted into Eq. (15) to obtain the distance travelled to complete evaporation, x_{max} :

$$x_{max} = K\left(\frac{t_{max}}{2}\right)D_o^2 \qquad \text{. . . Eq. (16)}$$

Table 3. Settling Time and Corresponding Settling Distance for Complete Evaporation of 100 μm diameter Saliva Droplets Containing Different Volume Fractions of SARS-CoV-2 Virions at 23 °C, 1 atmosphere and 30% RH.

Volume Fraction of Virions	Initial Droplet Diameter D (μm)	Effective Evaporation Rate C* (m^2/sec)	K ($m^{-1}s^{-1}$)	Maximum settling time, $t_{max} = -\pi Do^2/C^*$ (sec)	Maximum settling distance, $x = KDo^2t_{max}/2$ (m)
0.00	100	-3.42E-09	2.98E+07	9.19	1.37
0.01		-3.45E-09		9.09	1.35
0.05		-3.60E-09		8.73	1.30
0.25		-4.56E-09		6.89	1.03
0.50		-6.84E-09		4.59	0.68

The values for settling distances and times in Figure 4 agree with the end values shown in Table 3. Furthermore, and importantly, the curves for $f_V \sim 0$ and $f_V = 0.05$ are very close. But, a simple calculation shows that at a volume fraction of 0.05, the number of virions in a droplet is 50 x 10^6. However, for the conditions being considered, the curve for 100 μm diameter droplets in Figure 9 (one virion) of Ref. [1] is the same as that in Figure 4 of the current paper. *This means that the results of Ref. [1] are representative of the behavior of droplets for falling times and distances up to extremely high numbers of virions.*

(b) Application to 10 μm Saliva Droplets at Various Volume Fractions of Virions

Clearly similar calculations to those in the preceding section can be carried out for different sized droplets under different conditions of temperature and RH by using the formulations and data given in this and in Ref. [1]. In Ref. [1] it was shown that a significant number of droplets from a sneeze has a diameter around 10 μm. Because the smaller droplets are expected to be even more impervious to masking barriers and release virions more quickly after shorter falling distances it is therefore important to consider this size range of droplets. Given these properties, the smaller particles may be the most critical in a typical sneeze distribution. The analysis given above for the 100 μm droplets was repeated for 10 μm droplets and the results are shown below in Figure 5.

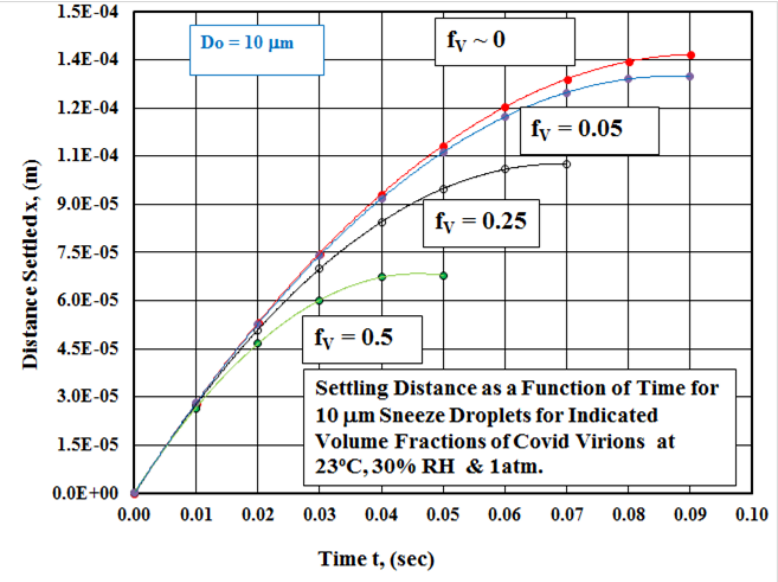

Fig. 5. Distance settled as a function of time for saliva droplets of 10 μm initial diameter containing different volume fractions of virions sneezed at 23°C, in 30% RH air and 1 atmosphere. At maximum distance and zero velocity, (i.e. dx/dt =0) the virions have all been released and are "dry".

