Engineering design problems and non-parametric statistical results

I. Tension/compression spring design

Consider: $X = [x_1\ x_2\ x_3] = [d\ D\ N]$,

Minimize: $f(X) = (x_3 + 2)x_2x_1^2$,

Subject to: $g_1(X) = 1 - \dfrac{x_3x_2^3}{71785x_1^4} \leq 0$,

$$g_2(X) = \frac{4x_2^2 - x_1x_2}{12566\left(x_2x_1^3 - x_1^4\right)} + \frac{1}{5108x_1^2} - 1 \leq 0,$$

$$g_3(X) = 1 - \frac{140.45x_1}{x_3x_2^2} \leq 0,$$

$$g_4(X) = \frac{x_1 + x_2}{1.5} - 1 \leq 0,$$

Variable range: $0.05 \leq x_1 \leq 2$,

$0.25 \leq x_2 \leq 1.3$,

$2 \leq x_3 \leq 15$.

II. Pressure vessel design

Consider: $X = [x_1\ x_2\ x_3\ x_4] = [T_s\ T_h\ R\ L]$,

Minimize: $f(X) = 0.6224x_1x_2x_3 + 1.7781x_2x_3^2 + 3.1661x_4x_1^2 + 19.84x_3x_1^2$,

Subject to: $g_1(X) = -x_1 + 0.0193x_3 \leq 0$,

$g_2(X) = -x_3 + 0.00954x_3 \leq 0$,

$$g_3(X) = -\pi x_4x_3^2 - \frac{4}{3}\pi x_3^3 + 1296000 \leq 0,$$

$g_4(X) = x_4 - 240 \leq 0$,

Variable range: $0 \leq x_1 \leq 99$,

$0 \leq x_2 \leq 99$,

$10 \leq x_3 \leq 200$,

$10 \leq x_4 \leq 200$.

III. Welded-beam design

Consider: $X = [x_1\ x_2\ x_3\ x_4] = [h\ l\ t\ b]$,

Minimize: $f(X) = 1.10471x_2x_1^2 + 0.04811x_3x_4(14 + x_2)$,

Subject to: $g_1(X) = \tau(X) - \tau_{max} \le 0$,

$g_2(X) = \sigma(X) - \sigma_{max} \le 0$,

$g_3(X) = x_1 - x_4 \le 0$,

$g_4(X) = 1.10471x_1^2 + 0.04811x_3x_4(14 + x_2) - 5 \le 0$,

$g_5(X) = 0.125 - x_1 \le 0$,

$g_6(X) = \delta(X) - \delta_{max} \le 0$,

$g_7(X) = P - P_c(X) \le 0$,

Variable range: $0.1 \le x_1 \le 2$,

$0.1 \le x_2 \le 10$,

$0.1 \le x_3 \le 10$,

$0.1 \le x_4 \le 2$,

where $\tau(X) = \sqrt{(\tau')^2 + 2(\tau')(\tau'')\frac{x_2}{2R} + (\tau'')^2}$,

$\tau' = \frac{P}{\sqrt{2}x_1x_2},\ \tau'' = \frac{MR}{J},\ M = P\left(L + \frac{x_2}{2}\right)$,

$R = \sqrt{\frac{x_2^2}{4} + \left(\frac{x_1 + x_3}{2}\right)^2}$,

$J = 2\left\{\sqrt{2}x_1x_2\left[\frac{x_2^2}{4} + \left(\frac{x_1 + x_3}{2}\right)^2\right]\right\}$,

$\sigma(X) = \frac{6PL}{x_4x_3^2},\ \delta(X) = \frac{6PL^3}{Ex_4x_3^2}$,

$P_c(X) = \frac{4.013E\sqrt{\frac{x_3^2x_4^6}{36}}}{L^2}\left(1 - \frac{x_3}{2L}\sqrt{\frac{E}{4G}}\right)$,

$\tau_{max} = 13600$ psi, $\sigma_{max} = 30000$ psi, $\delta_{max} = 0.25$ in.,

$P = 6000$ lb, $L = 14$ in., $E = 30 \times 10^6$ psi, $G = 12 \times 10^6$ psi.

