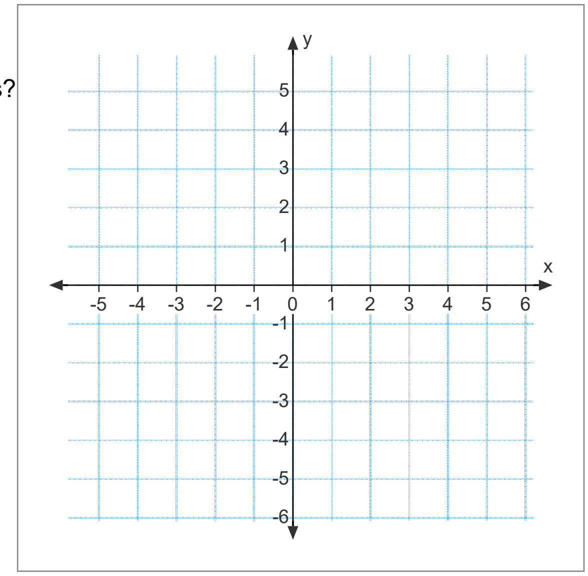
1. Suppose $A = (1, 3)$. On one set of axes, plot these points.
   a. $2A$ = (2, 6) b. $3A$ c. $5A$
   d. $(-1)A$ e. $(-3)A$ f. $(-6.5)A$

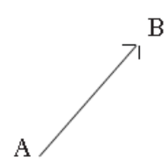
What do you notice about all of these points?



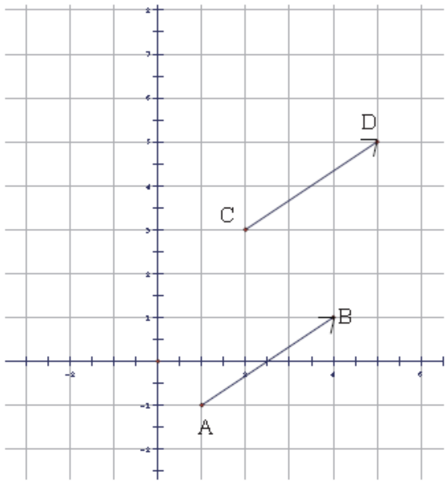
Definition

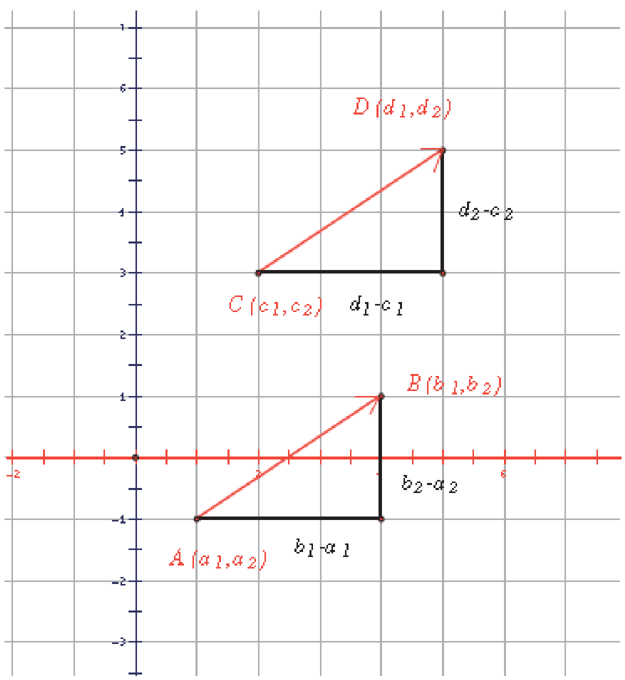
If A and B are points in $\mathbb{R}^n$, the **vector** with tail A and head B is the ordered pair of points (A, B). You can denote the vector (A, B) by $\overrightarrow{AB}$.

A vector is a directed line segment that is usually represented by drawing an arrow. The arrow has a length (or magnitude), and one end has an arrowhead that denotes the direction the arrow is pointing. In this figure, the two endpoints of the line segment are labeled A and B. If you know the two endpoints, you can completely describe the vector. This vector starts at A and ends at B, so it is denoted by $\overrightarrow{AB}$. The point A is called the **tail** (or **initial point**) of $\overrightarrow{AB}$, and the point B is called the **head** (or **terminal point**) of $\overrightarrow{AB}$.



