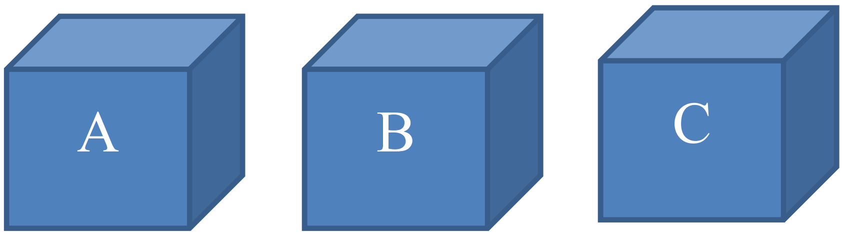
Conditional Probability

Let's Make a Deal!!

Suppose I have three boxes.
In one box is $1,000,000. In the other two there is nothing.
Suppose you choose box C.
What is the probability that you chose correctly? ______
What is the probability that you chose wrongly? ______



Now let us suppose that I open box, B, and its empty. I ask you is you want to switch your choice.

Here is the thought process going on in the brillian head of yours:

What is the probability that C is the wrong box? Not 1/2, but still 2/3.
Since we now know that B is not the right box, the probability that C is the wrong box and the probability that A is the wrong box must add up to 1. Therefore, the probability that A is the wrong box is 1/3. Which means the probability that A is the right box is 2/3. SWITCH TO A!!!!!

Good job! This problem is called the Monty Hall problem. A statistician named Steve Selvin in the *American Statistician* in 1975 (Selvin 1975a). The problem was answered by Marilyn vos Savant in a column in *Parade* magazine in 1990. Her answer upset so many mathematician, that Parade magazine received 10,000 letters, 1,000 from PhD's in math, exclaiming how wrong she was. Even Paul Erdos, an extraordinarily renowned mathematician was utterly unconvinced she was wrong, until a computer simulation performed 10,000 trials and supported her claim. If you are interested in trying a simulation, go to the websites

http://math.ucsd.edu/~crypto/cgi-bin/MontyKnows/monty2?0+4338

http://www.nytimes.com/2008/04/08/science/08monty.html?_r=0

The problem above is an example of conditional probability. The conditional probability of an event **B** given an event **A**, is the probability that **B** will occur given the knowledge that event **A** has already occurred. This probability is written $P(B \mid A)$. There are two cases in conditional probability

- **B** and **A** are independent events: $P(B \mid A) = P(B)$
- **B** and **A** are dependent events: $P(B \mid A) = \dfrac{P(B \text{ and } A)}{P(A)} = \dfrac{P(B \cap A)}{P(A)}$

Example: Suppose I have two cards. Card A has a blue side and a red side. Card B has two red sides. I choose one at random and place it on the table. The top is red. What is the probability that the other side is red?

Solution: If you said 1/2, read the first page again. Go on. I'll wait.
What is **B**?
B = {the possibile outcomes in which both sides are red}. **A** = { a given side is red}
$P(B \cap A) = \frac{|B \cap A|}{|S|} = \frac{1}{2}$ and the $P(A) = \frac{|A|}{|S|} = \frac{3}{4}$. Therefore, the probability of **B** given **A** is

$$P(B|A) = \frac{P(B \cap A)}{P(A)} = \frac{\frac{1}{2}}{\frac{3}{4}} = \frac{2}{3}.$$

Exercise: Bag X has 5 white marbles and 2 black marbles. Bag Y has 3 white marbles and 5 black marbles. A bag is chosen at random and amarble taken from the bag. The marble is white. What is the probability that the bag was bag X?

Solution: A = { events that the ball is white }, B = { events that Bag X is chosen }

Step 1: Compute $P(B \cap A)$.

Step 2: Compute $P(B) = P(\text{ white marble from bag X}) + P(\text{ white marble from bag Y})$

Step 3: Compute $P(B|A) = \frac{P(B \cap A)}{P(A)} =$

Problems:

1) In the World Series, twoteams play each other repeatedly until one team has won a total of 4 games, then the series ends. If each team is equally likely to win each game, what is the probability that the series ends in exactly 6 games?

2) If 3 successive rolls of a die are all greater than three, what is the probability that they are all the same?

3) If a number is selected at random from the set of all five digit natural numbers in which the sum of the digits equal to 43, what is the probability that this number will be divisibly by 11?

4) A point P is chosen at random in the coordinate plane. What is the probability that the unit circle with center P contains exactly two points with integer coordinates (lattice points).

5) Three points A, B, and C are selected at random on the circumference of a circle. Find the probability that the points lie on a semicircle.

6) Three number are chosen at random between 0 and 1. What is the probability that the difference between the greatest and the least is less than $\frac{1}{3}$?