IV. Speed reducer design

Consider: $X = [x_1\ x_2\ x_3\ x_4\ x_5\ x_6\ x_7]$,

Minimize: $f(X) = 0.7854x_1x_2^2(3.3333x_3^2 + 14.9334x_3 - 43.0934) - 1.508x_1(x_6^2 + x_7^2) + 7.4777(x_6^3 + x_7^3) + 0.7854(x_4x_6^2 + x_5x_7^2)$,

Subject to:
$$g_1(X) = \frac{27}{x_1x_2^2x_3} - 1 \le 0,$$
$$g_2(X) = \frac{397.5}{x_1x_2^2x_3^2} - 1 \le 0,$$
$$g_3(X) = \frac{1.93x_4^3}{x_2x_3x_6^4} - 1 \le 0,$$
$$g_4(X) = \frac{1.93x_5^3}{x_2x_3x_7^4} - 1 \le 0,$$
$$g_5(X) = \frac{\left(\left(\frac{745x_4}{x_2x_3}\right)^2 + 16.9 \times 10^6\right)^{1/2}}{110x_6^3} - 1 \le 0,$$
$$g_6(X) = \frac{\left(\left(\frac{745x_5}{x_2x_3}\right)^2 + 157.5 \times 10^6\right)^{1/2}}{85x_7^3} - 1 \le 0,$$
$$g_7(X) = \frac{x_2x_3}{40} - 1 \le 0,$$
$$g_8(X) = \frac{5x_2}{x_1} - 1 \le 0,$$
$$g_9(X) = \frac{x_1}{12x_2} - 1 \le 0,$$
$$g_{10}(X) = \frac{1.5x_6 + 1.9}{x_4} - 1 \le 0,$$
$$g_{11}(X) = \frac{1.1x_7 + 1.9}{x_5} - 1 \le 0,$$

Variable range:
$$2.6 \le x_1 \le 3.6,$$
$$0.7 \le x_2 \le 0.8,$$
$$17 \le x_3 \le 28,$$
$$7.3 \le x_4 \le 8.3,$$
$$7.3 \le x_5 \le 8.3,$$
$$2.9 \le x_6 \le 3.9,$$
$$5 \le x_7 \le 5.5.$$

V. Three-bar truss design

Consider: $X = [x_1\ x_2] = [A_1\ A_2]$,

Minimize: $f(X) = (2\sqrt{2}x_1 + x_2) \times l$,

Subject to: $$g_1(X) = \frac{\sqrt{2}x_1 + x_2}{\sqrt{2}x_1^2 + 2x_1x_2} P - \sigma \leq 0,$$

$$g_2(X) = \frac{x_2}{\sqrt{2}x_1^2 + 2x_1x_2} P - \sigma \leq 0,$$

$$g_3(X) = \frac{1}{\sqrt{2}x_2 + x_1} P - \sigma \leq 0,$$

Variable range: $0 \leq x_1,\ x_2 \leq 1$,

where $l = 100$ cm, $P = 2$ KN/cm^2, $\sigma = 2$ KN/cm^2.

Table A.1: Wilcoxon signed-rank test simulation results: artificial protozoa optimizer and 16 comparative algorithms on CEC2022

