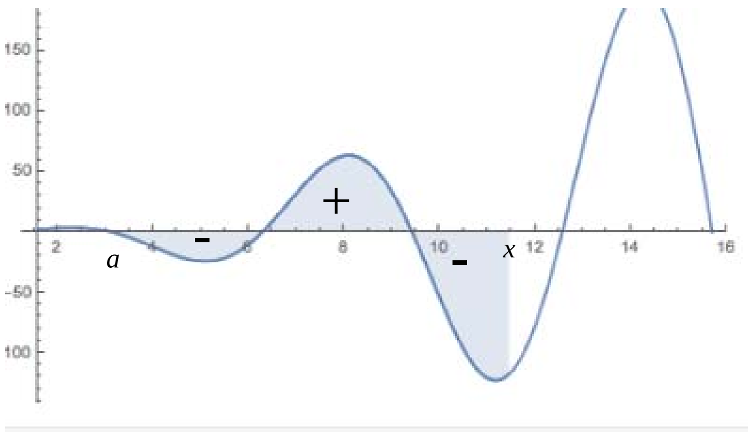
Fundamental Theorem of Calculus

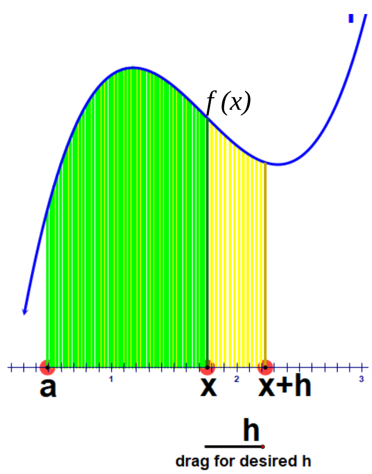
Let $A(x) = \lim_{n\to\infty} \sum_{k=1}^{n} \Delta x\, f(x_k) = \lim_{n\to\infty} \sum_{k=1}^{n} \frac{x-a}{n} f(a + k(\frac{x-a}{n}))$ be the function which gives the signed area between *f* and the *x*-axis from *a* to *x*.



Then the derivative of the signed area function is

$$A'(x) = \lim_{h\to 0} \frac{A(x+h) - A(x)}{h}$$

What does $\frac{A(x+h)-A(x)}{h}$ represent? It's approximately the average height of the area in yellow. Meaning, if we made a rectangle with width, h , whose area was the area of in yellow, then when we divide by *h*, we would have the height left over.



As *h* approaches 0, then $\frac{A(x+h)-A(x)}{h} \to f(x)$. This means thath $A'(x) = f(x)$! So, *A*, the signed area function is an antiderivative of *x*.

The signed area under a curve and the antiderivative:

Let $F(x)$ be a differentiable function and

$F'(x) = f(x)$ on an interval $[a,b]$

$$F(b) - F(a) = \int_a^b f(x)dx$$

Example 1: Compute $\int_0^3 (2x+3)dx$. This is the signed area between 2*x* + 3 and the *x*-axis on the interval [0,3].

2. Compute the area between the graph of $f(x) = 3x^2 + 2x - 1$ and the x-axis on the interval $[1,3]$.