Integration Technique Cheat Sheet

Integration Technique	***When to use this method***	***The Method***	***Example***
U-sub	When you have reverse chain rule $\int f(u(x))u'(x)dx$	Choose u and compute du so that $\int f(u)du$ is integrable.	$\int \frac{x^2}{\sqrt{4x^3+2}}dx \quad u=4x^3+2 \quad du=12x^2dx$ $=\frac{1}{12}\int \frac{12x^2}{\sqrt{4x^3+2}}dx=\frac{1}{12}\int\frac{du}{\sqrt{u}}$
Trigonometric Identities	When you are given $\int \sin^k(x)dx$ or $\int \cos^k(x)dx$	k is even = use the half angle formulas $\cos^2(x)=\frac{1+\cos(2\theta)}{2}$ $\sin^2(x)=\frac{1-\cos(2\theta)}{2}$ k is odd = "save" one of the functions for the du, and substitute the rest with the identities $\cos^2(x)=1-\sin^2(x)$ $\sin^2(x)=1-\cos^2(x)$	$\int \cos^4(x)dx=\int(\frac{1+\cos(2\theta)}{2})^2 d\theta$ $=\int\frac{1}{4}(1+2\cos(2\theta)+\cos^2(2\theta))d\theta$ $=\int\frac{1}{4}(1+2\cos(2\theta)+\frac{1+\cos(4\theta)}{2})d\theta$ or $\int\cos^3(x)dx=\int(1-\sin^2(x))\cos(x)dx$ $=\int 1-u^2du$
Integration by Parts (traditional)	When you have the product of two functions but neither is the derivative of the other $\int f(x)g(x)dx$	Choose $u = f(x)$ and $dv = g(x)$ $du = f'(x) \qquad v=\int g(x)dx$ $\int udv = uv-\int vdu$	Choosing u (log, inverse trig, algebraic, trig, exp) $\int \sin(x)e^x dx=\sin(x)e^x-\int e^x\cos(x)dx$ $u_1=\sin(x) \quad dv_1=e^x \qquad u_2=\cos(x) \quad dv_2=e^x$ $du_1=\cos(x) \quad v_1=e^x \qquad du_2=-\sin(x) \quad v_2=e^x$ $\int\sin(x)e^xdx=\sin(x)e^x-\cos(x)e^x-\int\sin(x)e^xdx$ $2\int\sin(x)e^xdx=\sin(x)e^x-\cos(x)e^x$ $\int\sin(x)e^xdx=\frac{1}{2}e^x(\sin(x)-\cos(x))$