Fun.	Index	GA	DE	BSA	Jaya	PSO	GWO	WOA	PPE	GSA	MVO	SCA	AOA	TLBO	SDO	SPO	SPBO
F1	p-value	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	7.81E-03	1.73E-06	1.73E-06
	R^+/R^-	465/0	465/0	465/0	465/0	465/0	465/0	465/0	465/0	465/0	465/0	465/0	465/0	465/0	0/212	465/0	465/0
	Winner	+	+	+	+	+	+	+	+	+	+	+	+	+	-	+	+
F2	p-value	1.73E-06	2.07E-02	1.19E-03	1.73E-06	3.61E-03	5.22E-06	3.06E-04	2.77E-03	1.63E-02	4.39E-03	1.73E-06	1.73E-06	2.58E-03	2.39E-04	1.73E-06	2.11E-03
	R^+/R^-	465/0	120/345	397/58	465/0	91/374	454/11	408/57	378/87	59/328	94/371	465/0	465/0	86/379	23/406	465/0	382/83
	Winner	+	-	+	+	-	+	+	+	-	-	+	+	-	-	+	+
F3	p-value	1.73E-06	1.95E-03	3.86E-02	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06
	R^+/R^-	465/0	255/0	245/49	465/0	465/0	465/0	465/0	465/0	465/0	465/0	465/0	465/0	465/0	465/0	465/0	465/0
	Winner	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+
F4	p-value	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06
	R^+/R^-	465/0	465/0	465/0	465/0	465/0	465/0	465/0	465/0	465/0	465/0	465/0	465/0	465/0	465/0	465/0	465/0
	Winner	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+
F5	p-value	1.73E-06	1.73E-06	1.17E-02	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	7.10E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06
	R^+/R^-	465/0	465/0	299/30	465/0	465/0	465/0	465/0	465/0	0/420	465/0	465/0	465/0	465/0	465/0	465/0	465/0
	Winner	+	+	+	+	+	+	+	+	-	+	+	+	+	+	+	+
F6	p-value	1.73E-06	1.73E-06	6.27E-02	1.73E-06	6.98E-06	1.73E-06	1.73E-06	2.13E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	8.47E-06	1.73E-06	1.73E-06
	R^+/R^-	465/0	465/0	323/142	465/0	451/14	465/0	465/0	463/2	465/0	465/0	465/0	465/0	465/0	449/16	465/0	465/0
	Winner	+	+	=	+	+	+	+	+	+	+	+	+	+	+	+	+
F7	p-value	1.73E-06	3.41E-05	4.05E-01	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06
	R^+/R^-	465/0	434/31	273/192	465/0	465/0	465/0	465/0	465/0	465/0	465/0	465/0	465/0	465/0	465/0	465/0	465/0
	Winner	+	+	=	+	+	+	+	+	+	+	+	+	+	+	+	+
F8	p-value	1.73E-06	1.73E-06	2.84E-05	1.73E-06	1.73E-06	1.73E-06	1.73E-06	2.35E-06	1.73E-06	1.13E-05	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06
	R^+/R^-	465/0	465/0	436/29	465/0	465/0	465/0	465/0	462/3	465/0	446/19	465/0	465/0	465/0	465/0	465/0	465/0
	Winner	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+
F9	p-value	1.73E-06	5.00E-01	5.00E-01	1.73E-06	1.71E-06	1.73E-06	1.73E-06	1.73E-06	1.00E+00	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.00E+00	1.73E-06	1.73E-06
	R^+/R^-	465/0	59/0	59/0	465/0	465/0	465/0	465/0	465/0	0/0	465/0	465/0	465/0	465/0	0/0	465/0	465/0
	Winner	+	=	=	+	+	+	+	+	+	+	+	+	+	=	+	+
F10	p-value	1.73E-06	1.31E-01	1.73E-06	1.73E-06	5.79E-05	1.92E-06	1.73E-06	7.71E-04	1.73E-06	3.41E-05	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06
	R^+/R^-	465/0	306/159	465/0	465/0	428/37	464/1	465/0	396/69	465/0	434/31	465/0	465/0	465/0	465/0	465/0	465/0
	Winner	+	=	+	+	+	+	+	+	+	+	+	+	+	+	+	+
F11	p-value	1.73E-06	7.91E-01	6.10E-05	1.73E-06	1.89E-04	1.73E-06	1.15E-04	1.74E-04	1.73E-06	1.73E-06	1.73E-06	1.73E-06	3.50E-02	3.91E-02	1.73E-06	1.24E-05
	R^+/R^-	465/0	205/140	0/345	465/0	414/51	465/0	420/45	415/50	465/0	465/0	465/0	465/0	335/130	208/26	465/0	445/20
	Winner	+	=	-	+	+	+	+	+	+	+	+	+	+	+	+	+
F12	p-value	1.73E-06	1.04E-02	1.11E-03	1.73E-06	1.73E-06	1.92E-06	1.92E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	6.89E-05
	R^+/R^-	465/0	108/357	74/391	465/0	465/0	464/1	464/1	465/0	465/0	465/0	465/0	465/0	465/0	465/0	465/0	426/39
	Winner	+	-	-	+	+	+	+	+	+	+	+	+	+	+	+	+
Win (+)/Draw (=)/Lost (-)		**12/0/0**	**7/3/2**	**7/3/2**	**12/0/0**	**11/0/1**	**12/0/0**	**12/0/0**	**12/0/0**	**10/0/2**	**11/0/1**	**12/0/0**	**12/0/0**	**11/0/1**	**9/1/2**	**12/0/0**	**12/0/0**

Table A.2: Friedman test simulation results: artificial protozoa optimizer and 16 comparative algorithms on CEC2022