Figure 5 shows that the times and distances of 10 μm saliva droplets containing virions are orders of magnitude less that for the 100 μm droplets considered previously. The implications of this are very important and can be summarized as follows:

- Small droplets can release their viral load in hundredths of seconds as opposed to many seconds for the larger droplets.
- The virions are released in very short distances which means that they will not have time to land on surfaces.
- The smaller droplets (and most of the larger droplets) will discharge their virions long before reaching any surface. Thus, any transmission, is airborne.

Given the fact that SARS-CoV-2 virions die within minutes in air, and as shown in Ref. [1], the actual virions float forever in the air, while most of them are dead.

III. Calculation of the Shedding Rate of SARS-CoV-2 Virions from 100 μm and 10 μm Saliva Droplets

It is of interest to determine the shedding rates of the virions from droplets as a function of the virion concentration within the saliva drops. Formally, the rate of saliva emission (#/sec) is given by the basic equation:

$$\frac{dN}{dt} = -\frac{dV}{dD} \bullet \frac{dD}{dt} \bullet \frac{dN}{dV} \qquad \ldots \text{(Eq.17)}$$

The negative sign in Equation (17) arises from the fact that *dN* virions *leave* the droplet in *dt* seconds; for clarity of presentation the virion emission rate is presented as a positive quantity.

Where N = number of virions in the droplet at any time t
V = volume of saliva droplet

All other symbols have been defined.

Equation (17) may be further developed noting that the last term is simply N_V (a constant) and inserting the results of Equations (10 and 12) to yield:

$$\frac{dN}{dt} = -\frac{DC^*}{4} \bullet N_V \quad \ldots$$

Eq. (18)

Substituting Eq. (11) into Eq. (18) gives:

$$\frac{dN}{dt} = -\frac{C^* N_V}{4}\left(D_o^2 + \frac{C^* t}{\pi}\right)^{\frac{1}{2}} \quad \ldots$$

Eq. (19)

A. Virion Shedding from 100 μm Diameter Saliva Droplets

The emission rate of virions as a function of time for various volume fractions is shown in Fig. 6 while the data from which Figure 6 was developed are provided in Table 4.

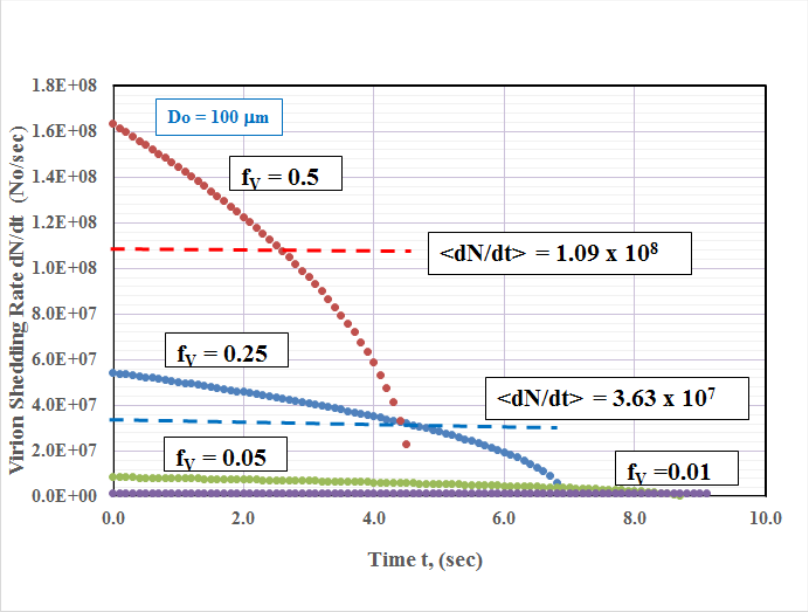

Figure 6. SARS-CoV-2 virion shedding rates from 100 μm saliva droplets as a function of time for 25°C and 30% RH. The virion volume fractions of 0.01, 0.05. 0.25 and 0.50 are indicated on the graph.

