<u>Rational Roots Theorem:</u>

Let p be the polynomial below.

$$p(x) = (3x-2)(5x+7)$$

We can see that the roots of $p(x)$ are ________________ and _______________

List the denominators of the roots: __________________________________.

List the numerators of the roots: ____________________________________.

Distribute the polynomial completely $p(x) = (3x-2)(5x+7)$

The leading coefficient of $p(x)$ is ________________ whose factors are ______________________.

The constant coefficient of $p(x)$ is ________________ whose factors are _____________________.

Note that the denominators of the roots are factors of the _________________coefficient,

The numerators of the roots are factors of the ____________________ coefficient.

Why do you think this is true? Discuss with your table, then write down reasons your table (including you) gave:

I ____

II ____

III ____

This can be generalized for any number of factors.

<u>Rational Roots Theorem:</u>

Let $f(x) = a_n x^n + a_{n-1}x^{n-1} + \ldots + a_2 x^2 + a_1 x + a_0$, then $\frac{p}{q}$ is a possible rational root of $f(x)$ if p is a factor of a_0 and q is a factor of a_n

Example: List all possible rational roots of the polynomial $15x^4 - 2x^3 + 14x^2 - 2x - 1$.

Example 2:

$$f(x) = x^3 - 4x^2 + 7x - 6$$

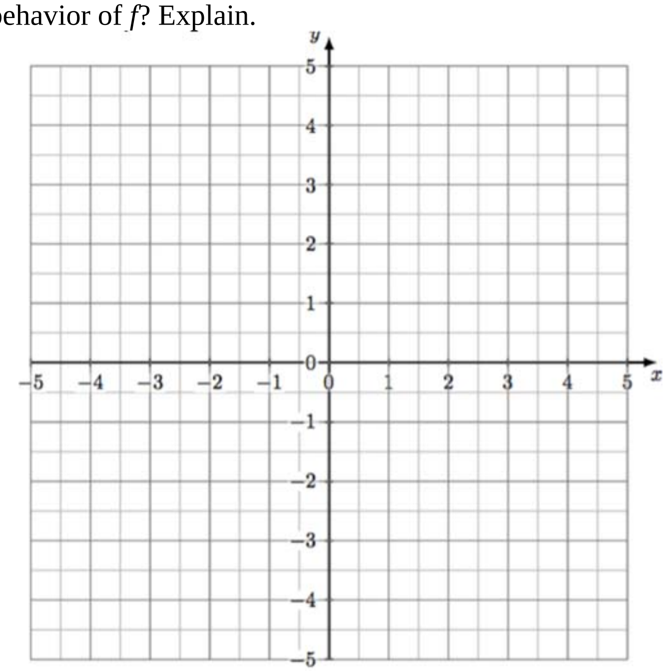
a) Use the Rational Roots Theorem to find all possible rational roots of *f*.

b) Find out which (if any) of the possible rational roots in part (a) are actual roots of *f* . Then use Synthetic Division to write *f* as the product of a linear factor (degree 1) and a quadratic factor (degree 2). In other words, you want to find *a, b, c, d* such that

$$f(x) = x^3 - 4x^2 + 7x - 6 = (x - d)(ax^2 + bx + c)$$

c) Find all real roots of *f* and list their multiplicities.

d) What is the end behavior of *f*? Explain.



Complex Conjugate Theorem:

The quadratic formula gives us the roots of quadratics are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

If the discriminant, $b^2 - 4ac$, is negative, then our roots are complex conjugates.

For example, if we needed to find the roots of $f(x) = x^2 + 4x + 5$, the quadratic formula would give us

$$x = \frac{-4 \pm \sqrt{4^2 - 4(1)(5)}}{2(1)}$$

What are the roots, when simplified?

Fundamental Theorem of Algebra:

The fundamental theorem of algebra states that if $p(x) = a_0 + a_1x + a_2x^2 + \cdots a_nx^n$ is a nonconstant polynomial (deg (*p*) > 1) with real coefficients, then it must have at least one complex root.

How does this show that $p(x) = a_0 + a_1x + a_2x^2 + \cdots a_nx^n$ is a *n*-degree polynomial with real coefficients, then it must have *n* roots (real or complex)?

Putting both theorems and the Factor Theorem together, we can find a polynomial with lowest degree whose roots are

$$2i, -3, \frac{1}{2}$$

By the Complex Conjugate Theorem, we know that if 2*i* is a root, then _______ must also be a root.

By the Factor Theorem, we know that if *x* = -3 is a root, then ______________ is a factor.

If ____________ is a root, then __________________ is a factor

If ____________ is a root, then __________________ is a factor

If ____________ is a root, then __________________ is a factor

Therefore, the polynomial must be
____\

Descartes Rules of Signs

I never paid much attention to this theorem, but it may help you guys. The Descartes Rules of Signs says that

- the number of sign changes in the coefficients of $f(x)$ tells you the maximum number of positive real roots of $f(x)$
- the number of sign changes in the coeffecients of $f(-x)$ tells you the maximum number of negative real roots that of $f(x)$

Advanced Theorems for Math Nerds: Coefficients and Roots

Suppose f is a polynomial with leading coefficient 1 of degree n with real coefficients. By the Fundamental Theorem of Algebra, it has n complex roots. Let us look at f in terms of its coefficients and its roots simultaneously.

$$f(x) = a_0 + a_1 x + a_2 x^2 + \cdots + x^n$$
$$f(x) = (x - r_1)(x - r_2)(x - r_3)\cdots(x - r_n)$$

If we multiply the roots out you will see that

$$f(x) = x^n - (r_1 + r_2 + r_3 + \cdots + r_n)x^{n-1} + (r_1 r_2 + r_1 r_3 + \cdots r_2 r_3 + \cdots)x^{n-2} + \cdots + (-1)^n (r_1 r_2 r_3 \cdots r_n)$$

The constant coefficient gives us the product of all the roots. The coefficient of n -1 gives us the sum of all the roots, and so forth. As you can see, there are signs to consider, but they are easily computed by taking -1 to the power of the number of roots you are multiplying together.

Challenge Problems: (2 pts extra credit each (on quiz or test) or hwk pass solution WITH explanations): Must be done on a separate sheet of paper.

1) The equations with roots $3+\sqrt{2}, 3-\sqrt{2}, 3+i\sqrt{2}, 3-i\sqrt{2}$ is in the form $x^4 + ax^3 + bx^2 + cx + d = 0$, find $a+b+c+d$,

2) A polynomial p contains only terms of odd degree. When p is divided by x - 3, the remainder is 6. What is the remainder when p is divided by $x^2 - 9$.

3) Suppose $p(x) = a_0 + a_1 x + a_2 x^2 + \cdots a_n x^n$ is a polynomial with integer coefficients. Show if $p(0)$ and $p(1)$ are both odd, then p has no integer roots

4) If $x^4 - 2x^3 + x^2 - 8x + r$ is divisible by $x - 2i$ and $x - 1$, then what is r ?

5) Solve the $(x+1)(x+2)(x+3)(x+4) = -1$.

6) Give the remainder when $x^{203} - 1$ is divided by $x^4 - 1$.