Fun.	Index	APO	GA	DE	BSA	Jaya	PSO	GWO	WOA	PPE	GSA	MVO	SCA	AOA	TLBO	SDO	SPO	SPBO
F1	Score	2	11	8	5	14	3	13	7	4	15	6	12	16	10	1	17	9
F2	Score	7	13	4	6	14	1	12	11	9	8	3	15	17	5	2	16	10
F3	Score	1	9	3	2	10	13	7	16	8	14	6	11	17	12	5	15	4
F4	Score	1	11	4	2	15	8	5	14	7	9	6	16	13	10	3	17	12
F5	Score	3	8	5	2	10	11	9	16	12	1	4	13	15	14	7	17	6
F6	Score	1	6	10	2	15	4	12	9	7	5	11	16	14	13	3	17	8
F7	Score	1	5	3	2	12	11	8	14	9	17	7	13	16	10	6	15	4
F8	Score	1	5	4	2	9	13	10	11	7	16	14	12	15	8	3	17	6
F9	Score	3	11	3	3	13	12	14	10	8	3	7	15	17	9	3	16	6
F10	Score	2	6	1	3	13	11	10	14	9	16	12	5	15	7	4	17	8
F11	Score	3	13	2	1	14	10	12	11	5	8	7	15	17	4	6	16	9
F12	Score	3	10	2	1	6	13	7	12	14	15	4	11	17	8	9	16	5
Mean Rank		2.33	9.00	4.08	2.58	12.08	9.17	9.92	12.08	8.25	10.58	7.25	12.83	15.75	9.17	4.33	16.33	7.25
Ranking		**1**	**8**	**3**	**2**	**13**	**9**	**11**	**13**	**7**	**12**	**5**	**15**	**16**	**9**	**4**	**17**	**5**

Table A.3
Parameter setting of the comparison algorithms

Ref.	Algorithm	Parameter settings	Year
[55]	Aquila Optimizer (AO)	$\alpha, \delta = 0.1, r_1 = 10, U = 0.00565, \omega = 0.005$	2021
[56]	Artificial Rabbits Optimization (ARO)	no special parameters	2022
[57]	Coati Optimization Algorithm (COA)	$I = 1$ or 2, $ps = \frac{1}{2}\cdot$ setting ps, because each generation uses $(2 \times ps + 1)$ *FEs*.	2023
[58]	Dandelion Optimizer (DO)	$randn = 1.5$	2022
[59]	Electric Eel Foraging Optimization (EEFO)	no special parameters	2024
[60]	Five Phases Algorithm (FPA)	no special parameters	2023
[61]	Gannet Optimization Algorithm (GOA)	$r = 0.5, q = 0.5, c = 0.2$	2022
[62]	Hippopotamus Optimization (HO)	$T = 0.6, r_6 = 0.5$, $ps = \frac{1}{3}\cdot$ setting ps, because each generation uses $(3 \times ps)$ *FEs*.	2024
[63]	Mountain Gazelle Optimizer (MGO)	$r_i = 1$ or 2, $ps = \frac{1}{4}\cdot$ setting ps, because each generation uses $(4 \times ps)$ *FEs*.	2022
[64]	Marine Predators Algorithm (MPA)	$P = 0.5, FADs = 0.2$, $ps = \frac{1}{2}\cdot$ setting ps, because each generation uses $(2 \times ps)$ *FEs*.	2020
[65]	QUasi-Affine TRansformation Evolutionary (QUATRE)	$F = 0.7$, (QUATRE/Best/1)	2016
[66]	Reptile Search Algorithm (RSA)	$\alpha = 0.1, \beta = 0.1$	2022
[67]	Sea-horse Optimizer (SHO)	$u, v, l = 0.05, r_1 = 0, r_2 = 0.1$, $ps = \frac{2}{3}\cdot$ setting ps, because each generation uses $(1.5 \times ps)$ *FEs*.	2023
[68]	Slime Mould Algorithm (SMA)	$z = 0.03$	2020
[69]	Tunicate Swarm Algorithm (TSA)	$P_{min} = 1, P_{max} = 4$	2020
[70]	Zebra Optimization Algorithm (ZOA)	$R = 0.01, P_s = 0.5$, $ps = \frac{1}{2}\cdot$ setting ps, because each generation uses $(2 \times ps)$ *FEs*.	2022

Table A.4: Experimental results of artificial protozoa optimizer and 16 comparative algorithms on CEC2022

