Many problems in physics and in real-world applications involve particles moving through space. How would we model the path of a particle through space with a function? In order to define any function, it is necessary, first, to define the domain and range of the function. Since the particle is moving through space, then it makes sense to think of the position of the particle in space to be dependent on the time at which the particle is in said position. Therefore, the set of input values would have to be continuous, real-valued, and finite in length, just like an interval of time. The set of output values would have to include three-coordinates to give the position of the particle.

Hence, if *r* is a function which models a path of a particle in space, then it must be

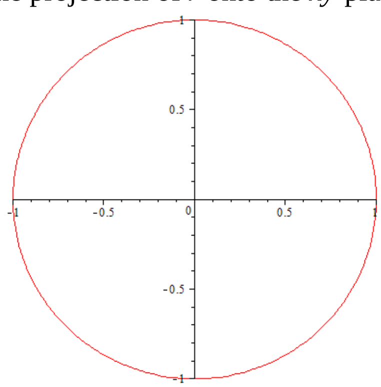
$$r:[a,b]\to\mathbb{R}^3$$

and $r(t)=(x(t),y(t),z(t))$ would be a vector-valued function on an interval $I=[a,b]$ (it is convention to use the capital letter *I* for a finite interval). The points $\{(x,y,z)\mid x=x(t),y=y(t),z=z(t),t\in\mathbb{R}\}$ consist of the ***curve*** of the particle's ***path.*** The equation and the interval are said to ***parametrize*** the curve. The coordinate functions are called ***component functions*** of the position vector.

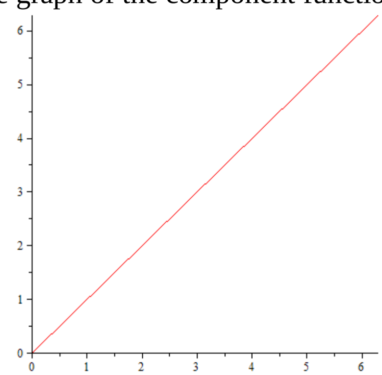
Example 1: The Helix

Consider the function $r:[0,6\pi]\to\mathbb{R}^3$ be defined by $r(t)=(\cos(t),\sin(t),t)$. In order to realize the graph of this function, we could analyze the behavior by graphing the projection onto the *xy*-plane (giving us a 2-dimensional graph), and graphing the component function *z*(*t*).

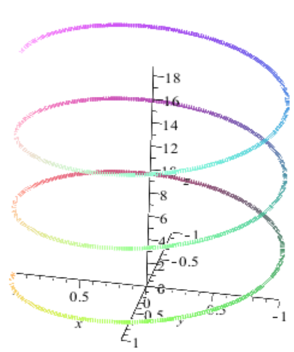
The projection of *r* onto the *xy*-plane



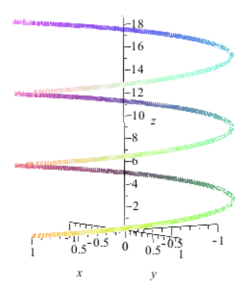
The graph of the component function *z*



The top view of the graph



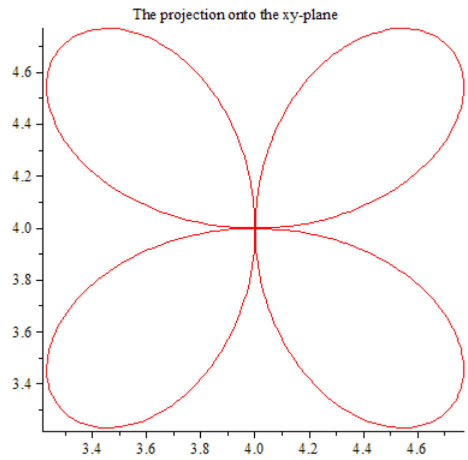
The side view of the graph



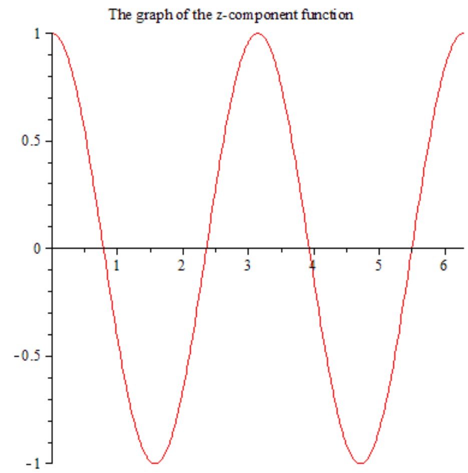
As we can see from the graph of the projection onto the *xy*-plane, the position function is circular from the top-view and rotates counter-clockwise around the origin as *t* increases. From the graph of the component function, we can see that the graph rises continuously as *t* increases. Therefore, the path is moving upward in a circular motion.