Table 4. Data Used to Construct the Curves Shown in Fig. 6. All Data at 25°C and 30% RH

Droplet Radius R (m)	Initial Droplet Diameter D_o (μm)	Initial Volume of Droplet $V_{Do} = (4/3)\,\pi\,R^3$ (m^3)	Virion Diameter d (m)	Volume of Virion V_v (m^3)	Initial Total Number of Virions N_o	Volume Fraction of Virions f_V	Area Fraction of Virions $f_A = f_V$	Evaporation Rate C* (m^2/sec)	Concentration of Virions $N_v = N_o/V_{Do}$ (#/m^3)
5.00E-05	100	5.236E-13	1.000E-07	5.236E-22	1.000E+00	0.000	0.000	**-3.42E-09**	1.910E+12
5.00E-05	100	5.236E-13	1.000E-07	5.236E-22	1.000E+07	0.010	0.010	**-3.45E-09**	1.910E+19
5.00E-05	100	5.236E-13	1.000E-07	5.236E-22	5.000E+07	0.050	0.050	**-3.60E-09**	9.549E+19
5.00E-05	100	5.236E-13	1.000E-07	5.236E-22	2.500E+08	0.250	0.250	**-4.56E-09**	4.775E+20
5.00E-05	100	5.236E-13	1.000E-07	5.236E-22	5.000E+08	0.500	0.500	**-6.84E-09**	9.549E+20

As expected, the virion emission rate for all volume fractions is a decreasing function of time with the rate of decrease increasing with time, again an intuitively reasonable result. As a check on the self-consistency of the results, integration of the emission rate by the active period of evaporation (i.e. t_{max} in Table 3) should yield the total number of virions released (i.e., the initial number of virions in the saliva droplet, *No*). This may be stated as:

$$N_{calc} = \int_0^{t_{max}} \frac{dN}{dt} dt \qquad \ldots \text{(Eq. 20)}$$

The average emission rate, $\langle \frac{dN}{dt} \rangle$, over the period of active evaporation (i.e., see Table 3), is of interest. The average value is the line which makes the areas above and below the line bounded by the rate curve equal. Mathematically this is:

$$\langle \frac{dN}{dt} \rangle = \frac{1}{t_{max}} \int_0^{t_{max}} \frac{dN}{dt} dt \qquad \ldots \text{(Eq. 21)}$$

By definition, the average shedding rate *must* be:

$$\langle \frac{dN}{dt} \rangle = \frac{N_o}{t_{max}} \qquad \ldots \text{(Eq. 22)}$$

To verify that this is indeed the case, Equation (19) can be integrated with some substitutions to make the expression less cumbersome:

$$\langle \frac{dN}{dt} \rangle = \int_0^{t_{max}} \frac{A}{t_{max}} (\mathrm{B} + Et)^{\frac{1}{2}} dt \quad \ldots \text{(Eq. 23)}$$

Where: $A = -\frac{C^* N_V}{4}$ ... (Eq. 24)

$B = D_o^2$ ... (Eq. 25)

$E = \frac{C^*}{\pi}$ ... (Eq. 26)

The integral in Eqquation (23) is of an elementary form and is calculated to be:

$$\langle \frac{dN}{dt} \rangle = \frac{2A}{3Et_{max}} \{ \left(\mathrm{B} + Et_{tmax} \right)^{\frac{3}{2}} - B^{\frac{3}{2}} \} \quad \ldots \text{(Eq. 27)}$$

Substituting Equations (24-26) as well as Eq. (13) into Eq. (27) gives Eq. (22):

$$\langle \frac{dN}{dt} \rangle = \frac{N_o}{t_{max}} \quad \ldots \text{Eq. (22)}$$

as would be expected and as is required.