Fun.	Index	APO	AO	ARO	COA	DO	EEFO	FPA	GOA	HO	MGO	MPA	QUATRE	RSA	SHO	SMA	TSA	ZOA
F1	Mean	1.51580E-14	2.80673E+00	1.04213E-13	3.82766E+04	1.80875E-04	2.91796E-13	5.68430E-15	1.61056E-13	1.03217E+03	9.94760E-13	0.00000E+00	0.00000E+00	2.06745E+04	1.04908E+04	1.93063E-04	2.36763E+04	6.86306E+03
	Std	2.5567E-14	1.0919E+00	5.7976E-14	1.2560E+04	1.5411E-04	2.8974E-13	1.7345E-14	6.1700E-14	8.3219E+02	2.0143E-12	0.0000E+00	0.0000E+00	3.0813E+03	3.5408E+03	7.7635E-05	8.5155E+03	3.2833E+03
F2	Mean	4.89099E+01	5.39557E+01	5.59627E+01	1.99017E+03	4.07589E+01	3.40660E+01	4.65683E+01	2.77197E+01	7.05873E+01	3.22845E+01	3.95339E+01	4.33382E+01	1.57356E+03	2.00120E+02	4.59214E+01	4.48160E+02	1.45563E+02
	Std	9.5627E-01	1.6731E+01	9.7287E+00	8.1538E+02	2.2589E+01	2.2150E+01	1.3317E+01	2.3203E+01	3.2444E+01	2.3011E+01	1.8386E+01	1.4789E+01	6.9304E+02	8.4791E+01	1.1771E+01	2.3861E+02	5.5329E+01
F3	Mean	7.57910E-14	1.60093E+01	1.75802E-04	7.83757E+01	1.22864E+01	6.13492E-02	1.34615E-05	2.30424E-02	4.89175E+01	4.20141E+00	4.56277E-04	1.36510E-06	7.36156E+01	1.72944E+01	8.77963E-02	5.82123E+01	4.12801E+01
	Std	5.4509E-14	4.8665E+00	1.8045E-04	7.4403E+00	8.2638E+00	1.0272E-01	5.1569E-05	8.5735E-02	8.6233E+00	3.5565E+00	7.0529E-04	2.7649E-06	8.6904E+00	9.7931E+00	7.1663E-02	8.7687E+00	6.4715E+00
F4	Mean	5.80800E+00	6.04659E+01	3.99973E+01	1.70608E+02	8.15269E+01	3.69129E+01	9.35730E+00	4.08927E+01	6.96469E+01	4.44082E+01	2.74277E+01	3.92434E+01	1.60594E+02	8.25339E+01	5.71534E+01	9.94145E+01	5.71925E+01
	Std	1.9764E+00	1.4983E+01	9.4404E+00	1.3699E+01	2.5113E+01	1.3454E+01	2.4732E+00	1.7935E+01	6.5479E+00	1.5346E+01	7.0144E+00	1.1964E+01	1.0243E+01	1.3093E+01	2.1015E+01	1.2472E+01	1.1917E+01
F5	Mean	2.98430E-03	5.52765E+02	1.07062E+01	2.36441E+03	1.02214E+03	1.37644E+01	1.65694E-01	1.05784E+00	1.13979E+03	2.62082E+02	1.24626E+00	6.95290E-02	2.03567E+03	1.21284E+03	1.98903E+01	1.89342E+03	8.10888E+02
	Std	1.6346E-02	2.3131E+02	1.4042E+01	3.9320E+02	4.0295E+02	1.3117E+01	2.1702E-01	2.1408E+00	2.5079E+02	2.4036E+02	1.0907E+00	1.5764E-01	2.4498E+02	2.4643E+02	5.3544E+01	1.8029E+02	1.7942E+02
F6	Mean	3.51793E+01	1.09985E+04	2.11024E+03	1.98734E+09	3.51628E+03	9.01319E+02	2.82725E+03	2.60956E+03	5.08706E+03	4.32902E+03	8.19354E-01	3.55101E+01	1.27018E+09	1.29429E+05	2.10827E+04	7.32768E+07	4.53003E+06
	Std	2.2617E+01	7.5117E+03	1.9423E+03	1.0137E+09	5.2749E+03	1.1064E+03	4.2855E+03	2.5374E+03	5.5155E+03	4.9431E+03	6.4034E-01	2.2984E+01	1.0251E+09	3.5903E+05	5.1499E+03	8.8395E+07	1.4157E+07
F7	Mean	1.29106E+01	5.73204E+01	3.57952E+01	2.07650E+02	6.04318E+01	3.09663E+01	2.85322E+01	4.60548E+01	1.34070E+02	6.27599E+01	1.91344E+01	2.80398E+01	2.02843E+02	7.79182E+01	3.33001E+01	1.96622E+02	1.03601E+02