Example 2:

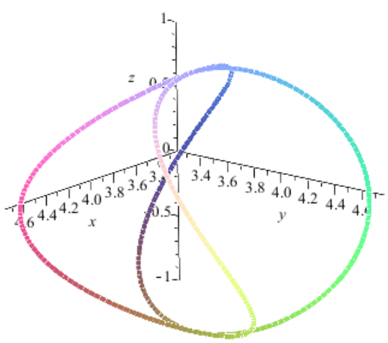
If r is defined by the component functions, $r(t) = (4 + \sin(2t)\cos(t), 4 + \sin(2t)\sin(t), \cos(2t))$, but on the same interval, how will it's graph change?

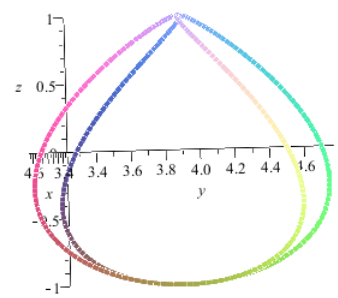


The top view of the graph



The side view of the graph





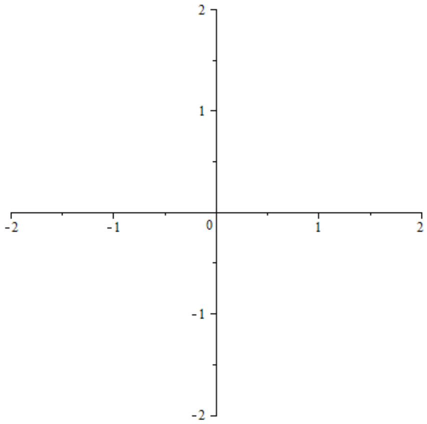
As you can see from the projection onto the xy-plane, the graph is a four-petal flower wrapped around the line $x = y = 4$. The z-coordinate oscillates between -1 and 1 as t increases, reaching a maximum at 0, π and 2π. As you can see, this is an efficient way to learn how to graph space curves. In the next two exercises, graph the projection of the space curves onto the xy-plane, then graph the z-component function. Finally, try to incorporate all the information into the graph of the space-curve.

After you are done with the graph in the axes below, use maple to verify your work. You will need to write the following commands:

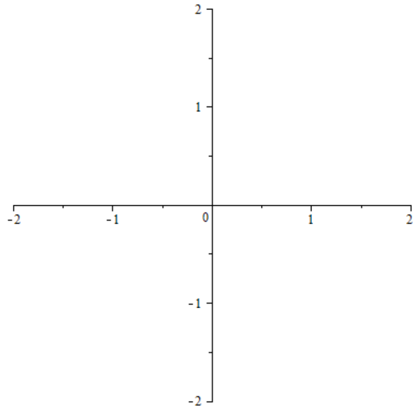
Exercises: Assume all vector-valued functions are defined on the interval $[0, 2\pi]$.

1) $r(t) = (\sin(3t)\cos(t), \sin(3t)\sin(t), t)$

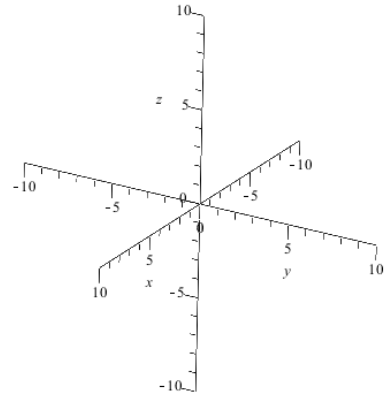
The projection onto the xy-plane



The graph of the z-component function

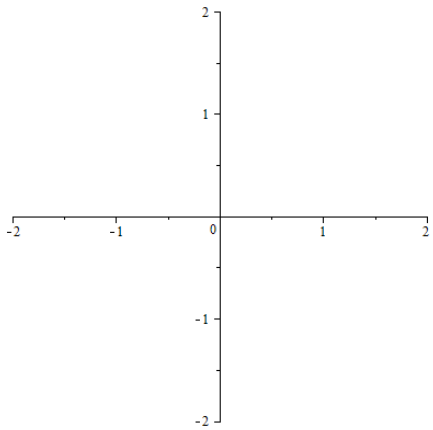


The graph of the curve



2) $r(t) = (\cos(t), \sin(t), \sin(2t))$

The projection onto the xy-plane



The graph of the z-component function