The total number of virions emitted can be calculated by integration of Eq. (17) with substitutions, as done above, yielding:

$$N_o^{calc} = \frac{2A}{3E} \{ \left(\mathrm{B} + Et_{tmax} \right)^{\frac{3}{2}} - B^{\frac{3}{2}} \} = N_o \quad \ldots \text{Eq. (28)}$$

Again, the result in Eq. (28) is both expected and required.

The preceding analysis demonstrates that the approach to calculating shedding rates is entirely consistent.

As an additional check to demonstrate the consistency of the model, both N_o^{calc} and $\langle \frac{dN}{dt} \rangle$ were computed numerically using Mathcad software. Results are shown in Table 5. A sample Mathcad work sheet is shown in Appendix 1. Values of $\langle \frac{dN}{dt} \rangle$ are shown in Figure 6 as dotted

lines for purposes of illustration at volume fractions of 0.25 and 0.50. All values of *Ncalc* and $\langle \frac{dN}{dt} \rangle$ are provided in Table 5.

Table 5. Calculated Number of Virions and Average Virion Emission Rate for 100 μm Dimeter Saliva Droplets

Volume Fraction of Virions f_V	Initial Total Number of Virions No	Calculated Number of Virions Ncalc	Calculated Average Virion Emission Rate <dN/dt> (#/sec)
0.010	1.000E+07	1.000E+07	1.100E+06
0.050	5.000E+07	5.000E+07	5.729E+06
0.250	2.500E+08	2.500E+08	3.629E+07
0.500	5.000E+08	5.000E+08	1.089E+08

It is seen that exact agreement between the assumed and calculated numbers of virions is obtained, demonstrating the reliability of the model.

B. Virion Shedding from 10 μm Diameter Saliva Droplets

The exact same analysis that was done for falling and shedding of 100 μm droplets was repeated for 10 μm droplets and the results are shown in Fig. 7. In all cases, as for the 100 μm droplets, the analysis was internally consistent. It is interesting that in comparing the results of Figs. 6 and 7, the larger drops have a larger (~ 10X) emission rate for the same volume fraction of virions than the smaller drops. We can understand this by combining Equations (13) and (22) to write the average emission rate as:

$$\langle \frac{dN}{dt} \rangle = \dot{N} = \frac{N_o}{t_{max}} = \frac{N_v C^* D_o}{6} \qquad \text{Eq. . . . (29)}$$

In Equation (29), for a given concentration (i.e., N_v) C^* is constant and the average emission rate is directly proportional to the initial droplet size D_o. Thus, in comparing the average emission rates of droplets containing the same concentration of virions one obtains:

$$\frac{\dot{N}_S}{\dot{N}_L} = \frac{D_o^S}{D_o^L} \qquad \text{Eq.. . . (30)}$$

Where 'S' and 'L' in Eq. (30) refer to small and large droplets respectively.