	Std	7.5497E+00	1.7728E+01	1.1625E+01	3.5356E+01	3.0013E+01	1.3580E+01	5.0098E+00	1.9940E+01	1.1836E+01	2.0278E+01	7.2363E+00	9.7152E+00	1.9204E+01	2.9663E+01	1.2299E+01	7.0591E+01	1.9709E+01
F8	Mean	1.97540E+01	2.96207E+01	2.11938E+01	2.21562E+02	2.77653E+01	2.06542E+01	2.90710E+01	2.21137E+01	3.66654E+01	2.50722E+01	1.52027E+01	2.17917E+01	2.32881E+02	2.50750E+01	2.13751E+01	1.54614E+02	7.79129E+01
	Std	2.3634E+00	3.9844E+00	4.8156E-01	1.3823E+02	6.2005E+00	1.2223E+00	2.4007E+00	1.1533E+00	7.8004E+00	6.7030E+00	8.3047E+00	4.8323E+00	4.5427E+02	2.1335E+00	4.7497E-01	1.2087E+02	8.4937E+01
F9	Mean	1.80781E+02	1.81119E+02	1.80781E+02	1.00268E+03	1.80786E+02	1.80781E+02	1.80781E+02	1.80781E+02	1.91524E+02	1.80781E+02	1.80781E+02	1.80781E+02	9.56129E+02	2.53010E+02	1.80783E+02	4.21138E+02	2.87188E+02
	Std	8.6723E-14	4.1544E-01	5.1690E-13	2.9544E+02	4.0214E-03	6.0196E-12	8.6723E-14	1.1523E-07	9.4072E+00	3.8939E-10	8.6723E-14	8.6723E-14	2.6806E+02	3.7268E+01	1.8647E-03	1.0166E+02	4.7032E+01
F10	Mean	1.00333E+02	1.22910E+02	1.06795E+02	3.46798E+03	1.38312E+02	1.00398E+02	1.08809E+02	9.70411E+02	1.10538E+03	5.51224E+02	1.00394E+02	1.41029E+02	1.08865E+03	3.34078E+02	2.01316E+02	2.40773E+03	6.11292E+02
	Std	3.6067E-02	5.7092E+01	2.6339E+01	1.6850E+03	9.8466E+01	7.5916E-02	3.2437E+01	7.6908E+02	1.0164E+03	6.6588E+02	6.7109E-02	9.1257E+01	1.3137E+03	3.8601E+02	1.6105E+02	1.2226E+03	8.4394E+02
F11	Mean	3.03333E+02	3.13384E+02	3.13333E+02	5.43145E+03	3.60729E+02	3.23333E+02	3.06667E+02	3.30000E+02	3.45111E+02	3.16667E+02	3.06667E+02	2.93333E+02	5.80540E+03	2.17930E+03	3.56674E+02	2.94355E+03	1.64564E+03
	Std	1.8257E+01	6.2269E+01	7.3030E+01	7.5494E+02	2.4920E+02	4.3018E+01	2.5371E+01	7.9438E+01	1.1904E+02	7.4664E+01	1.4606E+02	1.0807E+02	6.3812E+02	7.8876E+02	8.1720E+01	1.0219E+03	9.6660E+02
F12	Mean	2.37106E+02	2.57582E+02	2.58202E+02	8.77182E+02	2.68072E+02	2.57503E+02	2.42121E+02	2.61334E+02	3.41579E+02	2.56421E+02	2.35626E+02	2.40344E+02	6.40781E+02	3.36473E+02	2.42546E+02	4.82775E+02	7.24071E+02
	Std	3.6438E+00	1.0960E+01	1.3905E+01	2.2915E+02	1.5383E+01	9.2043E+00	3.6629E+00	2.4689E+01	5.5610E+01	1.2973E+01	3.1766E+00	5.6090E+00	3.4921E+02	3.5836E+01	2.8848E+00	1.2846E+02	1.2579E+02
Win/Draw/Lost		**compared**	**12/0/0**	**11/1/0**	**12/0/0**	**11/1/0**	**10/2/0**	**7/4/1**	**11/0/1**	**12/0/0**	**11/1/0**	**6/4/2**	**6/4/2**	**12/0/0**	**12/0/0**	**11/1/0**	**12/0/0**	**12/0/0**
Ranking		**1**	**10**	**6**	**17**	**11**	**5**	**4**	**7**	**12**	**8**	**2**	**3**	**16**	**13**	**9**	**15**	**14**

