A **sequence** is a function $a_n : \mathbb{N} \to \mathbb{R}$ whose domain is the positive or nonnegative integers (Natural Numbers) and whose range is the real numbers. Usually, sequences are denoted by (a_n), where we assume *n* runs from 0 to infinity, or from 1 to infinity.

- A sequence is said to be defined **recursively** if it is defined by its previous terms.
- A sequence is said to be defined **inductively** (closed form) if it is defined only by the number of its term, *n*.

Practice Problem #1

The **Fibonacci Sequence** given below is a sequence given recursively.
Write down the first five terms.

$$a_0 = 1,\ a_1 = 1,\ a_n = a_{n-1} + a_{n-2} \text{ when } n \geq 2$$

$a_1 =$

$a_2 =$

$a_3 =$

$a_4 =$

$a_5 =$

Practice Problem #2

Consider the inductive (closed) sequence: $a_n = \frac{(-1)^n}{n^2}$ where $n \in \mathbb{Z}^+$. Write down the first five terms.

$a_1 =$

$a_2 =$

$a_3 =$

$a_4 =$

$a_5 =$

Problem Practice Problem #3

Consider the sequence: $3, \frac{5}{4}, \frac{7}{9}, \frac{9}{16}, \frac{11}{25}, \cdots$. Starting with $n = 1$, define this sequence inductively. (Find its generating or closed form)

What do you notice about the numerators?

What do you notice about the denominators?

Put these thoughts together.

Practice Problem #4
Consider the sequence: 1, 3, 6, 10, 15, $\cdots$

a. Starting with $a_1 = 1$, find a recursive form of the sequence.

b. Starting with $n = 1$, find a generating form or closed form of the sequence.

A sequence, a_n, is said to **converge** if $\lim_{n\to\infty} a_n$ equals a real number.

A sequence, a_n, is said to **diverge** if $\lim_{n\to\infty} a_n$ does not exist or equals ∞ or $-\infty$.

Practice Problem #5
Determine whether the sequence $a_n = \frac{2^n}{n^2}$ converges or diverges. If the sequence converges, state its limit.

Practice Problem #6
Determine whether the sequence $a_n = \frac{3n^2 + 5n + 2}{1 + n^2}$ converges or diverges. If the sequence converges, state its limit.

Practice Problem #7

Determine whether the sequence $a_n = \frac{(-1)^n}{n}$ converges or diverges. If the sequence converges, state its limit.