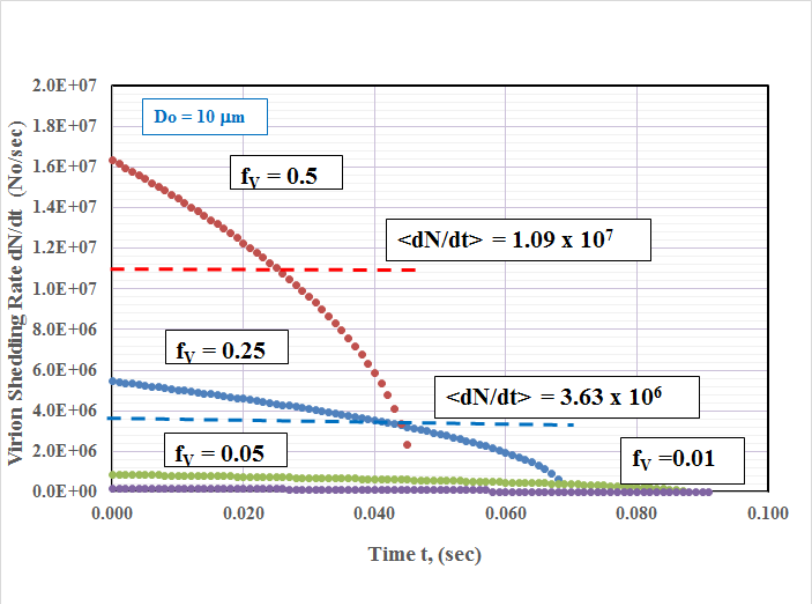

Figure 7. SARS-CoV-2 virion shedding rates from 10 µm saliva droplets as a function of time for 25°C and 30% RH. The virion volume fractions of 0.01, 0.05. 0.25 and 0.50 are indicated on the graph. Compare results to those shown in Figure 6.

C. Comments on Shedding Rates

Equation (30) implies that the virion shedding rate of a large droplet is larger than that of a small droplet. This is correct in as much as a single large particle has more virions at a given volume fraction f_v than do small droplets. However, care must be taken in interpreting this result. The proper way to compare large and small droplets would be on an equal mass basis. For example, consider a 100 µm droplet and an equivalent mass of 10 µm droplets. There are obviously 1000X more small droplets of the same concentration. From Figs. 6 and 7 the shedding rates of the 100 µm and 10 µm droplets at, for example, a volume fraction of 0.5 are 1.09×10^8 and 1.09×10^7 virions/sec respectively. The 100 µm droplets completely evaporate in 4.59 seconds (see Table 3). Using this value and Equation (13), the evaporation time for the 10 µm droplets is only 4.59×10^{-2} seconds. Thus, in this example, the same number of virions are released from the same *mass* of small 10 µm droplets in only 1/100 th of the time it would take to release the same number from the larger 100 µm droplet. This approach can be used to compare droplets of any two sizes. In fact, one need only use Equation (13) with appropriate data to show that for equivalent masses, small particles will always be more efficient in releasing virions into the surroundings.

IV. Comments on Mechanical Masking

A. Calculation of Mask Porosity

In the previous paper, it was demonstrated that the material used for currently available masks is insufficient to filter out the Covid virions [1]. The openings were shown to be many orders of magnitude larger than the virions which are, on average, around 100 nm in diameter. The question then arises if masks were made using current fibers whether it is practical to make the openings in the masks small enough to exclude the virions but still allow for normal breathing? This does not include gaps between the face and the as-fabricated mask that result from the overall design of the mask. Clearly such gaps represent a very easy path for both breathing and for virions to pass from the human subject to surroundings and vice versa. In this section we do not address mask design but consider only the mask fabric without consideration of these gaps. A characteristic element of the mask fabric is shown in Figure 8.

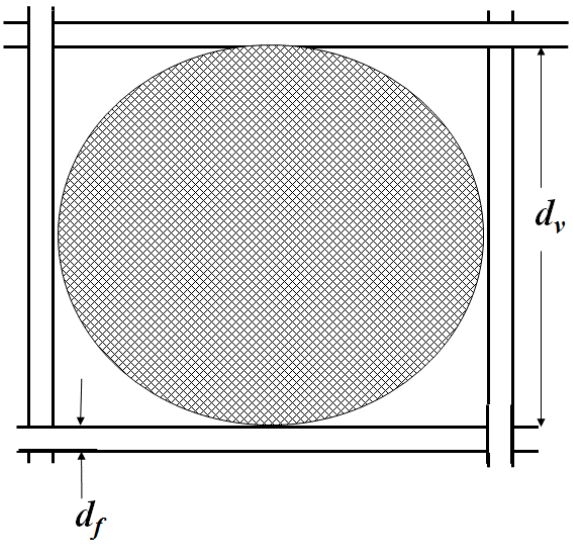

Figure 8. Characteristic element of mask fabric. The gap is just small enough to block the virions, shown as a cross-hatched circle which is the cross section of a sphere.