Table A.5: Wilcoxon signed-rank test simulation results: artificial protozoa optimizer and 16 comparative algorithms on CEC2022

Fun.	Index	AO	ARO	COA	DO	EEFO	FPA	GOA	HO	MGO	MPA	QUATRE	RSA	SHO	SMA	TSA	ZOA
F1	p-value	1.73E-06	3.52E-06	1.73E-06	1.73E-06	2.33E-06	1.80E-01	2.19E-06	1.73E-06	1.70E-06	7.81E-03	7.81E-03	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06
	R^+/R^-	465/0	459/0	465/0	465/0	464/0	52/182	464/0	465/0	465/0	0/212	0/212	465/0	465/0	465/0	465/0	465/0
	Winner	+	+	+	+	+	=	+	+	+	-	-	+	+	+	+	+
F2	p-value	2.43E-02	1.12E-05	1.73E-06	8.94E-01	1.20E-01	2.53E-01	1.96E-03	1.15E-04	3.82E-01	3.78E-01	1.74E-03	1.73E-06	1.73E-06	9.10E-01	1.73E-06	1.73E-06
	R^+/R^-	342/123	446/19	465/0	239/226	157/308	244/101	82/383	420/45	190/275	190/275	56/373	465/0	465/0	238/227	465/0	465/0
	Winner	+	+	+	=	=	=	-	+	=	=	-	+	+	=	+	+
F3	p-value	1.73E-06	2.56E-06	1.73E-06	1.73E-06	1.73E-06	5.57E-06	6.19E-06	1.73E-06	1.73E-06	1.73E-06	1.24E-04	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06
	R^+/R^-	465/0	464/0	465/0	465/0	465/0	459/0	455/0	465/0	465/0	465/0	404/25	465/0	465/0	465/0	465/0	465/0
	Winner	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+
F4	p-value	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	6.89E-05	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06
	R^+/R^-	465/0	465/0	465/0	465/0	465/0	426/39	465/0	465/0	465/0	465/0	465/0	465/0	465/0	465/0	465/0	465/0
	Winner	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+
F5	p-value	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	2.46E-04	1.73E-06	1.73E-06	1.73E-06	3.50E-06	7.46E-01	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06
	R^+/R^-	465/0	465/0	465/0	465/0	465/0	373/26	465/0	465/0	465/0	459/5	165/264	465/0	465/0	465/0	465/0	465/0
	Winner	+	+	+	+	+	+	+	+	+	+	=	+	+	+	+	+
F6	p-value	1.73E-06	1.92E-06	1.73E-06	1.73E-06	1.92E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	9.10E-01	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06
	R^+/R^-	465/0	464/1	465/0	465/0	464/1	465/0	465/0	465/0	465/0	0/465	238/227	465/0	465/0	465/0	465/0	465/0
	Winner	+	+	+	+	+	+	+	+	+	-	=	+	+	+	+	+
F7	p-value	1.73E-06	1.73E-06	1.73E-06	1.73E-06	3.52E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.59E-03	6.34E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06
	R^+/R^-	465/0	465/0	465/0	465/0	458/7	465/0	465/0	465/0	465/0	386/79	452/13	465/0	465/0	465/0	465/0	465/0
	Winner	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+
F8	p-value	1.73E-06	1.92E-06	1.73E-06	1.73E-06	2.83E-04	1.73E-06	4.73E-06	1.73E-06	1.73E-06	1.78E-01	1.83E-03	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06
	R^+/R^-	465/0	464/1	465/0	465/0	409/56	465/0	455/10	465/0	465/0	167/298	384/81	465/0	465/0	465/0	465/0	465/0
	Winner	+	+	+	+	+	+	+	+	+	=	+	+	+	+	+	+
F9	p-value	1.73E-06	1.14E-06	1.73E-06	1.73E-06	4.17E-07	1.00E+00	6.01E-05	1.73E-06	1.73E-06	4.32E-08	1.00E+00	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06
	R^+/R^-	465/0	459/0	465/0	465/0	465/0	0/0	399/0	465/0	465/0	465/0	0/0	465/0	465/0	465/0	465/0	465/0
	Winner	+	+	+	+	+	=	+	+	+	+	=	+	+	+	+	+
F10	p-value	1.73E-06	1.97E-05	1.73E-06	9.84E-03	3.59E-04	4.28E-02	4.07E-05	1.73E-06	1.02E-05	6.64E-04	4.90E-04	1.73E-06	1.73E-06	5.32E-03	1.73E-06	1.73E-06
	R^+/R^-	465/0	440/25	465/0	358/107	406/59	134/331	432/33	465/0	447/18	398/67	402/63	465/0	465/0	368/97	465/0	465/0
	Winner	+	+	+	+	+	-	+	+	+	+	+	+	+	+	+	+
F11	p-value	2.83E-04	1.09E-01	1.73E-06	1.60E-04	7.03E-02	6.25E-01	5.65E-04	1.48E-04	2.97E-05	4.51E-01	8.36E-01	1.73E-06	1.73E-06	3.11E-05	1.73E-06	1.73E-06
	R^+/R^-	409/56	203/52	465/0	416/49	186/26	85/29	399/30	417/48	435/30	280/140	164/110	465/0	465/0	435/30	465/0	465/0
	Winner	+	=	+	+	=	=	+	+	+	=	=	+	+	+	+	+
F12	p-value	1.73E-06	1.92E-06	1.73E-06	1.73E-06	1.92E-06	2.61E-04	1.73E-06	1.73E-06	1.73E-06	1.31E-01	2.43E-02	1.73E-06	1.73E-06	2.16E-05	1.73E-06	1.73E-06
	R^+/R^-	465/0	464/1	465/0	465/0	464/1	410/55	465/0	465/0	465/0	159/306	342/123	465/0	465/0	439/26	465/0	465/0
	Winner	+	+	+	+	+	+	+	+	+	=	+	+	+	+	+	+
Win (+)/Draw (=)/Lost (-)		**12/0/0**	**11/1/0**	**12/0/0**	**11/1/0**	**10/2/0**	**7/4/1**	**11/0/1**	**12/0/0**	**11/1/0**	**6/4/2**	**6/4/2**	**12/0/0**	**12/0/0**	**11/1/0**	**12/0/0**	**12/0/0**

