LAW OF DETACHMENT

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Now that we are familiar with construction of conditional statements in the language of logic, we can begin to identify which conditional statements are logically equivalent. The "If, then" statements we have been working on are called *conditional statements* , or *implications*. The reason why they are called implications, is because the **hypothesis** implies the **conclusion**. Let us look at an example.

Example 1. If you are 30, then you are too old to go to Disneyland.

The hypothesis is *you are* 30
The conclusion is *you are too old to go to Disneyland*

Another way to say this is:
You are 30 **implies** that you are too old to go to Disneyland.

In symbolic logic, we try to condense the notation, so we can see in general, when two logical statements are equivalent.

Let
$p =$ You are 30.
$q =$ You are too old to go to Disneyland.

Then the statement above is written

$$p \implies q$$

In the previous activity, we started to study the correlation between the truth value of the hypothesis, conclusion and the entire statement. In order to assess the validity of any arbitrary logical statement, it is necessary that we create a rule for this relation.

0.1. **Law of Detachment.** The Law of detachment says that
if the hypothesis is true,
then the only way that an implication can be true,
is if the conclusion is also true.
A false hypothesis has no bearing on the truth value of the implication.

In symbolic logic, the above statement is as follows:

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Hyp: p	Conc: q	Implication: $p \implies q$
T	T	T
T	F	F
F	T	T
F	F	T

From this table we see if the hypothesis is true, then the conclusion has to be true, or the whole implication is false.

Example 2. Let us look at a simple mathematical statement:

If you multiply two negatives together, their product will be positive.

Suppose I give you the numbers -1 and -2.
Do they satisfy the hypothesis?
Is their product positive?

Suppose I give you the numbers 1 and 4.
Do they satisfy the hypothesis? Is their product positive?
Suppose I give you the numbers -1 and 4.
Do they satisfy the hypothesis? IS their product positive?

Now let us look at an example which fails the truth table.

Example 3. If a number is odd, then it is prime.

Suppose I gave you the number 3.
Does it satisfy the hypothesis?
Is it prime?

Suppose I gave you the number 2.
Does it satisfy the hypothesis?
Is it prime?

Suppose I gave you the number 9.
Does it satisfy the hypothesis?
Is it prime?

This last number is a counterexample to the statement and therefore shows that the statement is not true for all numbers, and therefore, is not a true statement.

Problems

Using the Law of Detachment, determine whether the following statements are true or false.
If they are true, explain why. Determine the validity of the hypothesis and conclusion.
If it is false, give a counterexample.

(A) If one is in shock, one will get a headache.

(B) All doggies are cute.

(C) Dreamers often lie. (Shakespeare)

(D) If you build it, he will come. (Field of Dreams)

(E) If you study hard, you will pass the test.

(F) If you pass the test, then you studied hard.

(G) If you eat your breakfast, you will not be hungry in the morning.

Assume each conditional statement is true. Then read the statement following it. Determine whether you can conclude anything from the next statement.

(A) If one is in shock, one will get a headache.
Paul has a headache.

(B) All doggies are cute.
Jacobi is a dog.

(C) Dreamers often lie. (Shakespeare)
Robert isn't lying.

(D) If you build it, he will come. (Field of Dreams)
You didn't build anything.

(E) If you study hard, you will pass the test.
Alice studied very hard.

(F) If you pass the test, then you studied hard.
Alice still studied very hard.

(G) If you eat your breakfast, you will not be hungry in the morning.
Dan isn't hungry.

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