If the masks are to have any possibility of filtering out Covid virions, the characteristic hole dimension must be no larger than the diameter of the virion, denoted d_v. If the fiber diameter is denoted d_f, then the % porosity P is easily calculated to be:

$$P = 100\left(\frac{d_f}{d_v} + 1\right)^{-2} \qquad \text{Eq. . . (31)}$$

All the terms have been defined above. Equation (31) is plotted in Figure 9 for a typical virion diameter of about 100 nm (0.1 μm). The previous paper [1] showed that a typical R95 fiber diameter is about 20 μm.

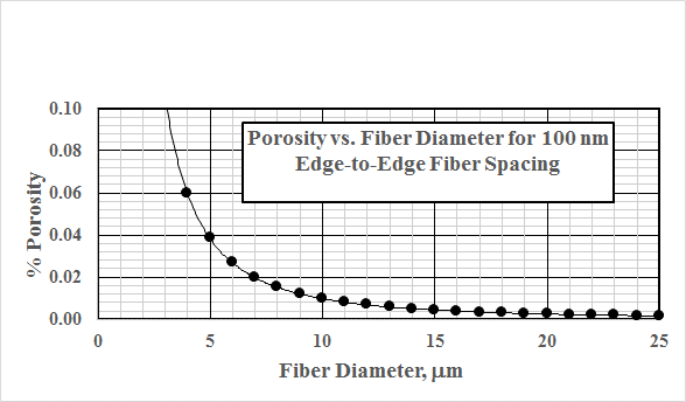

Figure 9. Percent porosity in mask fabric as a function of fiber diameter for filtering of 100 nm diameter Covid virions. It is seen that using 20 μm diameter fibers results in essentially no porosity which would cause suffocation if used.

Under these conditions the porosity P is about 0.0022%. Such limited porosity makes such a mask impossible to breathe through. Recall that in the previous paper [1] the conditions for normal breathing were cited:

> *"Human lung pressures and lung compliance are defined by the bulk air flowing into and out of the lungs along pressure gradients created in between the external environment and the alveoli [12,.13,.14]. The intra-pleural pressure at the commencement of inhalation is approximately -1.84 mm Hg (-2.5 cm H_2O) [.15]. On average, a human has a respiration rate of 28.3 liters per minute (1 cubic ft./min) [11, 12, 13].*

One might well ask what fiber diameter is needed to obtain 50% porosity and still block out virions 100 nm or larger. That question is easily answered though used of Equation (31). It turns out that a fiber diameter of 0.0414 μm or 41.4 nm would be required to both filter out 100 nm virions and provide 50% porosity to avoid suffocation. However, even at 50% porosity it is not clear that normal breathing would be viable. For 90% porosity, which would provide relatively little obstruction, a fiber diameter of 0.005 μm (5 nm) would be needed. Attaining such fine fibers (on the size scale of molecules) commercially seems highly improbable. Further detailed study is required. However, what can be stated unambiguously is that the use of currently available materials is called into question.

To summarize, based on a straightforward analysis, it seems improbable that masks, while effective for dust, pollens and large germs, will ever be useful against virions because of the extremely small size of the virions.. If exclusion of virions is the goal, then the most effective means is a high-grade hazmat suit such as shown in Fig. 10.

