

Due: January 21<sup>st</sup>

On a separate sheet of paper, complete 3 out of the 5 problems below. Make sure to write sentences with your equations to explain what you are doing and why you are doing it.

The set of Fibonacci numbers are constructed by

$$F_0 = 1$$

$$F_1 = 1$$

$$F_{k+1} = F_k + F_{k-1}$$

The first 12 Fibonacci numbers are

1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, ...

1) Show that  $\lim_{n \rightarrow \infty} \frac{F_{n+1}}{F_n} = \phi = \frac{1 + \sqrt{5}}{2}$

2) Show that  $F_n^2 - F_{n+1}F_{n-1} = \pm 1$ . Find the conditions on  $n$  on which it is 1, and when is -1.

3) Show that the sum of the first  $n$  Fibonacci numbers with odd indices is given by the formula

$$F_1 + F_3 + F_5 + \cdots + F_{2n-1} = F_{2n}$$

4) Show that the sum of the first  $n$  Fibonacci numbers with even indices is given by the formula

$$F_2 + F_4 + F_6 + \cdots + F_{2n} = F_{2n+1} - 1$$

5) Show that, for each integer  $m$ , the sequence  $F_n \pmod{m}$  repeats