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<td>Integration by Parts (tabular)</td>
<td>When one function is a polynomial and the other is NOT a log function</td>
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<tr><td>Sign</td><td>u (der)</td><td>dv (int)</td></tr>
<tr><td>+</td><td>u</td><td>dv</td></tr>
<tr><td>-</td><td>u'</td><td>v</td></tr>
<tr><td>+</td><td>u''</td><td>$\int v$</td></tr>
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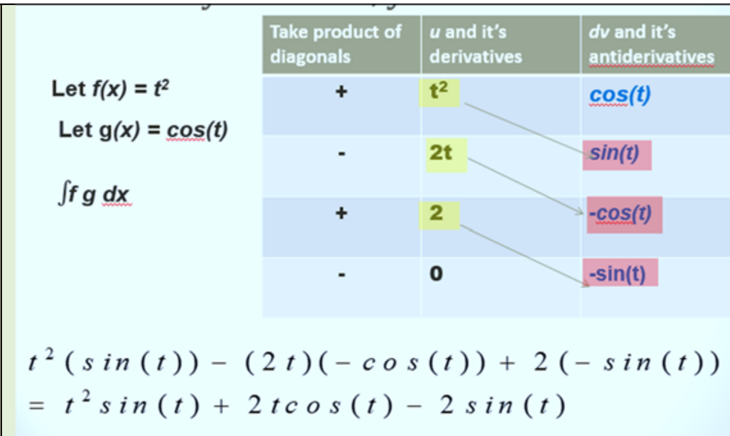
Let $f(x) = t^2$
Let $g(x) = \cos(t)$
$\int f\, g\, dx$
<table>
<tr><td>Take product of diagonals</td><td>u and it's derivatives</td><td>dv and it's antiderivatives</td></tr>
<tr><td>+</td><td>t²</td><td>cos(t)</td></tr>
<tr><td>-</td><td>2t</td><td>sin(t)</td></tr>
<tr><td>+</td><td>2</td><td>-cos(t)</td></tr>
<tr><td>-</td><td>0</td><td>-sin(t)</td></tr>
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$t^2(\sin(t)) - (2t)(-\cos(t)) + 2(-\sin(t))$
$= t^2\sin(t) + 2t\cos(t) - 2\sin(t)$
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<td>Partial Fractions</td>
<td>When you are integrating a rational function in which the numerator is not the derivative of the denominator</td>
<td>You are given A and B and factor the denominator into
$\frac{Ax+B}{(x-r_1)(x-r_2)} = \frac{M}{x-r_1} + \frac{N}{x-r_2}$
$Ax + B = M(x-r_2) + N(x-r_1)$
Plug in $x = r_1$ and $x = r_2$</td>
<td>$\int \frac{4x+37}{x^2+11x+28}dx$
$\frac{4x+37}{x^2+11x+28} = \frac{A}{x+7} + \frac{B}{x+4}$
$4x+37 = A(x+4) + B(x+7)$
$x = -4 \rightarrow B = 7 \quad x = -7 \rightarrow A = -3$
$\int \frac{4x+37}{x^2+11x+28}dx = \int -\frac{3}{x+7}dx + \int \frac{7}{x+4}dx$
$= -3\ln|x+7| + 7\ln|x+4| + C$</td>
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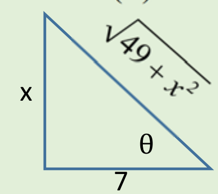
<tr>
<td>Trigonometric Substitution</td>
<td>When you have
$\sqrt{a-x^2}$ or
$\sqrt{x^2-a}$ or
$\sqrt{x^2+a}$</td>
<td>Using the trig identities
$1-\sin^2(\theta) = \cos^2(\theta)$
$1+\tan^2(\theta) = \sec^2(\theta)$
$\sec^2(\theta) - 1 = \tan^2(\theta)$
Choose x to be one of the trig functions so that the square root is cancelled.</td>
<td>$\int \sqrt{49+x^2}dx \quad x = 7\tan(\theta) \quad dx = 7\sec^2(\theta)$
$\int \sqrt{49+49\tan^2(\theta)}7\sec^2(\theta)d\theta$
$\int 47\sec^3(x)dx$


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Tricky Integrals:

1. $\int \sec(x)dx$ $\int \sec(x)dx = \int \sec(x)\cdot\frac{\sec(x)+\tan(x)}{\sec(x)+\tan(x)}dx$ $= \int \frac{\sec^2(x)+\sec(x)\tan(x)}{\sec(x)+\tan(x)}dx$ $u = \sec(x)+\tan(x) \quad du = \sec(x)\tan(x)+\sec^2(x)$ $= \int \frac{du}{u} = \ln\|\sec(x)+\tan(x)\| + C$	2. $\int \frac{x}{x+1}dx$ $\int \frac{x}{x+1}dx = \int \frac{x+1-1}{x+1}dx$ $= \int \frac{x+1}{x+1} - \frac{1}{x+1}dx$ $= \int 1 - \frac{1}{x+1}dx$ $= x - \ln\|x+1\| + C$
3. $\int \sec^3(x)dx$ $\int \sec^3(x)dx = \int \sec^2(x)\sec(x)dx$ Integration by Parts $u = \sec(x) \qquad dv = \sec^2(x)dx$ $du = \sec(x)\tan(x) \qquad v = \tan(x)dx$ $\int \sec^3(x)dx = \sec(x)\tan(x) - \int \tan^2(x)\sec(x)dx$ $\int \sec^3(x)dx = \sec(x)\tan(x) - \int (\sec^2(x)-1)\sec(x)dx$ $\int \sec^3(x)dx = \sec(x)\tan(x) - \int \sec^3(x)dx + \int \sec(x)dx$ $\int \sec^3(x)dx = \sec(x)\tan(x) - \int \sec^3(x)dx + \ln\|\sec(x)+\tan(x)\| + C$ $2\int \sec^3(x)dx = \sec(x)\tan(x) + \ln\|\sec(x)+\tan(x)\| + C$ $\int \sec^3(x)dx = \frac{1}{2}(\sec(x)\tan(x) + \ln\|\sec(x)+\tan(x)\|) + C$	4. $\int \ln(x)dx$ $\int \ln(x)dx$ Integration by Parts $u = \ln(x) \quad dv = dx$ $du = \frac{1}{x}dx \qquad v = x$ $\int \ln(x)dx = x\ln(x) - \int dx$ $= x\ln(x) - x + C$