A polynomial is a function which is the sum of **real multiples** of **nonnegative integral** powers of *x*. For example,

$$p(x) = 5x^4 - \sqrt{2}x^3 + \pi x - \frac{4}{3}$$

is a polynomial. However, $f(x) = 2^x$, $g(x) = \frac{x-2}{x+3}$, $h(x) = x^{\frac{1}{2}} - x^{-2}$ are NOT polynomials.

The function *f* is not a polynomial because ____

The function *g* is not a polynomial because ____

The function *h* is not a polynomial because ____

In general we write

$$p(x) = a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \cdots + a_2x^2 + a_1x + a_0$$

$$p(x) = 5x^4 - \sqrt{2}x^3 + \pi x - \frac{4}{3}$$

In our example above, $n = 4$ because the highest power of *x* is 4.

We call the highest power of *x* in a polynomial, the **degree** of the polynomial, and write $\deg(p) = 4$

The *a*'s with subscripts are the real number multiples of the powers of *x*, we call these the **coefficients** of the powers of *x*.

$$p(x) = 5x^4 - \sqrt{2}x^3 + \pi x - \frac{4}{3}$$

For example, the coefficient of *x* in the example above is π . So, $a_1 = \pi$.

The coefficient of x^3 is $-\sqrt{2}$, so $a_3 = -\sqrt{2}$.

Identify: $a_4 =$ $a_0 =$

Checking for understanding:

Which of the functions below are polynomials?	Identify the degrees of the polynomials below.
$a)\ f(x) = \sqrt{x}$	$a)\ f(x) = x^5 - x^7 + 3x^2 - 1$
$b)\ g(x) = 3x - 1$	$b)\ g(x) = 5x + 2$
$c)\ h(x) = \frac{x^2 - 3x + 2}{5}$	$c)\ h(x) = (x-1)(7x+2)$
$d)\ p(x) = \frac{2x^3 - 3x^2 + 1}{x - 1}$	$d)\ p(x) = (x+1)(x-5)^2(x-10)$

The **leading coefficient** is the coefficient of the highest power of *x*.

The **constant coefficient** is the coefficient that corresponds to x^0 .

	Leading coefficient	*Constant Coefficient*
$a)\ f(x) = x^5 - x^7 + 3x^2 - 1$		
$b)\ g(x) = 5x + 2$		
$c)\ h(x) = (x-1)(7x+2)$		
$d)\ p(x) = (x+1)(x-5)^2(x-10)$		