Fig. 10. Ideal means for avoiding contact with SARS-COV-II virions. Source: from internet publicly available image.
(https://3.bp.blogspot.com/-F5bzzY_gDVM/WAHyHMXiQ_I/AAAAAAAAASo/_OjwW4RqUX4PutwT80zZPoXrANnbgQrjwCLcB/s1600/Hazmat-Suits-Ebola-Dupont004.jpg)

B. Suggested Experiment

The United States has a network of outstanding national laboratories. It should be very straight forward to use these laboratory assets to carry out a set of experiments using masking fabric, complete masks, irradiated SARS-COV-2 virions and strategically placed sensors. Such tests could be used to determine the ability of the masking material and mask design to filter these virions. The first step should be to definitively determine the effectiveness of current mask materials and designs and put a definitive end to the speculation and contradictory statements that continue to plague this field. Based on these results a rational policy for improving or abandoning masking can be developed.

V. Summary and Conclusions

A quantitative analysis of emission rates of SARS-CoV-2 virions from saliva droplets was performed as a function of droplet size, volume fraction of virions and time. In addition, the possibility of masking as a strategy for filtering these virions was also considered. The following conclusions were drawn:

1. The emission rate of SARS-CoV-2 virions from saliva droplets at a given volume fraction increases with droplet diameter.
2. The emission rate, regardless of the initial droplet size, is a decreasing function of time.

3. It was rigorously shown that the specific evaporation rate *C* in Reference [1] can be replaced by an effective volume-fraction-mediated evaporation rate *C** when virions are uniformly distributed throughout the droplet.
4. The time for complete evaporation was consistent with prior results and increases with increasing droplet size.
5. In comparing the virion emission rates from large and small droplets, the appropriate basis of comparison is populations having the same total mass.
6. When the above approach is used, *populations* of small droplets emit virions much more rapidly than a population containing an equivalent mass of large droplets.
7. It was shown that an equivalent mass and volume fraction of virions of 10 nm droplets emit their virions 100X more rapidly than 100 nm droplets.
8. Based on a simple geometric analysis of a 2D fabric using current 20 μm fibers, it is not possible to filter SARS-CoV-2 virions without suffocating the wearer.
9. Further analysis suggests that in order to have a mask fabric porosity of 50%, the fiber diameter would have to be around 41 nm. If 90% porosity is required to maintain appropriate breathing rates and volumes, then the fiber diameter on the order of 5nm is required. Attaining these sizes presents difficult scientific, technical and economic challenges.
10 Full hazmat suits provide full safety from SARS-CoV-2 virions.
11. An experimental approach for determining the utility of masking as a rational policy is presented. This approach envisions using the advanced capabilities of US national laboratories to address this important issue.

Appendix A
Mathcad Worksheet Showing Calculation of Average Vision Emission Rates for 100 μm saliva Droplets at $f_V = 0.25$

Emission Rates of SARS-COV-2 from Saliva Droplets at RT for Volume Fraction of 0.50

Notes: 1. All inputs are indicated in green
2. Intermediate results are in blue
3. Final results are in red
4. The ORIGIN :=1 command allows subscripting to start at 1,1 instead of t
Mathcad default of 0,0

Input data (all in SI units)

Material: Saliva Droplets

$C1 := -6.84 \cdot 10^{-9}$ $Do := 10^{-4}$ $fv := 0.50$ $tmax := 4.59$ $Nv := 9.549 \times 10^{20}$

Intermediate Calculations

$A := \frac{-C1 \cdot Nv}{4}$ $B := Do^2$ $C := \frac{C1}{\pi}$

$A = 1.633 \times 10^{12}$ $B = 1 \times 10^{-8}$ $C = -2.177 \times 10^{-9}$

$$Ravg := \int_0^{tmax} \frac{A}{tmax} \cdot (B + C \cdot t)^{0.5} \, dt$$

$Ravg = 1.089 \times 10^{8}$

$$Ncalc := \left[\int_0^{tmax} \left(-C1 \cdot \frac{Nv}{4} \right) \cdot \left(Do^2 + C1 \cdot \frac{t}{\pi} \right)^{0.5} dt \right]$$

$Ncalc = 5 \times 10^{8}$

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