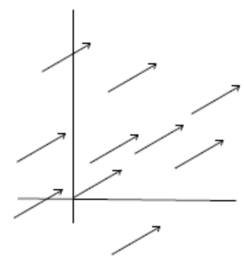
In $\mathbb{R}^2$ or $\mathbb{R}^3$, two vectors are called **equivalent** if they have the same magnitude (length) and the same direction. For example, in the figure below, $A = (1, -1), B = (4, 1), C = (2, 3)$, and $D = (5, 5)$. $\overrightarrow{AB}$ is equivalent to $\overrightarrow{CD}$.





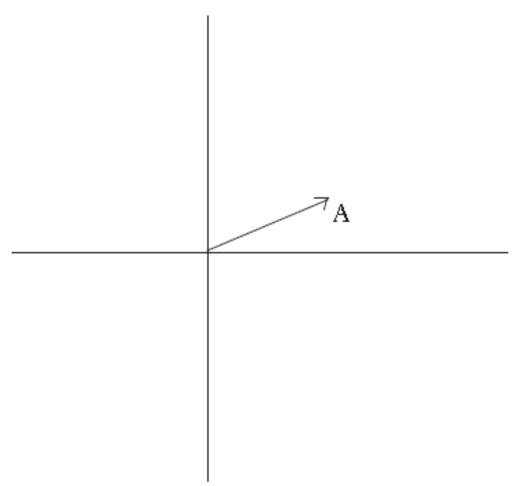
$D - C = B - A$

We break up vectors into equivalence classes. All vectors which have the same length and direction are said to be equivalent.
They are all represented by the vector which starts at the origin.
(Whose tail is at the origin)



Which vector in the picture above is the representative of this class?

2. Here's a picture of a point A, with an arrow drawn from the origin to A.



Draw these vectors, all on the same axes:

a. $2A$ b. $3A$ c. $5A$
d. $(-1)A$ e. $(-3)A$ f. $(-6.5)A$

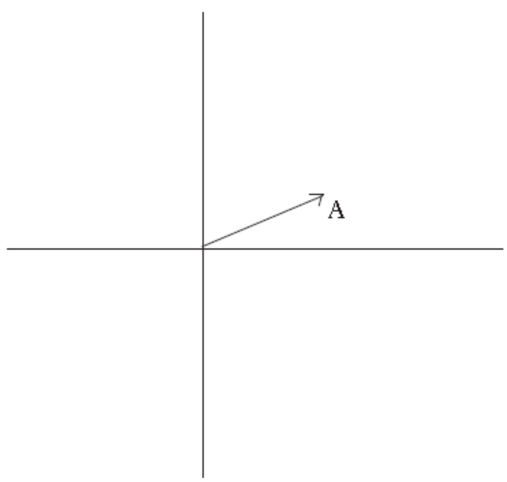
<u>Getting the Brain Ready for Class: Discussion</u>

1) What was the definition of vector?

2) Is the vector from (1,3) to (4,5) the same or different than the vector (3,2)? Explain.

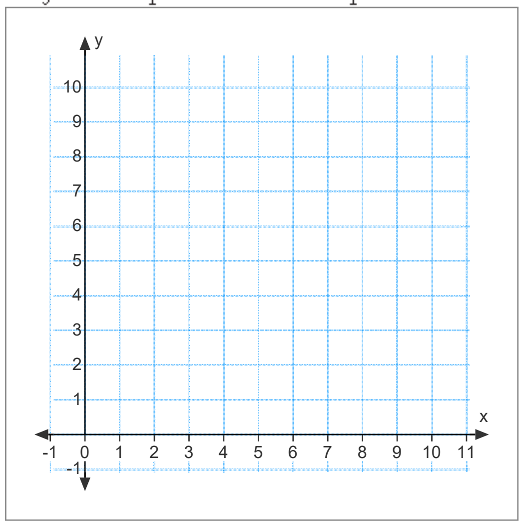
3) If A is a vector, what does 5A do to it? 1/2A?

Draw the set $L = \{ tA \mid t \in \mathbb{R} \}$



a. Describe in words the set of all multiples tA where t ranges over all real numbers.

4. Suppose $O = (0, 0)$, $A = (5, 3)$ and $B = (3, -1)$. Show that O, A, B, and $A + B$ lie on the vertices of a parallelogram. It may be helpful to draw a picture.



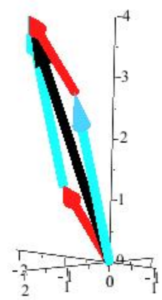
Now suppose (a,b) and (c,d) are vectors on the plane.

What can you say about (a,b), (c,d), (0,0), and (a+c, b+d) ?

How would you be able to verify this?

