Using Cauchy-Riemann Equations to Prove Differentiation Formulas

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For these exercises you can assume everything you learned in single variable calculus and what we discussed about multivariable calculus.
We can now say we know that the following are equivalent:

- $ru_r = v_\theta$ and $u_\theta = -rv_r$
- $u_x = v_y$ and $u_y = v_x$
- $f'(z)$ exists
- $f'(z) = u_x + iv_x$
- $f'(z) = e^{-i\theta}(u_r + iv_r)$

Problem 1. Let $f(z) = c$ where c is a constant complex number.
Show f is analytic and show that $f'(z) = 0$.

Problem 2. Let $f(z) = z^n$.
Show f is analytic and compute $f'(z)$.

Problem 3. Let $f(z)$ and $g(z)$ be analytic functions.
Let $h(z) = f(z) + g(z)$.
Show that h is analytic and find $h'(z)$.

Problem 4. Let $f(z)$ and $g(z)$ be analytic functions.
Let $h(z) = f(z)g(z)$.
Show that h is analytic and find $h'(z)$.

Problem 5. Let $f(z)$ and $g(z)$ be analytic functions.
Let $h(z) = \frac{f(z)}{g(z)}$.
Show that h is analytic and find $h'(z)$.

Problem 6. Let $f(z)$ and $g(z)$ be analytic functions.
Let $h(z) = f(g(z))$.
Show that h is analytic and find $h'(z)$.

Problem 7. Let $f(z)$ and $g(z)$ be analytic functions.
Suppose $f(a) = g(a) = 0$.
Show that

$$\lim_{z \to a} \frac{f(z)}{g(z)} = \lim_{z \to a} \frac{f'(z)}{g'(z)}$$