Worksheet 7.6.
Improper Integrals

Determine whether the improper integral converges, and if it does, evaluate it.

1. $\displaystyle\int_1^\infty \frac{dx}{x^{20/19}}$

2. $\displaystyle\int_{20}^\infty \frac{dt}{t}$

3. $\displaystyle\int_0^5 \frac{dx}{x^{19/20}}$

4. $\displaystyle\int_1^3 \frac{dx}{\sqrt{3-x}}$

5. $\displaystyle\int_{-2}^4 \frac{dx}{(x+2)^{1/3}}$

Solutions to Worksheet 7.6

Determine whether the improper integral converges and if it does, evaluate it.

1. $\displaystyle\int_1^\infty \frac{dx}{x^{20/19}}$

First evaluate the integral over the finite interval $[1, R]$ for $R > 1$:

$$\begin{aligned}\int_1^R \frac{dx}{x^{20/19}} &= -19x^{-1/19}\Big|_1^R = \frac{-19}{R^{1/19}} - (-19) = 19 - \frac{19}{R^{1/19}} \\ \int_1^\infty \frac{dx}{x^{20/19}} &= \lim_{R\to\infty} \int_1^R \frac{dx}{x^{20/19}} = \lim_{R\to\infty}\left(19 - \frac{19}{R^{1/19}}\right) \\ &= 19 - 0 = 19\end{aligned}$$

2. $\displaystyle\int_{20}^\infty \frac{dt}{t}$

First evaluate the integral over the finite interval $[20, R]$ for $20 < R$:

$$\begin{aligned}\int_{20}^R \frac{dt}{t} &= \ln|t|\Big|_{20}^R = \ln R - \ln 20 \\ \int_{20}^\infty \frac{dt}{t} &= \lim_{R\to\infty} \int_{20}^R \frac{dt}{t} = \lim_{R\to\infty}(\ln R - \ln 20) = \infty\end{aligned}$$

The integral does not converge.

3. $\displaystyle\int_0^5 \frac{dx}{x^{19/20}}$

The function $x^{-19/20}$ is infinite at the endpoint 0, so we first evaluate the integral on the finite interval $[R, 5]$ for $0 < R < 5$:

$$\begin{aligned}\int_R^5 \frac{dx}{x^{19/20}} &= 20x^{1/20}\Big|_R^5 = 20\left(5^{1/20} - R^{1/20}\right) \\ \int_0^5 \frac{dx}{x^{19/20}} &= \lim_{R\to 0^+} \int_R^5 \frac{dx}{x^{19/20}} = \lim_{R\to 0^+} 20\left(5^{1/20} - R^{1/20}\right) \\ &= 20\left(5^{1/20} - 0\right) = 20 \cdot 5^{1/20}\end{aligned}$$

4. $\displaystyle\int_1^3 \frac{dx}{\sqrt{3-x}}$

The function $f(x) = 1/\sqrt{3-x}$ is infinite at $x = 3$ and is left continuous at $x = 3$, so we first evaluate the integral on the interval $[1, R]$ for $1 < R < 3$:

$$\begin{aligned}\int_1^R \frac{dx}{\sqrt{3-x}} &= 2\sqrt{3-x}\,|_1^R \\ &= -2\sqrt{3-R} + 2\sqrt{2} \\ \lim_{R\to 3}\int_1^R \frac{dx}{\sqrt{3-x}} &= 0 + 2\sqrt{2}\end{aligned}$$

Therefore the integral is equal to $2\sqrt{2}$.

5. $\displaystyle\int_{-2}^4 \frac{dx}{(x+2)^{1/3}}$

The function $(x+2)^{-1/3}$ is infinite at $x = -2$ and right-continuous at $x = -2$, so we first evaluate the integral on the interval $[R, 4]$ for $-2 < R < 4$:

$$\begin{aligned}\int_R^4 \frac{dx}{(x+2)^{1/3}} &= \frac{3}{2}(x+2)^{2/3}\Big|_R^4 = \frac{3}{2}\left(6^{2/3} - (R+2)^{2/3}\right) \\ \int_{-2}^4 \frac{dx}{(x+2)^{1/3}} &= \lim_{R\to -2^+}\int_R^4 \frac{dx}{(x+2)^{1/3}} = \lim_{R\to 2^+}\frac{3}{2}\left(6^{2/3} - (R+2)^{2/3}\right) \\ &= \frac{3}{2}\left(6^{2/3} - 0\right) = \frac{3\cdot 6^{2/3}}{2}\end{aligned}$$