Parallelgram Rule of Vector Addition

If **$\vec{v}$ and $\vec{w}$ are vectors, then $\overrightarrow{v+w}$ si the diagonal of the parallelogram generated by $\vec{v}$ and $\vec{w}$.**

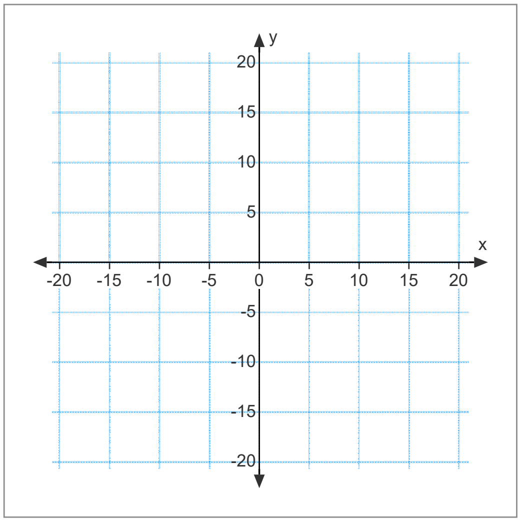


The sum of 2 vectors, showing the resultant in *black* and the parallelogram(s) of addition.

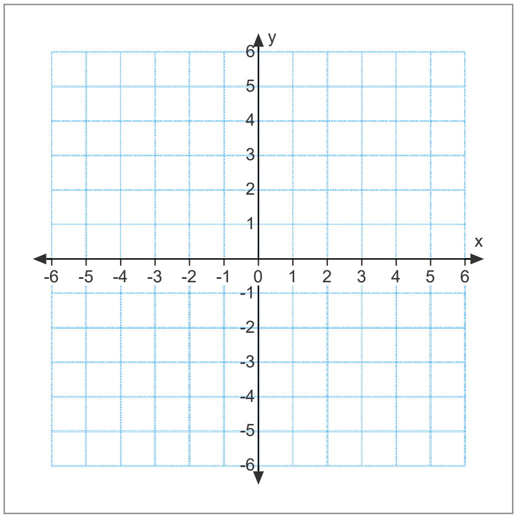
6. Suppose $A = (5, 3)$ and $B = (3, -1)$. Find and plot these points, all on the same axes:

a. $A + B$ b. $A + 3B$ c. $A + 5B$
d. $A + (-1B)$ e. $A + (-3B)$ f. $A + (-6.5B)$

Habits:
Draw a picture!!



7. Suppose $A = (5, 3)$ and $B = (3, -1)$. Find a coordinate equation for the set of points X that is generated by $A + tB$, where t ranges over all real numbers.



Theorem 1.2

(The Basic Rules of Arithmetic with Points)

Let $A = (a_1, a_2, \ldots, a_n)$, $B = (b_1, b_2, \ldots, b_n)$ *and* $C = (c_1, c_2, \ldots, c_n)$ *be points in* $\mathbb{R}^n$ *and let* d *and* e *be scalars. Then:*

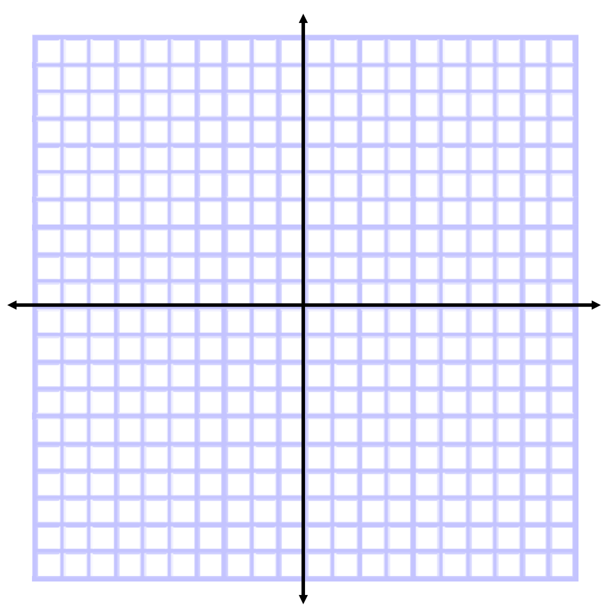
1. $A + B = B + A$
2. $A + (B + C) = (A + B) + C$
3. $A + O = A$
4. $A + (-1)A = O$
5. $(d + e)A = dA + eA$
6. $d(A + B) = dA + dB$
7. $d(eA) = (de)A$
8. $1A = A$