Table A.6: Friedman test simulation results: artificial protozoa optimizer and 16 comparative algorithms on CEC2022

Fun.	Index	APO	AO	ARO	COA	DO	EEFO	FPA	GOA	HO	MGO	MPA	QUATRE	RSA	SHO	SMA	TSA	ZOA
F1	Score	4	11	5	17	9	7	3	6	12	8	1.5	1.5	15	14	10	16	13
F2	Score	9	10	11	17	5	3	8	1	12	2	4	6	16	14	7	15	13
F3	Score	1	11	4	17	10	7	3	6	14	9	5	2	16	12	8	15	13
F4	Score	1	11	6	17	13	4	2	7	12	8	3	5	16	14	9	15	10
F5	Score	1	10	6	17	12	7	3	4	13	9	5	2	16	14	8	15	11
F6	Score	2	11	5	17	8	4	7	6	10	9	1	3	16	13	12	15	14
F7	Score	1	9	7	17	10	5	4	8	14	11	2	3	16	12	6	15	13
F8	Score	2	12	4	16	10	3	11	7	13	8	1	6	17	9	5	15	14
F9	Score	4.5	11	4.5	17	10	4.5	4.5	4.5	12	4.5	4.5	4.5	16	13	9	15	14
F10	Score	1	6	4	17	7	3	5	13	15	11	2	8	14	10	9	16	12
F11	Score	2	6	5	16	12	8	3.5	9	10	7	3.5	1	17	14	11	15	13
F12	Score	2	8	9	17	11	7	4	10	13	6	1	3	15	12	5	14	16
Mean Rank		2.54	9.67	5.88	16.83	9.75	5.21	4.83	6.79	12.50	7.71	2.79	3.75	15.83	12.58	8.25	15.08	13.00
Ranking		**1**	**10**	**6**	**17**	**11**	**5**	**4**	**7**	**12**	**8**	**2**	**3**	**16**	**13**	**9**	**15**	**14**