

FUN FACTS WITH DIHEDRAL GROUPS

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Let's see what we can figure out about the Dihedral Group by analyzing some Dihedral Groups.

Let $G = D_{10}$ be the Dihedral Group of a decagon.

Let ρ be the clockwise rotation of the decagon by 36° .

Let τ be the reflection across the vertex 1.

We know that

- $\text{ord}(\rho) = 10$
- $\text{ord}(\tau) = 2$
- $\rho\tau = \tau\rho^{-1}$.

Problem 1. Getting to know the rotations.

(a) Write the permutation ρ .

(b) Write the permutation ρ^2 .

(c) Write the permutation ρ^3 .

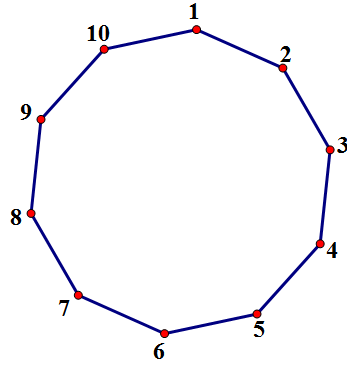
Note: The numbers in ρ^2 differ by _____ and the numbers in ρ^3 differ by _____.

(d) Write the permutation ρ^4 and ρ^5 .

(e) Write the permutation ρ^6 .

Note: Since ρ^6 is the inverse of ρ^{10} , to get the numbers in ρ^6 , we fix the number 1 and reverse the rest in ρ^4 .

(f) Write the rest of the powers of ρ .



Problem 2. Getting to know the reflections.

(a) Write the permutation for τ .

(b) Write the permutation for $\rho\tau$.

Note: The sum of the numbers in each transposition in τ is _____ mod 10.

The sum of the numbers in each transposition in $\rho\tau$ is _____ mod 10.

(c) Write the permutation for $\rho^2\tau$.

(d) Write the permutation for $\rho^4\tau$.

(e) Write the permutation for $\rho^8\tau$.

Problem 3. The product and inverse of reflections Using only

- $\text{ord}(\rho) = 10$
- $\text{ord}(\tau) = 2$
- $\rho\tau = \tau\rho^{-1}$.

prove the following statements.

(a) Show $\rho^2\tau = \tau\rho^8$.

(b) Show $\rho^3\tau = \tau\rho^7$.

(c) Show $\rho^3\tau\rho\tau = \rho^2$.

(d) Show $\rho^4\tau\rho^6\tau = \rho^8$.

Problem 4. All of the computations below must be written as powers of ρ or they must be of the form $\rho^k\tau$, where $k = 0, 1, 2, 3, \dots, 9$.

(a) $\rho^7\tau\rho^8$

(b) $\rho^3\tau\rho^7\tau$

(c) $\rho\tau\rho^5\tau$

(d) $\tau\rho^6\tau$