Bases

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Name:

Have you ever wondered what it means to be the number 17845? Your first grade teacher told you about 5 being in the *one's* place, and 4 being in the *hundreds* place, but why?

We think in base 10. What does this mean? All of the digits in the number 17845 signify a *coefficient* in front of a power of 10. For instance,

$$17845 = 1 \cdot (10^4) + 7 \cdot (10^3) + 8 \cdot (10^2) + 4 \cdot (10^1) + 5 \cdot (10^0)$$

Some theorize that we like base 10 because we have 10 fingers (at least, most of us do) and our toes are usually covered up.

The Mayans, however, use base 20. This may have been because in their hot weather, there toes were usually visible, and, hence, useful for counting.

Indeed, there was an African tribe that also used base 20, and whose word for 40 coincided with the word for *bed*. You can make your own inference from that.

What would our numbers be like if we had to use a different base. Programmers might require the use of base 2, 8 or 16.

The next exercise is designed to open your minds to other bases:

Example 1. Convert the number 113 base 2 into base 10.

$$\begin{aligned} 113 &= 1 \cdot 2^2 + 1 \cdot 2^1 + 3 \cdot 2^0 \\ &= 1 \cdot 4 + 1 \cdot 2 + 3 \\ &= 4 + 2 + 3 \\ &= 9 \end{aligned}$$

Convert the number 76 base 10 into base 3.
$3^0 = 1 \qquad 3^1 = 3 \qquad 3^2 = 9 \qquad 3^3 = 27 \qquad 3^4 = 81$
Since 81 is too large to divide 76, we will begin with 27.

$$\begin{aligned} 76 &= 2 \cdot 27 + 22 \\ &= 2 \cdot 3^3 + 2 \cdot 9 + 4 \\ &= 2 \cdot 3^3 + 2 \cdot 3^2 + 1 \cdot 3 + 1 \\ &= 2 \cdot 3^3 + 2 \cdot 3^2 + 1 \cdot 3^1 + 1 \cdot 3^0 \\ &= 2211 \text{ base } 3 \end{aligned}$$

Now you try:

Problem 1. Convert the following from their original base to base 10.

(a) 121 base 3

(b) 1011 base 2

(c) 1356 base 7

(d) 111 base 2

(e) 201 base 9

(f) 1110 base 2

Problem 2. Convert the following from base 10 to the base specified below.

(a) 100 to base 5

(b) 75 to base 2

(c) 123 to base 7

(d) 79 to base 3

(e) 421 to base 20