1. Let $A = (3, 1), B = (2, -4), C = (1, 0)$. Calculate and plot the following.

   a. $A + 3B$ b. $2A - C$

   c. $A + B - 2C$ d. $-A + \frac{1}{2}B + 3C$

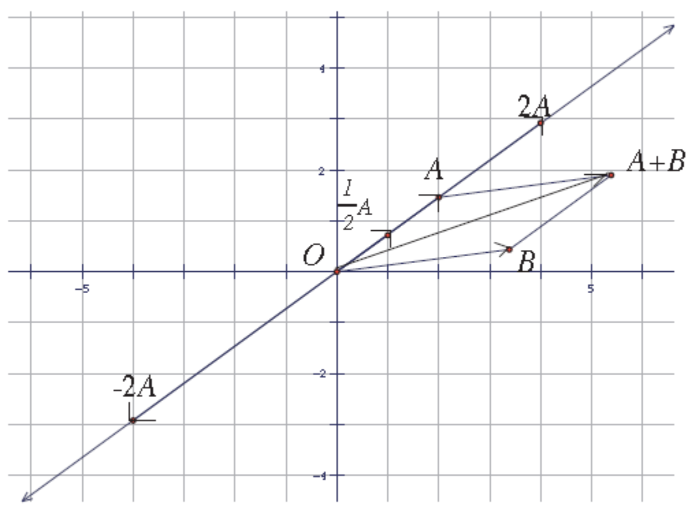
   e. $\frac{1}{2}(A + B) + \frac{1}{2}(A - B)$



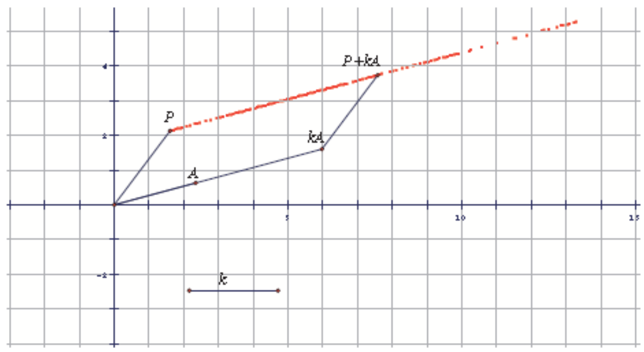
<u>Brain Break - </u>Get up and push in your chairs.

1. Wave to the person diagonally from you.
2. Turn to the person next to you.
3. Front high-five both hands.
4. Side low-five right hands.
5. Low fist-bump right hands.
6. Side low-five left hands.
7. Low fist-bump left hands.
8. Rinse, Lather, REPEAT (....faster!)

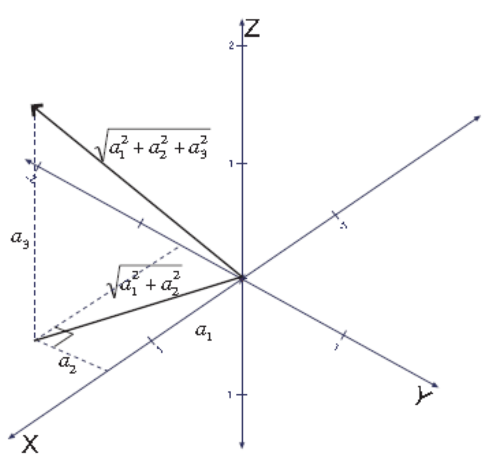
Use vectors to describe geometric ideas. If points in $\mathbb{R}^2$ and $\mathbb{R}^3$ are viewed as vectors, the geometric description of addition and scalar multiplication is much easier.



Use the vector equation as a point-generator. You can use the equation $X = P + kA$ to generate points: consider a "slider" of length k that you can control with your mouse. As you change the length of the slider, A gets scaled by k and added to P. The varying $P + kA$ traces out ℓ,



Every value of k generates a point on the line, so the equation $X = P + kA$ is a kind of "machine" that takes in numbers k and produces points on the line through P in the direction of A.



Definition

Let $A = (a_1, a_2, \ldots, a_n)$ be a vector in $\mathbb{R}^n$. The **length** of A, written $\|A\|$, is given by the formula

$$\|A\| = \sqrt{a_1^2 + a_2^2 + \cdots + a_n^2}$$

Notice that while A is a vector, $\|A\|$ is a **number**.

Theorem 1.5

Let A and B be vectors in $\mathbb{R}^n$ and let c be a real number. Then:

1. $\|A\| \geq 0$, *and* $\|A\| = 0$ *if and only if* $A = O$.
2. $\|cA\| = |c|\,\|A\|$.
3. $\|A + B\| \leq \|A\| + \|B\|$.

Theorem 1.6

Let A be a non-zero vector in $\mathbb{R}^n$. There is a vector in the same direction as A with length 1; in fact, this vector is $\frac{1}{\|A\|}A$.

Definition

If A and B are points in $\mathbb{R}^n$, the distance between A and B, written $d(A, B)$, is defined by the equation

$$d(A, B) = \|B - A\|$$