

Worksheet: Improper Integrals

1. Evaluate the following integrals.

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|---|--|--|
| (1) $\int_0^1 \frac{1}{x} dx$ | (2) $\int_1^{\infty} \frac{1}{x} dx$ | (3) $\int_0^{\infty} xe^{-x} dx$ |
| (4) $\int_0^{\infty} \frac{1}{1+x^2} dx$ | (5) $\int_5^{\infty} \frac{1}{\sqrt{x-1}} dx$ | (6) $\int_0^1 \frac{1}{1-x} dx$ |
| (7) $\int_1^{\infty} \ln x dx$ | (8) $\int_0^1 \frac{1}{\sqrt{x}} dx$ | (9) $\int_0^1 \frac{1}{\sqrt{1-x}} dx$ |
| (10) $\int_0^{\infty} e^{-x} dx$ | (11) $\int_{-\infty}^{\infty} \frac{e^x}{1+e^{2x}} dx$ | (12) $\int_{-\infty}^1 e^x dx$ |
| (13) $\int_{-\infty}^0 x^2 e^x dx$ | (14) $\int_{-\infty}^{\infty} xe^{-x^2} dx$ | (15) $\int_e^{\infty} \frac{1}{x(\ln x)^2} dx$ |
| (16) $\int_{-\infty}^{\infty} x^5 dx$ | (17) $\int_{-\infty}^{\infty} \frac{1}{x^2+16} dx$ | (18) $\int_1^{\infty} \ln x dx$ |
| (19) $\int_0^{\infty} e^{-2x} dx$ | (20) $\int_3^{\infty} \frac{1}{x^3} dx$ | (21) $\int_0^4 \frac{1}{(4-x)^{\frac{3}{2}}} dx$ |
| (22) $\int_0^2 \frac{1}{\sqrt{4-x^2}} dx$ | (23) $\int_0^4 \frac{x}{\sqrt{16-x^2}} dx$ | (24) $\int_0^{\frac{\pi}{2}} \tan \theta d\theta$ |
| (25) $\int_0^{\frac{\pi}{2}} \frac{1}{1-\sin x} dx$ | (26) $\int_0^1 \frac{1}{\sqrt{x}} dx$ | (27) $\int_0^{\infty} \frac{e^{-\sqrt{x}}}{\sqrt{x}} dx$ |
| (28) $\int_1^{\infty} \frac{1}{x^2} dx$ | (29) $\int_{-\infty}^{-1} \frac{2}{x^5} dx$ | (30) $\int_{-\infty}^{\infty} \frac{x}{(1+x^2)^2} dx$ |

2. Evaluate the following integrals.

$$\begin{array}{lll}
 (1) \int_{\frac{1}{2}}^{\infty} \frac{1}{x(\ln x)^{\frac{1}{5}}} dx & (2) \int_{-2}^2 \frac{1}{(x+1)^3} dx & (3) \int_0^2 \frac{1}{(x-1)^{\frac{2}{3}}} dx \\
 (4) \int_0^4 \frac{1}{x^2 - 2x - 3} dx & (5) \int_{-2}^0 \frac{1}{(x+1)^{\frac{1}{3}}} dx & (6) \int_0^2 \frac{x}{1-x} dx \\
 (7) \int_{-1}^2 \frac{1}{x^3} dx & (8) \int_0^{\pi} \frac{\sin x}{\sqrt[5]{\cos x}} dx & (9) \int_0^{\frac{\pi}{2}} \sec 2x dx \\
 (10) \int_0^{\frac{\pi}{4}} \frac{\sec^2 x}{\sqrt{1 - \tan x}} dx & (11) \int_{-1}^1 \frac{e^x}{\sqrt[5]{e^x - 1}} dx & (12) \int_1^4 \frac{1}{(x-2)^{\frac{2}{3}}} dx
 \end{array}$$

ANSWERS

1. ...

$$\begin{array}{lllll}
 (1) \text{ div} & (2) \text{ div} & (3) 1 & (4) \frac{\pi}{2} & (5) \text{ div} \\
 (6) \text{ div} & (7) \text{ div} & (8) 2 & (9) 2 & (10) 1 \\
 (11) \frac{\pi}{2} & (12) e & (13) 2 & (14) 0 & (15) 1 \\
 (16) \text{ div} & (17) \frac{\pi}{4} & (18) \text{ div} & (19) \frac{1}{2} & (20) \frac{1}{18} \\
 (21) \text{ div} & (22) \frac{\pi}{2} & (23) 4 & (24) \text{ div} & (25) \text{ div} \\
 (26) 2 & (27) 2 & (28) 1 & (29) -\frac{1}{2} & (30) 0
 \end{array}$$

2. ...

$$\begin{array}{llll}
 (1) \text{ div} & (2) \text{ div} & (3) 6 & (4) \text{ div} \\
 (5) 0 & (6) \text{ div} & (7) \text{ div} & (8) 0 \\
 (9) \text{ div} & (10) 2 & (11) 1.0614 & (12) 3 + 3\sqrt[3]{2}
 \end{array}$$