Change of Base: Logarithm Formula

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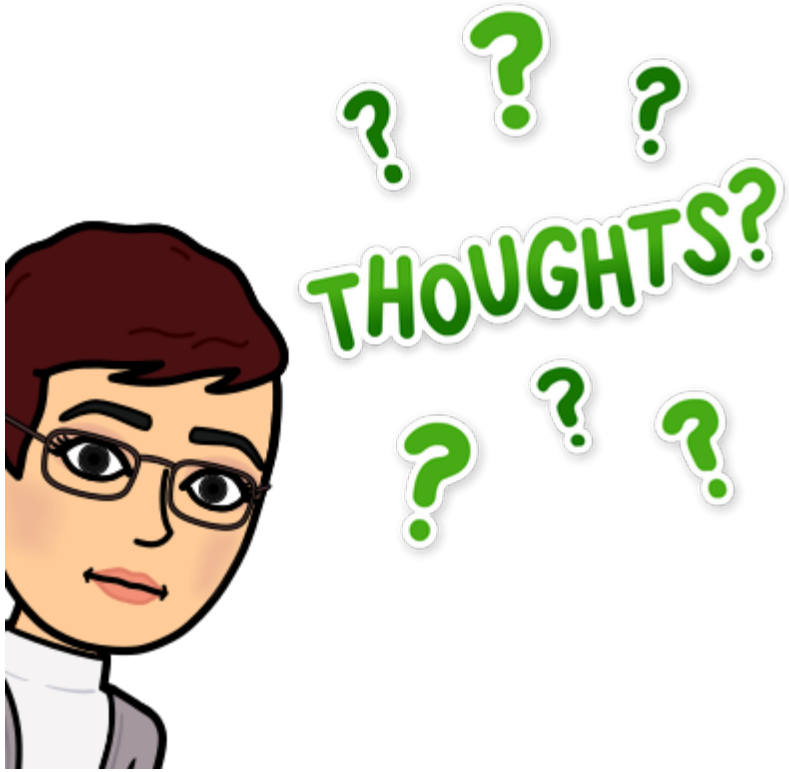
Calculator

Most Calculators only have buttons for log base 10, $\log(x)$
and natural log, $\ln(x)$.
So how do we compute all of the other bases of log in a calculator?



An Example First

Suppose I wanted to compute

$$\log_3(7)$$

How would I enter that in the calculator below

What to do?

I'm going to give this mysterious number a name:

$$\log_3(7) = L$$

I know that it has an equivalent exponential expression:

$$7 = 3^L$$

I know this sounds crazy

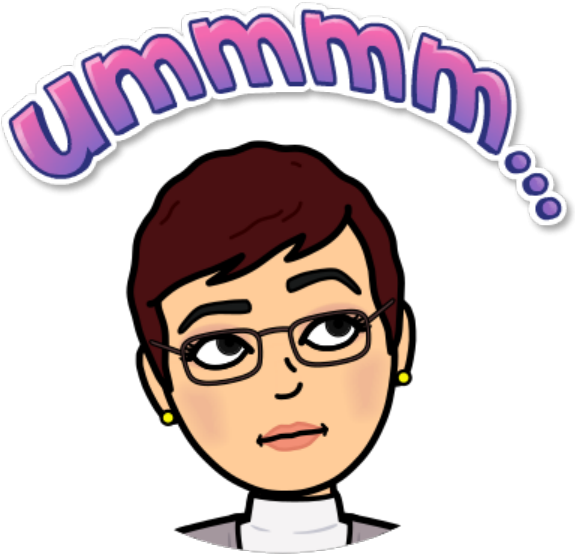
What if we took natural log of both sides?

$$7 = 3^L$$

would be

$$\ln(7) = \ln(3^L)$$

How does that help?



Isn't there a log property about powers?

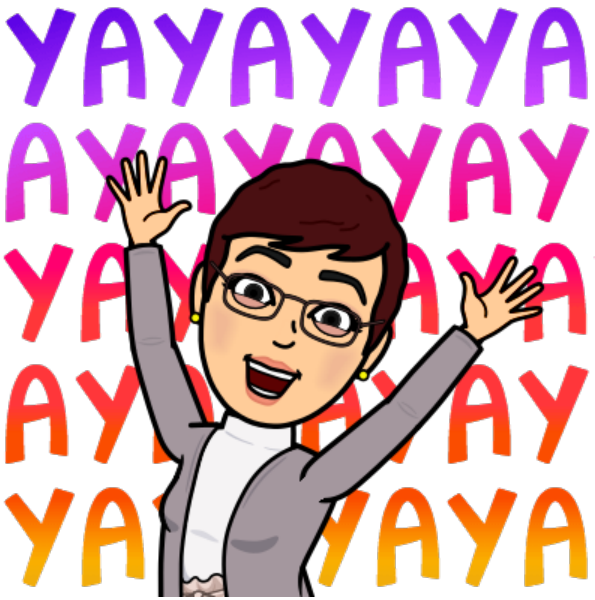
We could use the log property that a power in log moves to the front:

$$\ln(7) = \ln(3^L)$$

would be

$$\ln(7) = L\ln(3)$$

Now we can solve for L



Now we can use our calculator

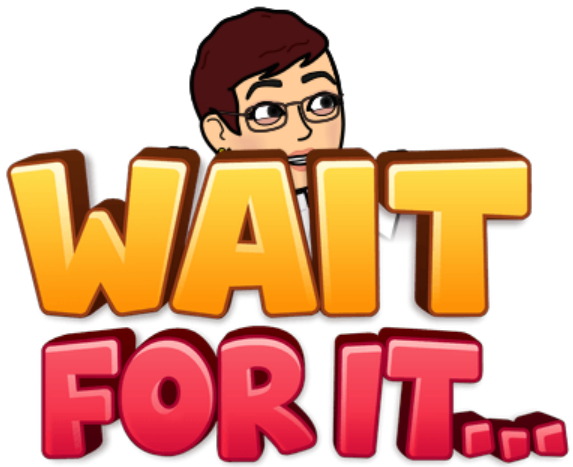
Divide both sides by $\ln(3)$

$$\ln(7) = L\ln(3)$$

and we have

$$\frac{\ln(7)}{\ln(3)} = L$$

So let's summarize



What did we start out with?

What was L originally?

$$\log_3(7) = L$$

and many slides later we got

$$\frac{\ln(7)}{\ln(3)} = L$$

The Take Away

which means

$$\log_3(7) = \frac{\ln(7)}{\ln(3)}$$

Since, the same thing works for any number, not just 7 and 3,

Change of Base Formula

$$\log_b(x) = \frac{\ln(x)}{\ln(b)}$$

Not just for Natural Log

Change of Base Formula

$$\log_b(x) = \frac{\log_a(x)}{\log_a(b)}$$

This means it also works for $\log_{10}(x) = \log(x)$.

Another Example

Example

Compute $\log_7(53)$ by changing the base to the common log.

Using the change of base formula, we get

$$\log_7(53) = \frac{\log(53)}{\log(7)} \approx 2.0403$$

Check for Understanding

Compute the following by using the change of base formula and entering it in the calculator:

Check for Understanding

- $\log_5(3451)$
- $\log_{11}(35)$
- $\log_2(100)$
- $\log_7(25)$

Check for Understanding

Compute the following by using the change of base formula and entering it in the calculator:

Check for Understanding

- $\log_5(3451) \approx 5.0617$
- $\log_{11}(35) \approx 1.0414$
- $\log_2(100) \approx 6.644$
- $\log_7(25) \approx 1.654$