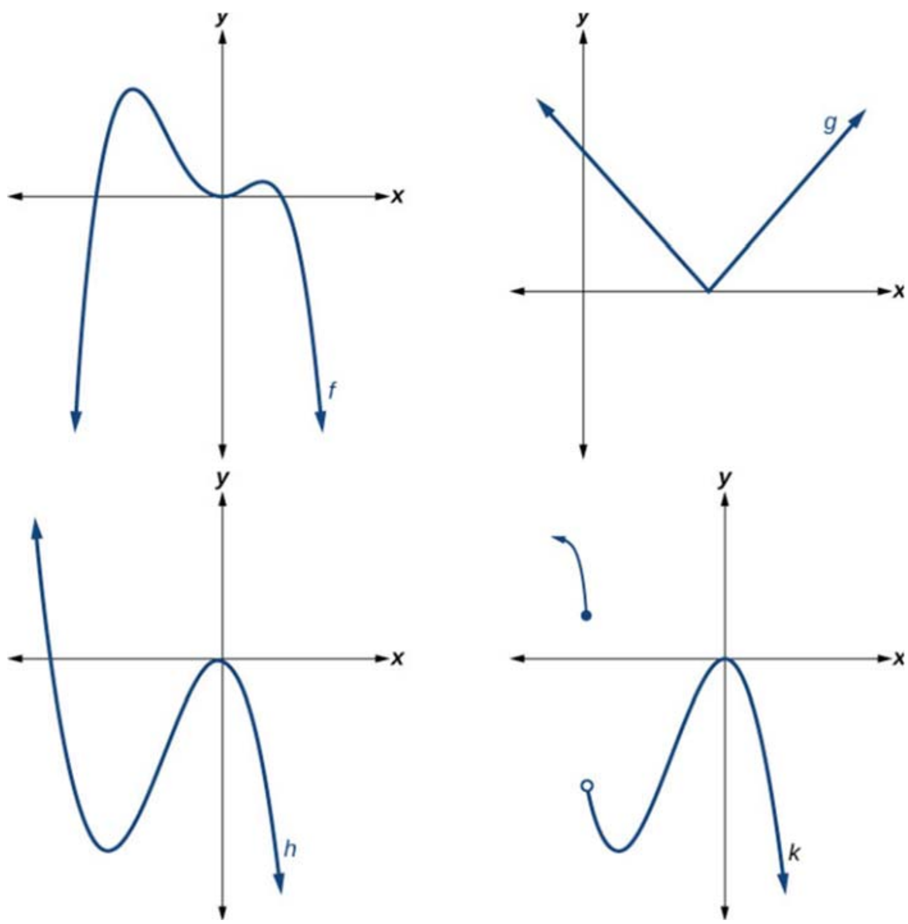
Graphing Polynomials

Since polynomials are created by multiplying nonnegative integer powers of *x* by real numbers, then adding those powers of *x* together, no math rules can be violated by a polynomial.

The domain of any polynomials is $\mathbb{R}$.

The graphs of polynomials with degree > 1, the graphs are smooth, rounded, do not break, and have no holes.

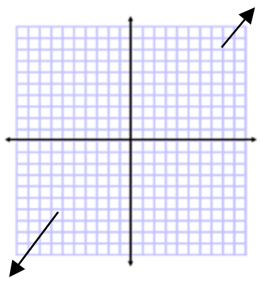
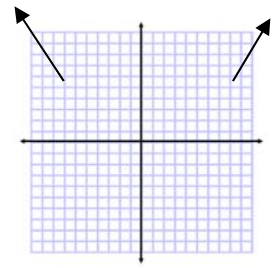
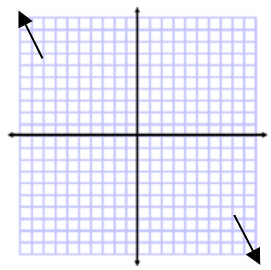
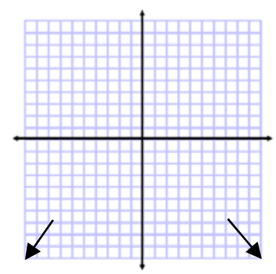
Which of the graphs below represent polynomials?



The **zeros** of a function, *f*, are the values of *x*, at which $f(x) = 0$.
Since $y = f(x)$, then the **zeros** of *f* are where $y = 0$.
This means that the **zeros** of *f* are also the ***x-intercepts*** of the graph of *f*.
The values of *x* at which a polynomial is 0 are also called the **roots** of the polynomial.

zeros = *x*-intercepts = roots

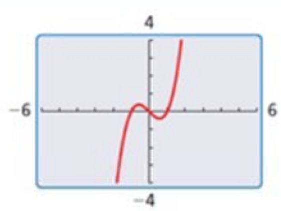
Lastly, the end behavior of the graph of a polynomial can be determined by its degree and the sign of its leading coefficient.

degree of polynomial	odd degree	even degree
Positive leading coefficient **End behavior**		
Negative leading coefficient **End Behavior**		

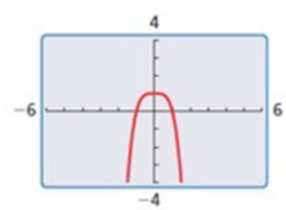
Match each polynomial function with its graph. Explain your reasoning.

1. $f(x) = x^3 - x$
2. $f(x) = -x^3 + x$
3. $f(x) = -x^4 + 1$
4. $f(x) = x^4$
5. $f(x) = x^3$
6. $f(x) = x^4 - x^2$

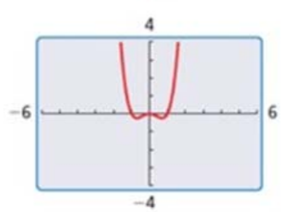
A.



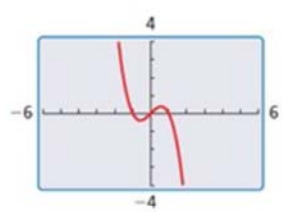
B.



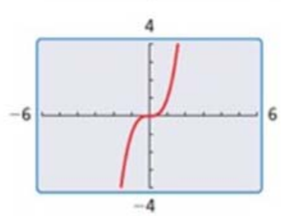
C.



D.



E.



F.